Perturbation Analysis for Spectrum Sharing in Cognitive Radio Networks

Hengameh Keshavarz, Ekram Hossain, Sima Noghanian, and Dong In Kim

***Abstract*—A primary ad-hoc network working in parallel with a secondary ad-hoc network is considered. The main challenge in operating cognitive ad-hoc networks is the lack of a centralized controller performing resource allocation for different users in the network. In this paper, a distributed power allocation scheme is considered for secondary users and its performance is analyzed when time average channel gains are substituted for instantaneous channel gains. In this way, it is not necessary to exchange instantaneous channel information; however, users' allocated power will be perturbed. It is of interest to analyze mathematically this perturbation and to show how it affects the network performance. In particular, an upper bound on perturbation of each user's allocated power, rate, and interference caused to a primary receivers by the secondary users is obtained. Then, it is shown that how this perturbation affects the transmission rate and the probability of interference constraint violation by the secondary users.**

***Index Terms*—Cognitive radio, power allocation, perturbation analysis.**

I. INTRODUCTION

COGNITIVE radio is a promising technology to solve the problem of the over-crowded spectrum. Federal Communications Commission's (FCC's) frequency allocation chart shows a heavily crowded spectrum with most frequency bands already assigned for some applications. However, measurements show that most of the time, many frequencies are unused. This fact inspired researchers to propose the notion of *cognitive radio* or using these temporary unused frequency bands referred to as spectrum *holes* to accommodate secondary (i.e. unlicensed) wireless devices for opportunistic communications.

In cognitive networks, secondary (i.e. cognitive) users are conditioned not to make unacceptable interference for the primary network. The problem of resource allocation for cognitive radio networks has been studied in the literature. Extensive research has already been focused on joint power/rate/channel allocation in cognitive networks with centralized controllers. For example, in [1], a near-optimal yet simple algorithm was derived with linear complexity targeting capacity maximization of a cognitive radio network while jointly optimizing power and channel allocation among secondary users and respecting total power constraints per individual users. In [2], the distributed multi-channel power allocation problem was studied for spectrum sharing in cognitive radio networks, where secondary transceiver pairs share the same spectrum with the primary system. The problem was formulated as a non-cooperative game with coupled constraints to address the interference temperature restrictions imposed by the primary system. Existence and uniqueness of the Nash Equilibrium for this power allocation game were investigated. In [3], a framework to perform joint admission control and rate/power allocation was developed in a dynamic spectrum sharing environment. In [3], instead of instantaneous channel gains, mean channel gains averaged over short-term fading were assumed to be available for resource allocation. In [4], a resource allocation framework was presented for spectrum underlay in cognitive wireless networks. Both interference constraints for primary users and quality of service (QoS) constraints for secondary users were considered. Admission control algorithms, which are performed jointly with power control, were proposed so that QoS requirements of all admitted secondary users are satisfied while keeping interference to primary users below the tolerable limit.

Research focusing on resource allocation in cognitive ad-hoc networks has been scanty because of non-convexity of sum-rate maximization problems in interference channels. Moreover, in cognitive ad-hoc networks, due to the lack of a centralized controller, distributed spectrum sharing methods are desirable. In [5], a novel joint power/channel allocation scheme was presented that uses a distributed pricing strategy to improve network performance. Here, the spectrum allocation problem was modeled as a non-cooperative game. A price-based iterative water-filling (PIWF) algorithm was proposed, which allows users to converge to the Nash Equilibrium (NE). This PIWF algorithm can be implemented distributively, with secondary users repeatedly negotiating their best transmission power and spectrum.

In this paper, a primary ad-hoc network working in parallel with a secondary ad-hoc network is considered. As shown in [5], controlling the interference caused by the secondary network to the primary users needs the knowledge of cross channel gains between the secondary and the primary network. To obtain this knowledge, channel information needs to be frequently exchanged between primary and secondary nodes which incurs large control overhead in the network. Therefore, a power allocation scheme is presented here for secondary users and its performance is analyzed when time average channel gains are substituted for instantaneous channel gains. In this way, it is not necessary to exchange instantaneous

Manuscript received May 16, 2009; revised October 24, 2009 and January 17, 2010; accepted February 10, 2010. The associate editor coordinating the review of this letter and approving it for publication was G. Li.

