

# A Novel Partial Decode-and-Forward Relaying with Multiple Antennas

*Invited Paper*

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**Abstract**—This paper proposes a novel partial decode-and-forward (DF) relaying strategy with multiple antennas. In the first phase, the source broadcasts data streams consisting of non-forwarding and forwarding data streams. In the second phase, a relay node forwards only forwarding data streams to a destination node, and a destination node decodes both non-forwarding and forwarding data streams by successive interference cancellation (SIC). We provide an analytical framework of achievable rate and design a linearly combined precoding matrix for rate maximization. Our results show that the proposed partial DF relaying with a linearly combined precoding matrix achieves substantially higher rate than a conventional DF relaying scheme.

## I. INTRODUCTION

A traditional role of relay was to help data transmission between source and destination. A relay node enhances the quality of received signal at a destination node when the channel gain between source and destination nodes is insufficient. So utilization of relays has focused on either reliable communication or coverage extension with low cost and low transmit power [1]-[6]. In these days, attention has moved to high data rate through relay communications. Motivated by the capacity promising of multiple-input multiple-output (MIMO) communications [15], MIMO techniques have been exploited in relay communications to achieve high data rate [7]-[14]. Several information theoretic bounds on achievable rate are derived in [7]-[10]. Some precoding techniques are introduced in [11]-[14] to deliver high data rate over MIMO links in relay communications.

Full duplex relays are practically infeasible since isolation between transmission and reception on the same antenna is difficult. So half-duplex relays are considered practical and inexpensive although they suffer from multiplexing loss due to time decoupling of transmission and reception. A partial decode-and-forward (DF) relaying is a way to improve spectral efficiency in half duplex relaying. It was originally proposed in [16] where relay forwards a part of decoded information to a destination based on superposition coding. The relay decodes both basic and superposed codes but forwards only the superposed part to destination. To further exploit the advantage of partial DF, Popovski and Carvalho adopted variable transmission slot length with the superposition coding in partial DF relaying [17]. The transmission time from relay to destination depends on the amount of information to be forwarded and the

relay-to-destination link condition. The authors investigated a proper power allocation for basic and superposed data to maximize overall rate. Recently, Popovski's work has been extended to multiple antenna relay networks when full channel state information (CSI) is not available at the transmitter [18].

In this paper, we propose a novel partial DF relaying protocol with multiple antennas, where forwarding information is constructed by stream control. Specifically, a source node broadcasts multiple data streams over multiple antennas in the first phase and a relay node forwards only a subset of the received streams in the second phase. A destination node decodes non-forwarded data streams after subtracting the forwarded data streams from the received data streams in the first phase using successive interference cancellation (SIC). In order to further exploit the advantage of MIMO, we propose a simple and intuitive linear precoding scheme for partial DF relaying using CSI. A proposed precoding matrix steers beams and allocates power for capacity maximization.

The rest of paper is organized as follows. We first introduce our system model in Section II. The analytical framework of the proposed stream control based partial DF relaying is provided in Section III. In Section IV, we design precoding for capacity maximization. Numerical results are presented in Section V, and conclusions are drawn in in Section VI.

*Notation:*  $(\cdot)^H$  represents the complex conjugate transpose and  $\mathbf{I}$  is the identity matrix.  $\mathbf{V}_{(M)}^{(max)}(\mathbf{A})$  (resp.  $\mathbf{V}_{(M)}^{(min)}(\mathbf{A})$ ) is the right singular vectors corresponding to the largest (resp. smallest)  $M$  eigenvalues of  $\mathbf{A}$ .

## II. SYSTEM MODEL

We consider a half duplex DF relaying system consisting of source, relay and destination nodes, as illustrated in Fig. 1. The source node and the destination node have  $M$ ,  $N$  antennas, respectively, and the relay node has  $L$  receive and  $K$  transmit antennas. The relay and the destination nodes are assumed to have SIC receivers.

