Overview of Spread Spectrum Communication

EEE3012 – Spring 2020
Digital Communication System

- **Source of information**
  - Message signal
  - Source encoder
    - Source code word
    - Channel encoder
      - Channel code word
      - Modulator
        - Waveform
      - Channel
  - Transmitter

- **Channel**
  - Received signal
  - Demodulator
    - Estimate of channel code word
  - Channel decoder
    - Estimate of source code word
  - Source decoder
    - Estimate of message signal
  - User of information

- **Receiver**
Spread Spectrum Communication

- **Techniques:**
  - Frequency-hopped (FH)
  - Direct-sequence (DS)
  - Hybrid (DS/FH)

- **Advantages:**
  - Low probability of signal detection/interception (LPDI)
  - Protection against hostile jamming (Anti-jamming)
  - Resistance to multipath fading (Anti-multipath)
  - Graceful performance degradation from interference (Soft capacity)
  - Frequency Reuse (processing gain)
Main Applications

- Military/secure communications
- Cellular telephone/Personal Communications Service
- Wireless Local Area Networks
- Unlicensed “Part 15” Devices
Basic Techniques

- **Direct-Sequence:**
  - Can often be coherently demodulated
  - Suffers from near-far problem (requires power control)
  - Very-resistant to multipath fading
  - Less expensive receivers

- **Frequency Hopping:**
  - Noncoherent modulation
  - No near-far problem
  - Less resistant to multipath fading
  - Requires synchronization with each hop
Direct-Sequence SS System

- **Transmitter:**
  - $b(t)$
  - $a(t) \times \sqrt{2P} \cos(\omega_c t)$
  - $s(t)$

- **Receiver:**
  - $r(t)$
  - $a(t) \times \sqrt{2P} \cos(\omega_c t)$
  - $\int_0^T dt$
  - $Z > 0 \Rightarrow \hat{b} = +1$
  - $Z < 0 \Rightarrow \hat{b} = -1$

Spreading operation

Despreading operation
Direct-Sequence SS Signal

user data

user-specific sequence

\( T \)

\( 2T \)

\( 3T \)
Power Spectral Density

\[ \Phi_{ss}(f) \]

Unspread Signal

Spread Signal

\(-\frac{1}{T} \quad \frac{1}{T} \quad \frac{1}{T_c} \quad \frac{1}{T_c} \)

\[ f(f) \text{ (Hz)} \]

filtered OUT

\[ \frac{P_j}{N} \]

\[ N = \text{Processing Gain} = \frac{T}{T_c} \]
Processing Gain

- Desired signal: \( s(t) = \sqrt{2} P a(t) b(t) \cos(\omega_c t) \)
- Tone jammer: \( J(t) = \sqrt{2} P_J \cos(\omega_c t) \) [Narrowband interference]
- Received signal: \( r(t) = s(t) + J(t) \)
- Correlation receiver: \( \int a(t) \cos(\omega_c t) r(t) \, dt \)
  - \( a(t) \cos(\omega_c t) s(t) = b(t) \sqrt{P/2} \) (despreads signal)
  - \( a(t) \cos(\omega_c t) J(t) = a(t) \sqrt{P_J/2} \) (spreads signal)
- Effective jamming power is reduced by \( \approx 1/N \)
- \( N \) is often called "Processing Gain."
Code Division Multiple Access

- We have considered only single communications links.
  - Practical systems need to support many users simultaneously.
- A common way of avoiding interference between multiple transmitting signal is to make the signals orthogonal to one another
  - Time Division Multiple Access (TDMA)
  - Frequency Division Multiple Access (FDMA)
- Spread Spectrum systems can support Code Division Multiple Access (CDMA).
Multiple Access Capability

- Assume baseband, synchronous system:
  \[ s_1(t) = a_1(t)b_1(t) \]
  \[ s_2(t) = a_2(t)b_2(t) \]

- \( a_1(t) \) and \( a_2(t) \) are distinct, pseudorandom waveforms:
  \[ a_1(t) = \sum_{j=0}^{N-1} a_{1,j} p_{T_c}(t-T_c) \]
  \[ a_2(t) = \sum_{j=0}^{N-1} a_{2,j} p_{T_c}(t-T_c) \]
Model \( \{a_{1,j}\} \) and \( \{a_{2,j}\} \) as IID random variables, equiprobable on \( \{-1,+1\} \).
\[
r(t) = s_1(t) + s_2(t)
\]
\[
T \int_0^T a_1(t)r(t)dt = Tb_1(t) + Tc b_2(t) \sum_{j=0}^{N-1} a_{1,j}a_{2,j} \quad \text{spread interference}
\]
\[
\text{As } N \to \infty , \text{ binomial distribution approaches Gaussian}
\]
CDMA for Cellular Telephone

- Spread-spectrum was proposed for original cellular telephone standard (Cooper 1978).