H. Keshavarz is currently with the School of Electrical and Computer Engineering, University of Sistan and Balouchestan, Zahedan, Iran (e-mail: keshavarz@ece.usb.ac.ir). This work was done when she was a Post-Doctoral Research Fellow at the Department of Electrical and Computer Engineering (ECE), University of Manitoba (UoM), Winnipeg, MB, Canada.

E. Hossain is with the Department of ECE, UoM, Winnipeg, MB, Canada (e-mail: ekram@ee.umanitoba.ca).

S. Noghanian is with the Department of Electrical Engineering, University of North Dakota, North Dakota, USA (e-mail: sima@mail.und.edu).

D. I. Kim is with the School of Information and Communication Engineering, Sungkyunkwan University (SKKU), Suwon, Korea (e-mail: dikim@ece.skku.ac.kr).

Digital Object Identifier 10.1109/TWC.2010.05.090715

channel information (which may reduce signaling overhead significantly when the scheme is implemented in a distributed manner); however, users' allocated power will be perturbed. It is of interest to analyze mathematically this perturbation and to provide an upper bound on it as a function of the number of secondary transmitters.

The rest of the paper is organized as follows: In Section II, the cognitive network model is introduced and the power allocation problem is mathematically formulated. Section III presents the perturbabtion analysis for spectrum sharing in cognitive ad-hoc networks. Simulation results are shown in Section IV. Finally, Section V concludes the paper.

II. System Model and Problem Formulation

A. Cognitive Ad-Hoc Network Model

Consider a primary ad-hoc network working in parallel with a secondary ad-hoc network. The nodes are located randomly on a plane and these are randomly divided into primary and secondary users. A transmitter communicates with a receiver using a point-to-point link. In other words, no multi-user channel is embedded in the primary and secondary network. Active primary transmitters send signals with constant power P, while secondary transmitters send information with power less than or equal to P. Hence, the achievable rate (in nats) of secondary link i can be written as

$$R_i \leq \log\left(1+\frac{P_i\left|h_{ii}\right|^2}{\sigma^2+\sum_{\substack{j=1\\ j\neq i}}^{M}P_j\left|h_{ji}\right|^2+P\sum_{k=1}^{K}\left|g_{ki}\right|^2}\right) \quad (1)$$

where P_i refers to transmitted power of cognitive transmitter i, $h_{ii}(t)$ denotes the secondary link fading channel (i.e., the fading channel between secondary transmitter i and secondary receiver i), $h_{ji}(t)$ represents an interference channel for secondary receiver i from other secondary transmitters, $g_{ki}(t)$ represents an interference channel between primary transmitter k and secondary receiver i, M and K denote the number of active secondary and primary links respectively, and σ^2 refers to the variance of background noise which follows complex Gaussian distribution (i.e., $\mathcal{CN}(0,\sigma^2)$).

B. Power Allocation: Problem Formulation

Since there is no centralized controller in these ad-hoc networks, resource allocation must be performed in a distributed fashion. Hence, in the secondary network, each transmitter maximizes its own utility function without considering the total performance of the network. Basically, this problem is a non-cooperative game in which each user behaves selfishly.

A distributed power allocation scheme is considered. That is, each secondary link maximizes its data rate subject to three constraints: 1) a power constraint for each secondary transmitter, 2) a minimum-rate constraint for secondary links, and 3) an interference constraint for the primary network. In other words, each secondary transmitter maximizes its own data rate such that the QoS for the corresponding link is guaranteed and it does not make unacceptable interference for the primary network.