A source node broadcasts multiple data streams over multiple antennas in the first phase. The data streams consist of non-forwarding and forwarding data streams. The relay node first decodes the non-forwarding data streams by treating the forwarding data streams as interference and then decodes

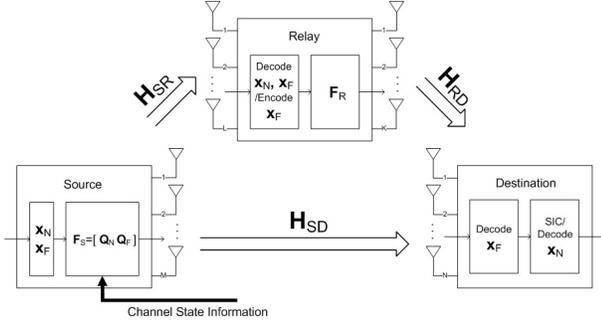


Fig. 1. System model

the forwarding data streams after subtracting out the non-forwarding data streams by SIC. In the second phase, the relay node transmits only the forwarding data streams to the destination node. A destination node first decodes the forwarding data streams in the second phase, and then decodes the non-forwarding from the full data streams received in the first phase by subtracting out the forwarding data streams using SIC.

To enhance the advantage of partial information relaying, this system adopts variable slot length in the second phase –  $T_1$  is the slot length in the first phase and fixed. On the other hand, the slot length  $T_2$  in the second phase varies according to the channel condition between relay and destination nodes and the amount of data streams to be forwarded to the destination node.

The MIMO channels between source-to-relay (S-R), source-to-destination (S-D) and relay-to-destination (R-D) are given by  $\mathbf{H}_{SR} \in \mathbb{C}^{L \times M}$ ,  $\mathbf{H}_{SD} \in \mathbb{C}^{N \times M}$  and  $\mathbf{H}_{RD} \in \mathbb{C}^{N \times K}$ , respectively. They have covariance matrices  $\sigma_{SR}^2 \mathbf{I}$ ,  $\sigma_{SD}^2 \mathbf{I}$ , and  $\sigma_{RD}^2 \mathbf{I}$ , respectively, and all components of channel matrices are independent and identically distributed (i.i.d.) zero mean complex Gaussian random variables. We assume that the channels are static during two phases and perfectly known at the transmitter (CSIT). Additive white Gaussian noises (AWGN) in S-R link  $\mathbf{n}_{SR}$ , S-D link  $\mathbf{n}_{SD}$ , and R-D link  $\mathbf{n}_{RD}$  are i.i.d complex Gaussian random variables  $\sim \mathcal{CN}(0, \mathbf{I})$ .

### III. PARTIAL DF RELAYING BY STREAM CONTROL

In this section, we provide an analytical framework of the proposed partial DF relaying protocol. The mutual information of non-forwarding and forwarding streams at the relay node and the destination node is analyzed.

#### A. Received Signal and Mutual Information

In the first phase, the source node broadcasts  $M$  independent data streams over  $M$  transmit antennas after multiplying precoding matrix  $\mathbf{F}_S$ . All  $M$  transmit data streams  $\mathbf{x}$  are independently encoded into two part:  $M - J$  non-forwarding streams  $\mathbf{x}_N$  which will not be forwarded to destination from relay and  $J$  forwarding data streams  $\mathbf{x}_F$  which will be

forwarded to destination from relay. Thus, the transmit data vector is given by

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_N \\ \mathbf{x}_F \end{bmatrix}, \quad (1)$$

where  $\mathbf{x}_N$  is a Gaussian distributed  $(M - J) \times 1$  vector with  $\mathbb{E}[\mathbf{x}_N \mathbf{x}_N^H] = \mathbf{I}_{(M-J) \times (M-J)}$  and  $\mathbf{x}_F$  is a Gaussian distributed  $J \times 1$  vector with  $\mathbb{E}[\mathbf{x}_F \mathbf{x}_F^H] = \mathbf{I}_{J \times J}$ .