- QUALCOMM has proposed a spread-spectrum standard for digital cellular telephone

- Rationale: Maximize # of Users K that can operate
User Capacity

- With powerful coding, single cell CDMA can support as many users as FDMA or TDMA.
- Voice Activity Monitoring:  
  - Speech signal has a duty factor approximately 3/8.
  - Vocoder can suppress transmission during periods of silence.
- Universal Frequency Reuse:
  - 4-6 times as many users.
- Conclusion: CDMA can support 12-16 times as many users than FDMA or TDMA.
Frequency Reuse

- TDMA and FDMA require a reuse pattern:

  narrowband signaling
to minimize the co-channel interference
  $\rightarrow$ reuse at least one cell away!
CDMA allows universal frequency reuse:
- assume power attenuates in proportion to $1/d^4$

50% in-cell interference from neighbor cells

$I_{tot} = (100\% + 50\%) I_{in-cell} = 1.5 I_{in-cell}$
Near-Far Problem

- Received Signal strength is proportional to $1/d^{\alpha}, \alpha \in [2,4]$
- Forward Channel:

- Reverse Channel:

- Base receives much more power from Y than from X.
- Solution: Power Control.

Hard to decode the signal from X!
Power Control

- **Open Loop Power Control.** Mobile makes estimates of received signal strength based on signal power received from base station. Assumes reciprocal channel.

  frequency-insensitive path loss and Shadowing (i.e., blocking loss)

- **Closed Loop Power Control.** Mobile receives instructions from base station to increase or decrease signal strength.

  frequency-sensitive multipath fading
Analysis of MA Capability

● Problem:
  ◆ Suppose we have $K$ simultaneous DS/SS users, each with a processing gain $N$ and unique spreading code, transmitting over a noisy channel.
  ◆ How can we calculate the probability of error?
  ◆ How many simultaneous users can transmit over the channel?

● Eventual Answer:

$$P_b = Q \left( \frac{1}{\sqrt{N_0 \sum_{k=2}^{K} \frac{P_k}{2E_{b,1}} + \frac{K}{3P_1 N}}} \right)$$

1. $\frac{2}{3N}$ (Asynchronous despreading)
2. $\frac{1}{N}$ (Synchronous despreading)

Note: $\frac{1}{N}$ (Synchronous despreading)

Co-channel interference

$\frac{1}{2}$ (coherent detection gain)
Gaussian Approximation

- We employ a “Gaussian approximation” for the multiple access interference.
- If the number of interferers is sufficiently large and all users are received with approximately identical powers, then the multiple access interference can be modeled as an additional noise contribution.
Transmitter Model

\[ b(t) \xrightarrow{X} a(t) \xrightarrow{X} \sqrt{2P} \cos(\omega_c t) \rightarrow s(t) \]
• There are $K$ simultaneous users
• Each user has identical processing gain of $N$
• The signal for the $k$th user is given by:
  $$s_k(t) = \sqrt{2P_k}b_k(t)a_k(t)\cos(2\pi f_c t + \phi_k), \quad k = 1, K, K'$$

  - $P_k =$ Power of the $k$th user
  - $b_k(t) =$ data signal of $k$th user
  - $a_k(t) =$ spreading or signature signal of $k$th user
  - $f_c =$ carrier frequency (same for all users)
  - $\phi_k =$ random phase of $k$th user - uniformly distributed on $[0, 2\pi)$ independent of all other variables
cont ...

- Data Signal: \( b_k(t) = \sum_{i=-\infty}^{\infty} b_{k,i} p_T(t-iT) \)

  where

- \( b_{k,i} \) is the \( i \)th data bit transmitted by the \( k \)th user
  - The data bit transmitted during the interval \([iT,(i+1)T)\)

- \( b_{k,i} \in \{+1,-1\} \) is an equiprobable random variable which is independent of all other random variables

- \( T = \) duration of one bit
  
  \[ p_T(t) = \text{unit pulse of duration} \ T = \begin{cases} 1, & t \in [0,T) \\ 0, & t \not\in [0,T) \end{cases} \]
Spreading Sequence

- Spreading Signal: \( a_k(t) = \sum_{j=\infty}^{\infty} a_{k,j} p_{T_c}(t-jT_c) \)
  where
  - \( a_{k,j} \) is the \( j \)th “chip of the \( k \)th user’s spreading sequence
    - The chip transmitted during the interval \([jT_c, (j+1)T_c)\)
  - \( a_{k,j} \in \{+1,-1\} \) is an equiprobable random variable
    which is independent of all other random variables
  - \( T_c = \) duration of one chip
  - \( p_{T_c}(t) = \) unit pulse of duration \( T_c \)
    - \( t \in [0, T_c) \)
    - \( t \not\in [0, T_c) \)
Here we assume that the pseudorandom sequence is generated according to a random process.

Each user has a unique spreading signal $a_k(t)$.