The aforementioned problem is mathematically formulated as

$$\begin{aligned} &\max_{P_i} R_i && (2)\\ \text{subject to}\quad &P_i \leq P \\ &R_i \geq R_{\min}, \quad i=1,\ldots,M \\ &\sum_{j=1}^{M} P_j\left|g_{jk}\right|^2 \leq \Gamma, \quad k=1,\ldots,K && (3)\end{aligned}$$

where $R_{\min}$ denotes the minimum rate required for each secondary link, and Γ represents the maximum tolerable interference at each primary receiver from the secondary network. This optimization problem is convex [6]. Considering the Lagrange duality method, the Lagrangian of this problem equals

$$\begin{aligned}\mathcal{L}(P_i,\lambda_i,\delta_i,\{\mu_k\}) &= \log\left(1+\frac{P_i\left|h_{ii}\right|^2}{\sigma^2+P_{Inf}}\right)-\delta_i(R_{min}-R_i)\\ &- \lambda_i(P_i-P)-\sum_{k=1}^{K}\mu_k\left(\sum_{j=1}^{M}P_j\left|g_{jk}\right|^2-\Gamma\right)\end{aligned}$$

where P_{Inf} denotes total interference power at cognitive receiver i, and λ_i, δ_i, and $\{\mu_k\}$, represent Lagrange multipliers. Note that $\{\mu_k\}$ denotes the set of μ_k for $k=1,\ldots,K$. In the Lagrange duality method, problem (2) is also called the primal problem. Then, the dual problem is written as

$$\min_{\lambda_i,\delta_i,\{\mu_k\}}\ \max_{P_i} \mathcal{L}(P_i,\lambda_i,\delta_i,\{\mu_k\}). \quad (4)$$

Since all constraints in (2) are affine, if there is a feasible point, the Slater's condition [6] is satisfied and thus, the duality gap is zero (i.e., strong duality holds). Hence, the solution of the dual problem equals the optimal solution of (2). It is known that for convex problems satisfying the Slater's condition, the Karush-Kuhn-Tucker (KKT) conditions provide necessary and sufficient conditions for optimality [6]. Suppose P_i^0 and λ_i^0, δ_i^0, and $\{\mu_k^0\}$ are primal and dual optimal solutions. Then, they satisfy the following KKT condition:

$$\nabla\mathcal{L}(P_i^0,\lambda_i^0,\delta_i^0,\{\mu_k^0\})=0 \quad (5)$$

where ∇ denotes the gradient with respect to P_i. Hence,

$$\frac{(1+\delta_i^0)\left|h_{ii}\right|^2}{P_i^0\left|h_{ii}\right|^2+\sigma^2+P_{Inf}}-\lambda_i^0-\sum_{k=1}^{K}\mu_k^0\left|g_{ik}\right|^2=0 \quad (6)$$

and

$$P_i^0=\left[\frac{(1+\delta_i^0)}{\lambda_i^0+\sum_{k=1}^{K}\mu_k^0\left|g_{ik}\right|^2}-\frac{\sigma^2+P_{Inf}}{\left|h_{ii}\right|^2}\right]_0^P, \quad i=1,\ldots,M \quad (7)$$

where $[x]_a^b$ represents the Euclidean projection of x on the interval $[a,b]$ [5]. Substituting (7) into the Lagrangian, the dual function is obtained which is a function of Lagrange multipliers. Then, different iterative optimization methods (e.g. the ellipsoid method [7]) can be utilized to obtain the optimal Lagrange multipliers minimizing the dual function.

As the objective function in non-cooperative game (2) is concave over $\{P_i\}$ and the action space corresponding to $\{P_i\}$ is a closed bounded convex set, there is at least one Nash Equilibrium (NE) for this game [8].

Note that the duration of power control adaptation interval does not affect power allocation as long as we are dealing with block fading channels remaining constant during each time period.

III. Perturbation Analysis for Spectrum Sharing

Assume the variance of background noise is known to all nodes in the primary and secondary network. Then, the total interference power at each receiver can be estimated using the energy detection methods in signal processing. The link channel gain can also be estimated by channel estimation algorithms. As indicated in (7), to allocate power to each secondary transmitter, the knowledge of instantaneous cross channel gains between the primary and secondary network is required. To obtain this knowledge, channel state information (CSI) needs to be exchanged between different nodes in the network frequently, which incurs large control overhead. Substituting the expected values for the instantaneous cross channel gains is one possible solution; however, any perturbation in the CSI results in perturbation of allocated power. In this paper, it is of interest to analyze theoretically this perturbation and to provide an upper bound on this perturbation as a function of the number of secondary transmitters.