The source precoding matrix  $\mathbf{F}_S$  is given by

$$\mathbf{F}_S = [ \mathbf{Q}_N \ \mathbf{Q}_F ] \in \mathbb{C}^{M \times M}, \quad (2)$$

where  $\mathbf{Q}_N$  is an  $M \times (M - J)$  matrix for non-forwarding streams  $\mathbf{x}_N$  and  $\mathbf{Q}_F$  is an  $M \times J$  matrix for forwarding streams  $\mathbf{x}_F$ , respectively. The precoding matrix satisfies a source power constraint:

$$\text{Tr}(\mathbf{F}_S \mathbf{F}_S^H) = \text{Tr}(\mathbf{Q}_N \mathbf{Q}_N^H) + \text{Tr}(\mathbf{Q}_F \mathbf{Q}_F^H) \leq P_S. \quad (3)$$

Then, the received signals at the relay and the destination nodes are given, respectively, by

$$\mathbf{y}_{SR} = \mathbf{H}_{SR} \mathbf{Q}_N \mathbf{x}_N + \mathbf{H}_{SR} \mathbf{Q}_F \mathbf{x}_F + \mathbf{n}_{SR}, \quad (4)$$

$$\mathbf{y}_{SD} = \mathbf{H}_{SD} \mathbf{Q}_N \mathbf{x}_N + \mathbf{H}_{SD} \mathbf{Q}_F \mathbf{x}_F + \mathbf{n}_{SD}. \quad (5)$$

The relay first decodes the non-forwarding streams  $\mathbf{x}_N$  treating the forwarding stream  $\mathbf{x}_F$  as noise. At the relay, the mutual information for  $\mathbf{x}_N$  is given by

$$\begin{aligned} & \mathcal{I}(\mathbf{x}_N; \mathbf{y}_{SR}) \\ &= \log_2 \frac{|\mathbf{I} + \mathbf{H}_{SR} \mathbf{Q}_N \mathbf{Q}_N^H \mathbf{H}_{SR}^H + \mathbf{H}_{SR} \mathbf{Q}_F \mathbf{Q}_F^H \mathbf{H}_{SR}^H|}{|\mathbf{I} + \mathbf{H}_{SR} \mathbf{Q}_F \mathbf{Q}_F^H \mathbf{H}_{SR}^H|}. \end{aligned} \quad (6)$$

Once the non-forwarding streams  $\mathbf{x}_N$  are successfully decoded, the effect of  $\mathbf{x}_N$  is subtracted from the received signal  $\mathbf{y}_{SR}$  by SIC. Then, the forwarding data streams  $\mathbf{x}_F$  are decoded. Consequently, the mutual information for  $\mathbf{x}_F$  at the relay node is given by

$$\mathcal{I}(\mathbf{x}_F; \mathbf{y}_{SR} | \mathbf{x}_N) = \log_2 |\mathbf{I} + \mathbf{H}_{SR} \mathbf{Q}_F \mathbf{Q}_F^H \mathbf{H}_{SR}^H|. \quad (7)$$

After decoding  $\mathbf{x}_N$  and  $\mathbf{x}_F$ , the relay forwards only the  $J$  forwarded streams  $\mathbf{x}_F$  to the destination over  $K$  antennas in the second phase. The received signal at the destination in the second phase is given by

$$\mathbf{y}_{RD} = \mathbf{H}_{RD} \mathbf{F}_R \mathbf{x}_F + \mathbf{n}_{RD} \quad (8)$$

where  $\mathbf{F}_R$  is a  $K \times J$  precoding matrix at the relay node and satisfies a transmit power constraint at the relay node  $\text{Tr}(\mathbf{F}_R \mathbf{F}_R^H) \leq P_R$ .