The spreading signal for a particular user is known at both the transmitter and receiver for that user.

There are a number of set of practical psuedorandom sequences which are easy to generate and have good properties (more on this later).
cont ...

- The cross-correlation between codes determines the multiple access performance:
  \[ C_{1,k}(n) = \sum_{i=0}^{N-1} a_{1,i} \cdot a_{k,i+n} \]

- Ideally, \( C_{1,k}(n) \approx 0, \forall n \).
  - This is possible for synchronous system
  - This is not possible for asynchronous systems
cont ...

- The autocorrelation of a code determines the multipath rejection capability:

$$C_{1,1}(n) = \sum_{i=0}^{N-1} a_{1,i} \cdot a_{1,i+n}$$

- Ideally,

$$C_{1,1}(n) = \begin{cases} N, & n = 0 \\ 0, & n \neq 0 \end{cases}$$
Example(1) : Walsh Codes

- Hadamard Matrices:
  \[ H_1 = [1], H_{2i+1} = \begin{bmatrix} H_{2i} & H_{2i} \\ H_{2i} & -H_{2i} \end{bmatrix} \]

- Examples:
  \[ H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \]

- Each user is assigned a row of a Hadamard matrix as a Walsh spreading code.

- If all users are synchronized, \( C_{k_1, k_2}(0) = 0 \)

- Cross-correlations for asynchronous systems are poor.
Example(2) : m-sequences

- Class of PN code which is efficient to generate
- Generated with a feedback shift register.
- An $m$ stage shift register can generate a sequence of length $m$ (hence, the term “maximal length”).
- m-sequences have nearly ideal autocorrelation properties:
  $$ C_{1,1}(n) = \begin{cases} N, & n = 0 \\ -1, & n \neq 0 \end{cases} $$
- For any given length $N$, there exists only a small number of m-sequences with good cross-correlations. (e.g., for $N=31$, there are 6 sequences)
Example (3/4) : Gold & Kasami

3) Gold Codes

- A much larger set of sequences with relatively good cross-correlation properties can be generated by combining the outputs of two shift-registers [source: Ziemer and Peterson, Introduction to Digital Communications, 1992]

4) Large and Small Sets of Kasami Sequences

This sequence generator produces 33 Gold sequences of $N=31$.

The preferred pair of primitive polynomials:

$G_1(D)=1+D^2+D^5$,  $G_2(D)=1+D+D^2+D^4+D^5$. 
cont ...

This sequence generator produces 8 Kasami sequences of $N=63$.

The two generator polynomials:

$G_1(D)=1+D+D^6$, $G_2(D)=1+D+D^3$. 
MA Channel Model

\[ s_1(t) \quad s_2(t) \quad \ldots \quad s_K(t) \]

\[ s_1(t - \tau_1) \quad s_2(t - \tau_2) \quad \ldots \quad s_K(t - \tau_K) \]

\[ n(t) \quad \sum \quad r(t) \]

K-user Asynchronous Transmission
Receiver Model

\[ r(t) = n(t) + \sum_{k=1}^{K} s_k(t - \tau_k) \]

where
- \( n(t) = \) additive white Gaussian noise with \( \Phi_{nn}(f) = \frac{N_0}{2} \)
- \( \tau_k = \) random delay of \( k \)th user
- \( \tau_k \in [0, T) \) is uniformly distributed and independent of other random variables
\begin{itemize}
  \item \[ s_k(t-\tau_k) = \sqrt{2P_k} b_k(t-\tau_k) a_k(t-\tau_k) \cos(2\pi f_c(t-\tau_k) + \phi_k) \]
  \[ = \sqrt{2P_k} b_k(t-\tau_k) a_k(t-\tau_k) \cos(2\pi f_c t + \theta_k) \]
  \item where \( \theta_k = [\phi_k - 2\pi f_c \tau_k] \mod 2\pi \)
  \item For \( f_c T \gg 1 \), the random phase \( \theta_k \) is uniformly distributed on \([0,2\pi)\), just like \( \phi_k \).
  \item We are assuming that because the signals originate at different locations, they arrive at the receiver with different delays and phases.
\end{itemize}
The $k$th user’s receiver is able to generate a perfect replica of the spreading signal $a_k(t)$. We assume that the receiver has already attained phase and timing synchronization.
Decision Statistics

- The $k$th user’s receiver generates a decision statistic $Z_{k,i}$ used for deciding about the bit $b_{k,i}$

$$Z_{k,i} = \int_{T+\tau_k}^{\tau_k} r(t)a_k(t-\tau_k)\cos(2\pi f_c t + \theta_k)dt$$

- Decision Rule:
  
  $Z_{k,i} \geq 0 \Rightarrow \overline{b}_{k,i} = +1$

  $Z_{k,i} < 0 \Rightarrow \overline{b}_{k,i} = -1$

- Note this is just the optimum correlation receiver