Cross channel gains are indeed random variables with a given distribution. Hence,

$$\sum_{j=1}^{M} P_j \left|g_{jk}\right|^2 = \sum_{j=1}^{M} P_j \mathbf{E}(\left|g_{jk}\right|^2) + \Delta_g \tag{8}$$

where $\mathbf{E}(x)$ denotes the expected value of random variable x, and Δ_g represents perturbation of interference power using instantaneous and expected cross channel gains. Substituting the expected values for the instantaneous channel gains in (3) changes the interference constraint as

$$\sum_{j=1}^{M} P_j \mathbf{E}(\left|g_{jk}\right|^2) \leq \Gamma \tag{9}$$

which is equivalent to

$$\sum_{j=1}^{M} P_j \left|g_{jk}\right|^2 \leq \Gamma + \Delta_g. \tag{10}$$

Now, to show how this perturbation affects power allocation and the network performance, we recall some results from the stability analysis of optimization problems.

Consider the following perturbed convex optimization problem [9]:

$$\begin{aligned} &\inf f(x) + \langle c, x\rangle \\ \text{subject to} \quad & g_t(x) \leq b_t, \quad t \in T \end{aligned} \tag{11}$$

where $\langle ., . \rangle$ represents the usual inner product, T is the index set, $f : \mathbb{R} \to \mathbb{R}$ and $g_t : \mathbb{R} \to \mathbb{R}$; $t \in T$, are given convex functions. Assume $\mathcal{F}^*(b, c)$ denotes the set of optimal solutions for perturbation parameters b and c, $\sigma(b) = \{g_t(x) \leq b_t, t \in T\}$ represents the constraint system, and $\mathcal{F}(b)$ refers to the feasible set of $\sigma(b)$. Then, the following corollary results from Theorem 1 in [9].

Corollary 3.1: For convex program (11), let $((b^*, c^*), x^*)$ belong to $\mathcal{F}^*$. If $\sigma(b^*)$ satisfies the Slater's condition and the KKT conditions are satisfied at $((b^*, c^*), x^*)$, $\mathcal{F}^*$ is strongly Lipschitz stable at (b^*, c^*). In other words, there exists a finite constant $c > 0$ such that

$$\sup_{x^* \in \mathcal{F}^*(b^*, c^*)} \text{dist}(x^*, \mathcal{F}^*(b^0, c^0)) \leq c \left\|u^* - u^0\right\| \tag{12}$$

where $u^* = (b^*, c^*)$, $u^0 = (b^0, c^0)$ refers to the unperturbed parameters, and $\text{dist}(x, S)$ denotes the distance from point x to set S and is defined as

$$\text{dist}(x, S) = \inf_{z \in S} \|x - z\|. \tag{13}$$

Substituting (9) for (3) and comparing optimization problem (2) to (11), it can be seen that $c^* = c^0 = 0$, $b^0 = \Gamma$, and $b^* = \Gamma + \Delta_g$. As the Slater's condition is still satisfied for this perturbed problem, the set of allocated power for different secondary transmitters is Lipschitz stable. That is,

$$\left|P_i^* - P_i^0\right| \leq c \left|\Delta_g\right| \quad \text{for} \quad i = 1, \ldots, M \tag{14}$$

where P_i^* and P_i^0 denote, respectively, the optimal solution of perturbed and unperturbed problem (2). As transmitted power is limited to P, perturbation in users' allocated power is also limited by the maximum transmitted power. However, to show the rate at which allocated power is perturbed as a function of the number of secondary users, we recall some results from the law of large numbers. According to the following strong law of large numbers for weighted sums of random variables [10], an upper bound can be obtained for perturbation of power allocation in the asymptotic case (i.e., as $M \to \infty$).