In the second phase, the mutual information of forwarding streams  $\mathbf{x}_F$  at the destination node is given by

$$\mathcal{I}(\mathbf{x}_F; \mathbf{y}_{RD}) = \log_2 |\mathbf{I} + \mathbf{H}_{RD} \mathbf{F}_R \mathbf{F}_R^H \mathbf{H}_{RD}^H|. \quad (9)$$

After decoding the forwarding streams  $\mathbf{x}_F$ , the destination node decodes  $\mathbf{x}_N$  by subtracting the effects of  $\mathbf{x}_F$  from  $\mathbf{y}_{SD}$  from the received signal in the first phase  $\mathbf{y}_{SD}$ . The mutual

information of  $\mathbf{x}_N$  at the destination node after SIC is obtained by

$$\mathcal{I}(\mathbf{x}_N; \mathbf{y}_{SD} | \mathbf{x}_F) = \log_2 \left| \mathbf{I} + \mathbf{H}_{SD} \mathbf{Q}_N \mathbf{Q}_N^H \mathbf{H}_{SD}^H \right|. \quad (10)$$

### B. Achievable Rate

The achievable rate of the forwarding streams  $\mathbf{x}_F$  in the second phase is given by

$$R_F^{RD} = \max_{\mathbf{F}_R: \text{Tr}(\mathbf{F}_R \mathbf{F}_R^H) \leq P_R} \left\{ \log_2 \left| \mathbf{I} + \mathbf{H}_{RD} \mathbf{F}_R \mathbf{F}_R^H \mathbf{H}_{RD}^H \right| \right\}. \quad (11)$$

Since the relay node knows the channel  $\mathbf{H}_{RD}$ , the optimal precoding matrix at the relay node is obtained by

$$\begin{aligned} \mathbf{F}_R^* &= \arg \max_{\mathbf{F}_R: \text{Tr}(\mathbf{F}_R \mathbf{F}_R^H) \leq P_R} \left\{ \log_2 \left| \mathbf{I} + \mathbf{H}_{RD} \mathbf{F}_R \mathbf{F}_R^H \mathbf{H}_{RD}^H \right| \right\} \\ &= \mathbf{V}_{RD} \Sigma_{RD}^{\frac{1}{2}}, \end{aligned} \quad (12)$$

where  $\mathbf{H}_{RD} = \mathbf{U}_{RD} \Lambda_{RD}^{\frac{1}{2}} \mathbf{V}_{RD}^H$  and  $\Sigma_{RD} = \text{diag} [P_{R,1}, \dots, P_{R,\min(K,J)}]$ . Based on the waterfilling algorithm, the optimal power allocation is given by

$$P_{R,i} = \left( \frac{1}{\nu} - \frac{N_0}{\lambda_i} \right)_+, \quad (13)$$

where  $\nu$  is determined by  $\sum P_{R,i} = P_R$  and  $(x)_+ := \max\{x, 0\}$ .

Since we adopt a variable slot length, the required transmission time in the second phase is represented as

$$T_2 = \frac{R_F^{SR}}{R_F^{RD}} T_1, \quad (14)$$

where  $T_1$  is the transmission duration for the first phase.  $R_F^{SR}$  denotes the achievable rate of the forwarding streams  $\mathbf{x}_F$  after SIC at the relay node in the first phase and is given by

$$R_F^{SR} = \max_{\mathbf{Q}_F} \log_2 \left| \mathbf{I} + \mathbf{H}_{SR} \mathbf{Q}_F \mathbf{Q}_F^H \mathbf{H}_{SR}^H \right|. \quad (15)$$

The overall achievable rate for the proposed partial DF relaying is obtained by

$$\begin{aligned} &R_N + R_F \\ &= \max_{\{\mathbf{Q}_N, \mathbf{Q}_F\}} \left\{ \frac{T_1}{T_1 + T_2} \min \left\{ \mathcal{I}(\mathbf{x}_N; \mathbf{y}_{SR}) + \mathcal{I}(\mathbf{x}_F; \mathbf{y}_{SR} | \mathbf{x}_N), \right. \right. \\ &\quad \left. \left. \mathcal{I}(\mathbf{x}_N; \mathbf{y}_{SD} | \mathbf{x}_F) + \mathcal{I}(\mathbf{x}_F; \mathbf{y}_{SR} | \mathbf{x}_N) \right\} \right\} \end{aligned} \quad (16)$$

where  $\text{Tr}(\mathbf{Q}_N \mathbf{Q}_N^H) + \text{Tr}(\mathbf{Q}_F \mathbf{Q}_F^H) \leq P_S$ . It should be noted that the achievable rate is bounded by the minimum of the achievable rate of non-forwarding streams  $\mathbf{x}_N$  at the relay node in the first phase and the achievable rate of non-forwarding streams  $\mathbf{x}_N$  at the destination after subtracting out the effect of forwarding streams  $\mathbf{x}_F$ .

Substituting (6), (7), (9), (10) and (14) into (16), the achievable rate is rewritten by

$$\begin{aligned} &R_N + R_F \\ &= \max_{\{\mathbf{Q}_N, \mathbf{Q}_F\}: \text{Tr}(\mathbf{Q}_N \mathbf{Q}_N^H) + \text{Tr}(\mathbf{Q}_F \mathbf{Q}_F^H) \leq P_S} \min \{R_1, R_2\} \end{aligned} \quad (17)$$

where

$$R_1 = \frac{\log_2 \left| \mathbf{I} + \mathbf{H}_{SR} \mathbf{Q}_N \mathbf{Q}_N^H \mathbf{H}_{SR}^H + \mathbf{H}_{SR} \mathbf{Q}_F \mathbf{Q}_F^H \mathbf{H}_{SR}^H \right|}{1 + \frac{1}{R_F^{RD}} \log_2 \left| \mathbf{I} + \mathbf{H}_{SR} \mathbf{Q}_F \mathbf{Q}_F^H \mathbf{H}_{SR}^H \right|}, \quad (18)$$

$$R_2 = R_F^{RD} \left( 1 + \frac{\log_2 \left| \mathbf{I} + \mathbf{H}_{SD} \mathbf{Q}_N \mathbf{Q}_N^H \mathbf{H}_{SD}^H \right| - R_F^{RD}}{\log_2 \left| \mathbf{I} + \mathbf{H}_{SR} \mathbf{Q}_F \mathbf{Q}_F^H \mathbf{H}_{SR}^H \right| + R_F^{RD}} \right). \quad (19)$$

Note that  $R_F^{RD}$  is a predetermined value according to (11) so that it is considered a constant in precoding matrix design.

## IV. PRECODING MATRIX DESIGN

The overall achievable rate of the proposed partial DF relaying is obtained by solving a max-min problem as given in (17). A sufficient condition of the optimal solution is  $R_1 = R_2$ . This section proposes a precoding matrix for capacity maximization.

### A. Optimal Directions

Because the overall achievable rate is determined by the minimum value of  $R_1$  and  $R_2$ , we first find the optimal directions in terms of  $R_1$  and  $R_2$ , respectively.

For  $R_1$  given in (18), the numerator is the achievable rate of full data streams  $\mathbf{x}$  over S-R link while the denominator is the achievable rate of forwarding data streams  $\mathbf{x}_F$  over S-R link. Since the numerator corresponds to the capacity of S-R link,  $R_1$  is a decreasing function with respect to  $\mathbf{x}_F$ . So the optimal beam direction for  $\mathbf{x}_F$  in terms of  $R_1$  is given by

$$\text{dir}(\mathbf{Q}_{F,1}) = \mathbf{V}_{(J)}^{(\min)}(\mathbf{H}_{SR}) \quad (20)$$

and correspondingly, the optimal beam direction for  $\mathbf{x}_N$  in terms of  $R_1$  is obtained by

$$\text{dir}(\mathbf{Q}_{N,1}) = \mathbf{V}_{(M-J)}^{(\max)}(\mathbf{H}_{SR}). \quad (21)$$

If the bottleneck of capacity is S-R link, that is, the minimum value of  $\{R_1, R_2\}$  is  $R_1$ , the overall capacity is maximized by steering beams as (21) and (20) at the source node.