Theorem 3.1: (See [10] for the proof) Let $\{X_i, i \geq 1\}$ be a sequence of independent and identically distributed (iid) random variables satisfying $\mathbf{E}(X) = 0$, and for any $h > 0$ and some $\gamma > 0$, $\mathbf{E}\left(\exp(h|X|^\gamma)\right) < \infty$. Let $\{a_{ni}, 1 \leq i \leq n, n \geq 1\}$ be a triangular array of constants satisfying

$$A_\alpha = \limsup_{n \to \infty} A_{\alpha,n} < \infty \tag{15}$$

where $A_{\alpha,n} = \sum_{i=1}^{n} |a_{ni}|^\alpha / n$ for some $1 < \alpha \leq 2$. Then, for $0 < \gamma < 1$ and $b_n = n^{\frac{1}{\alpha}} (\log n)^{\frac{1}{\gamma}}$,

$$\frac{\sum_{i=1}^{n} a_{ni} X_i}{b_n} \to 0 \quad \text{a.s.} \tag{16}$$

where a.s. denotes almost sure convergence. Moreover, for $\gamma > 1$ and $b_n = n^{\frac{1}{\alpha}} (\log n)^{\frac{1}{\gamma} + \delta}$, where $\delta = 1 - 1/\gamma - (\gamma - 1)/(1 + \alpha\gamma - \alpha)$, (16) is satisfied.

Now, let $\{P_j, 1 \leq j \leq M\}$ be an array satisfying

$$\limsup_{M \to \infty} \sum_{j=1}^{M} P_j^\alpha / M \leq \limsup_{M \to \infty} \sum_{j=1}^{M} P^\alpha / M = P^\alpha < \infty. \tag{17}$$

Assume cross channel gains are iid random variables taken from a distribution having a finite second-order moment. Note that in most common wireless channel models, channel gains are assumed to be iid random variables taken from a given distribution such as Rayleigh, Rician, and Nakagami. Then, substituting n with M in (16) in Theorem 3.1, we have

$$\Delta_g = \sum_{j=1}^{M} P_j \left(|g_{jk}|^2 - \mathbf{E}(|g_{jk}|^2) \right) = o\left(M^{\frac{1}{\alpha}} (\log M)^{\frac{1}{\gamma}+\delta} \right) \text{ a.s.}$$

where $o(\cdot)$ has the following interpretation: for any positive infinite sequences $f(n)$ and $g(n)$, $n = 1, 2, \ldots$, $f(n) = o(g(n))$ means $\lim_{n\to\infty} \left| \frac{f(n)}{g(n)} \right| = 0$. Now, we know the rate at which users' power is perturbed by perturbation in the knowledge of cross channel gains. It is also important to analyze the effect of this perturbation on the network performance; for example, how this perturbation changes the transmission rate and other constraints in problem (2). First, we need the following lemma.

Lemma 3.1: (See Lemma 1 in [9]) For every compact neighborhood of the optimal solution (i.e., x^*), U, the functions g_t, $t \in T$, are Lipschitzian on this neighborhood; in other words, for each $t \in T$, there exists a finite real constant $c > 0$ such that

$$|g_t(x_1) - g_t(x_2)| \le c\,|x_1 - x_2|, \qquad \forall\, x_1, x_2 \in U. \tag{18}$$

According to Lemma 3.1, we have

$$\begin{aligned} |R_i^* - R_i^0| &\le c\,|P_i^* - P_i^0| \quad (19) \\ \left| \sum_{j=1}^{M} P_j^* \,|g_{jk}|^2 - \sum_{j=1}^{M} P_j^0 \,|g_{jk}|^2 \right| &\le c\,|P_i^* - P_i^0| \end{aligned}$$

for $k = 1, \cdots, K$, where R_i^* and R_i^0 denote the rates corresponding to P_i^* and P_i^0, respectively. Hence, each secondary link rate and interference power made by the secondary network at each primary receiver are both perturbed in the same way as allocated power of each secondary transmitter. That is, for a large secondary network (i.e., $M \to \infty$) with cross channel gains taken from a distribution having a finite second-order moment, we have

$$\begin{aligned} |P_i^* - P_i^0| &= o\left(M^{\frac{1}{2}} (\log M)^{\frac{2}{3}} \right) \text{ a.s.} \\ |R_i^* - R_i^0| &= o\left(M^{\frac{1}{2}} (\log M)^{\frac{2}{3}} \right) \text{ a.s.} \\ \left| \sum_{j=1}^{M} P_j^* \,|g_{jk}|^2 - \sum_{j=1}^{M} P_j^0 \,|g_{jk}|^2 \right| &= o\left(M^{\frac{1}{2}} (\log M)^{\frac{2}{3}} \right) \text{ a.s.} \end{aligned}$$

for $k = 1, \cdots, K$, where $\alpha = 2$ and $\gamma = 2$.