For  $R_2$  given in (19), the optimal beam direction of non-forwarding streams  $\mathbf{x}_N$  in terms of  $R_2$  is easily obtained by

$$\text{dir}(\mathbf{Q}_{N,2}) = \mathbf{V}_{(M-J)}^{(\max)}(\mathbf{H}_{SD}) \quad (22)$$

since the numerator should be maximized for maximization of  $R_2$ . On the other hand, the optimal beam direction of forwarding streams  $\mathbf{x}_F$  in terms of  $R_2$  is determined by the achievable rate of  $R_F^{RD}$  and the achievable rate of the non-forwarding streams  $\mathbf{x}_N$  at the destination  $R_N^{SD}$  which is given by

$$R_N^{SD} = \log_2 \left| \mathbf{I} + \mathbf{H}_{SD} \mathbf{Q}_N \mathbf{Q}_N^H \mathbf{H}_{SD}^H \right|. \quad (23)$$

In other words, the optimal beam direction of forwarding streams  $\mathbf{x}_F$  in terms of  $R_2$  is determined by R-D link condition and the precoding matrix for non-forwarding streams  $\mathbf{Q}_N$ . If the numerator of (19) is negative, the denominator should be larger and vice versa. Hence, the optimal beam direction of  $\mathbf{Q}_F$  in terms of  $R_2$  is obtained by

$$\text{dir}(\mathbf{Q}_{F,2}) = \begin{cases} \mathbf{V}_{(J)}^{(max)}(\mathbf{H}_{SR}), & R_N^{SD} < R_F^{RD} \\ \mathbf{V}_{(J)}^{(min)}(\mathbf{H}_{SR}), & \text{otherwise} \end{cases}. \quad (24)$$

### B. A Linearly Combined Precoding Matrix

In the previous subsection, the optimal beam directions at the source node have been identified in terms of  $R_1$  and  $R_2$ , respectively. Motivated by the fact that the optimal beam direction at the source node might be a compromise between the optimal beam directions in terms of  $R_1$  and  $R_2$  since the overall achievable capacity is determined by  $\min\{R_1, R_2\}$ , we investigate a linearly combined precoding matrix with optimal beam directions for  $R_1$  and  $R_2$ .

A linearly combined precoding matrix for non-forwarding streams  $\mathbf{Q}_N$  is given by

$$\mathbf{Q}_N = (\alpha \mathbf{Q}_{N,1} \mathbf{Q}_{N,1}^H + (1 - \alpha) \mathbf{Q}_{N,2} \mathbf{Q}_{N,2}^H)^{\frac{1}{2}}, \quad (25)$$

where  $0 \leq \alpha \leq 1$  and  $\mathbf{Q}_{N,1} = \mathbf{V}_{(M-J)}^{(max)}(\mathbf{H}_{SR}) \mathbf{\Omega}_{N,1}$  and  $\mathbf{Q}_{N,2} = \mathbf{V}_{(J)}^{(min)}(\mathbf{H}_{SR}) \mathbf{\Omega}_{N,2}$ . The power allocation matrix  $\mathbf{\Omega}_{N,i}$ ,  $i = 1, 2$ , is obtained by the waterfilling algorithm with a constraint:

$$\text{Tr}(\mathbf{\Omega}_{N,i} \mathbf{\Omega}_{N,i}^H) \leq (1 - \gamma) P_S, \quad i = 1, 2 \quad (26)$$

where  $(1 - \gamma) P_S$  is the power allocated to non-forwarding data streams and  $0 \leq \gamma \leq 1$ . With (25), note that the  $R_1$  increases with  $\alpha$  while  $R_2$  decreases with  $\alpha$ . Thus, we balance  $R_1$  and  $R_2$  to maximize their minimum by changing  $\alpha$ .

For given  $\mathbf{Q}_N$ , a linearly combined precoding matrix for forwarding streams  $\mathbf{Q}_F$  is given by

$$\mathbf{Q}_F = (\beta \mathbf{Q}_{F,1} \mathbf{Q}_{F,1}^H + (1 - \beta) \mathbf{Q}_{F,2} \mathbf{Q}_{F,2}^H)^{\frac{1}{2}}, \quad (27)$$

where  $0 \leq \beta \leq 1$  and  $\mathbf{Q}_{F,1} = \mathbf{V}_{(J)}^{(max)}(\mathbf{H}_{SD}) \mathbf{\Omega}_{F,1}$  and

$$\mathbf{Q}_{F,2} = \begin{cases} \mathbf{V}_{(J)}^{(max)}(\mathbf{H}_{SR}) \mathbf{\Omega}_{F,2}, & R_N^{SD} < R_F^{RD} \\ \mathbf{V}_{(J)}^{(min)}(\mathbf{H}_{SR}) \mathbf{\Omega}_{F,2}, & \text{otherwise} \end{cases}. \quad (28)$$

The power allocation matrix  $\mathbf{\Omega}_{F,i}$ ,  $i = 1, 2$ , should satisfy a power constraint:

$$\text{Tr}(\mathbf{\Omega}_{F,i} \mathbf{\Omega}_{F,i}^H) \leq \gamma P_S, \quad i = 1, 2 \quad (29)$$

where  $\gamma P_S$  is the power allocated to the forwarding data streams. With (27), it should also be noted that  $R_1$  is an increasing function with respect to  $\beta$  while  $R_2$  is a decreasing function with respect to  $\beta$ . As shown in (26) and (29), the power division between non-forwarding and forwarding data streams is determined by a weighting factor  $\gamma$ .

Consequently, with the linearly combined precoding matrix, the achievable rate for the system is optimized by  $\alpha$ ,  $\beta$  and  $\gamma$ .

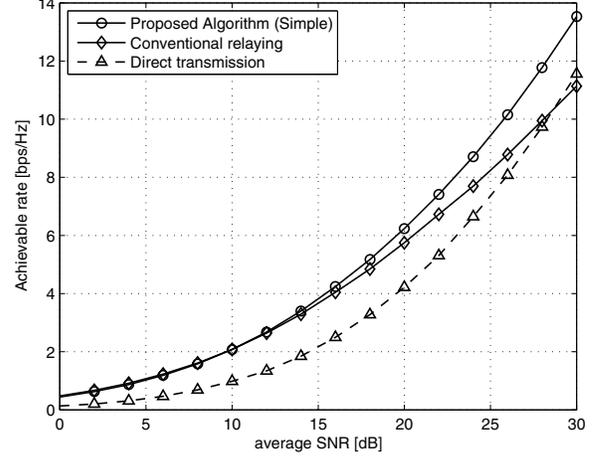


Fig. 2. Achievable rate for the system when  $\sigma_{SR}^2 = -10\text{dB}$ ,  $\sigma_{RD}^2 = -10\text{dB}$  and  $\sigma_{SD}^2 = -20\text{dB}$

For  $J = 0, \dots, M$ , the optimal weighting factors can be readily obtained by

$$\{\alpha^*, \beta^*, \gamma^*\} = \arg \min_{\alpha, \beta, \gamma} |R_1 - R_2|. \quad (30)$$

### C. A Simple Linearly Combined Precoding Matrix

Although complexity of the proposed algorithm depends on a search step size, three parameters,  $\{\alpha, \beta, \gamma\}$ , are required to be searched. Thus, we consider a much simpler algorithm which requires only one parameter search and compare its performance with that of the algorithm with three parameters. We use the same weighting factor  $\alpha$  for both  $\mathbf{Q}_N$  and  $\mathbf{Q}_F$ . Then, linearly combined precoding matrices are represented by

$$\mathbf{Q}_N = (\alpha \mathbf{Q}_{N,1} \mathbf{Q}_{N,1}^H + (1 - \alpha) \mathbf{Q}_{N,2} \mathbf{Q}_{N,2}^H)^{\frac{1}{2}}, \quad (31)$$

$$\mathbf{Q}_F = (\alpha \mathbf{Q}_{F,1} \mathbf{Q}_{F,1}^H + (1 - \alpha) \mathbf{Q}_{F,2} \mathbf{Q}_{F,2}^H)^{\frac{1}{2}}. \quad (32)$$

Instead of the power division by  $\gamma$ , we allocate a power proportional to the number of streams to reduce search complexity. Correspondingly, the power allocation matrix for non-forwarding streams  $\mathbf{\Omega}_{N,i}$  should satisfy

$$\text{Tr}(\mathbf{\Omega}_{N,i} \mathbf{\Omega}_{N,i}^H) \leq \left(1 - \frac{J}{M}\right) P_S, \quad i = 1, 2. \quad (33)$$

In the same manner, the power allocation matrix for forwarding streams  $\mathbf{\Omega}_{F,i}$  should satisfy a power constraint:

$$\text{Tr}(\mathbf{\Omega}_{F,i} \mathbf{\Omega}_{F,i}^H) \leq \frac{J}{M} P_S, \quad i = 1, 2. \quad (34)$$

For  $J = 0, \dots, M$ , the optimal weighting factor  $\alpha$  can be founded by

$$\alpha^* = \arg \min_{\alpha} |R_1 - R_2|. \quad (35)$$

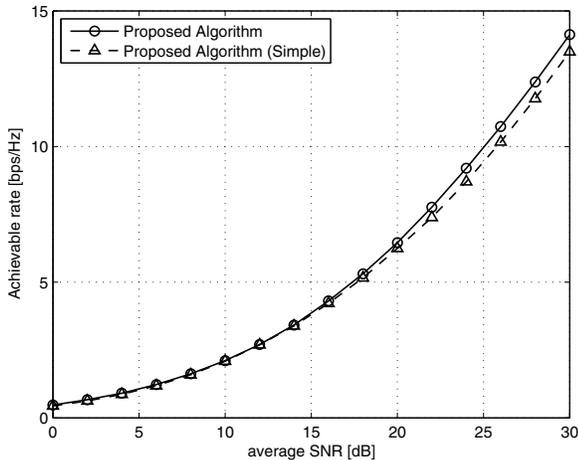


Fig. 3. Achievable rate for the system when  $\sigma_{SR}^2 = -10\text{dB}$ ,  $\sigma_{RD}^2 = -10\text{dB}$  and  $\sigma_{SD}^2 = -20\text{dB}$

## V. NUMERICAL RESULTS

In this section, we evaluate the achievable rate of the proposed partial DF relaying with linearly combined precoding and compare it with conventional relaying scheme and direct transmission without relaying. A conventional DF relaying scheme also adopts variable time slot length in the second phase. In a conventional relaying scheme, a relay node forwards full data streams received from a source node and the precoding matrices at a source node and a relay node are designed according to S-R link and R-D link, respectively, without considering S-D link.

In Fig. 2, we show achievable rate versus average SNR when  $\{\sigma_{SR}^2, \sigma_{RD}^2, \sigma_{SD}^2\} = \{-10, -10, -20\}\text{dB}$ . The antenna configurations are given by  $\{M, N, K, L\} = \{4, 4, 4, 4\}$ . It is shown that the proposed partial DF relaying with simple linearly combined precoding outperforms the other schemes in all SNR regime. The gain of the proposed partial DF relaying becomes larger as SNR is higher. It should be noted that the proposed partial DF relaying generalizes both full forwarding ( $J = M$ ) and direct transmission ( $J = 0$ ), since a proper number of data streams ( $J \in \{0, 1, \dots, M\}$ ) are forwarded by a relay according to a given SNR condition.

Fig. 3 compares the achievable rates of partial DF relaying for the linearly combined precoding with three search parameters and the simplified linearly combined precoding with a single search parameter. The average channel gains are given by  $\{\sigma_{SR}^2, \sigma_{RD}^2, \sigma_{SD}^2\} = \{-10, -10, -20\}\text{dB}$ . The number of antennas are assumed to be  $\{M, N, K, L\} = \{4, 4, 4, 4\}$ . Although the simplified linearly combined precoding significantly reduces search complexity, it achieves comparable data rate to the linearly combined precoding with three search parameters.

## VI. CONCLUSION

We proposed a novel partial DF relaying scheme with precoding, where a relay node forwards only a part of received

data streams from a source node after SIC and a destination node decodes non-forwarding data streams by subtracting forwarding data streams from the data streams in the first phase. We also provided an analytical framework to find achievable rate and proposed a linearly combined precoding matrix for rate maximization. Our numerical results showed that the proposed partial DF relaying with a linearly combined precoding matrix significantly outperforms a conventional DF relaying with precoding.

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