Fig. 1 shows perturbation of interference power made by the secondary network at each primary receiver (i.e., Δ_g) using instantaneous and expected cross channel gains for different numbers of secondary users. To draw this figure, channal gains are randomly taken from a Chi-square distribution with two degrees of freedom. Finding a feasible solution for problem (2) is very hard for a large number of users with random channel gains; therefore, to draw Fig. 1, secondary users' allocated powers are randomly chosen from a uniform distribution on interval $[0, 1]$. The theoretical upper bound provided by Theorem 3.1 is also shown.

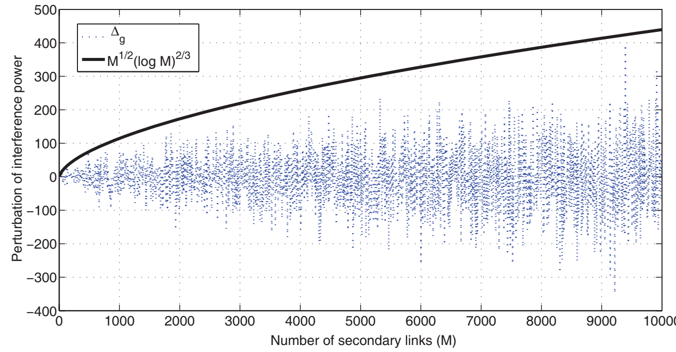

Fig. 1. Perturbation of interference power using instantaneous and expected cross channel gains.

IV. Numerical Results

Consider a primary ad-hoc network with K and a secondary ad-hoc network with M transmitter-receiver pairs. The minimum rate required for each active secondary link is 10 Kbps. It is assumed that all channel gains in both ad-hoc networks are taken from a Chi-square distribution with two degrees of freedom. The maximum transmitted power equals $P = 0.032$ watt (i.e., 15 dBm which is typical power for WiFi) for primary and secondary users. The spectrum bandwidth is 22 MHz (typical for WiFi) and the background noise variance is $\sigma^2 = 10^{-4}$. The maximum tolerable interference at each primary receiver is $\Gamma = 20\sigma^2$.

Note that total interference power at each receiver and direct link channel gains can be estimated using the energy detection and channel estimation methods in signal processing. Hence, they are assumed to be known at each secondary user in each time slot. To calculate perturbation, the optimization problem in (2) is solved using instantaneous and expected (or time-averaged) cross channel gains. In each time slot, power allocation is performed in a distributed fashion. If a feasible solution exists for a set of channel gains and system parameters, users' allocated powers converge in less than 5 iterations. We assume each user's allocated power is known to other secondary users. We consider both a fixed and varying power allocation order. For a fixed order, distributed power allocation always starts with secondary user 1 and ends with secondary user M, while for a varying order, power allocation starts with a different secondary user during each power allocation interval (i.e., time slot).

Fig. 2 shows perturbation in secondary users' allocated powers and rates for 200 time slots with a varying power allocation order for $M = 3$ and $K = 6$. Note that there is no feasible solution in some time slots; therefore, the number of feasible solutions is less than 200. In each time slot, channel gains are independently taken from Chi-square distribution with two degrees of freedom. This figure illustrates how each secondary link rate is perturbed by perturbation of user's allocated power. Since finding an optimal solution for problem (2) is very hard for a large secondary and primary network, we cannot numerically show the effect of perturbation in the asymptotic case and compare it with the theoretical bound presented by Theorem 3.1. Fig. 3 shows the same quantities with a fixed power allocation order. It can be seen that for a

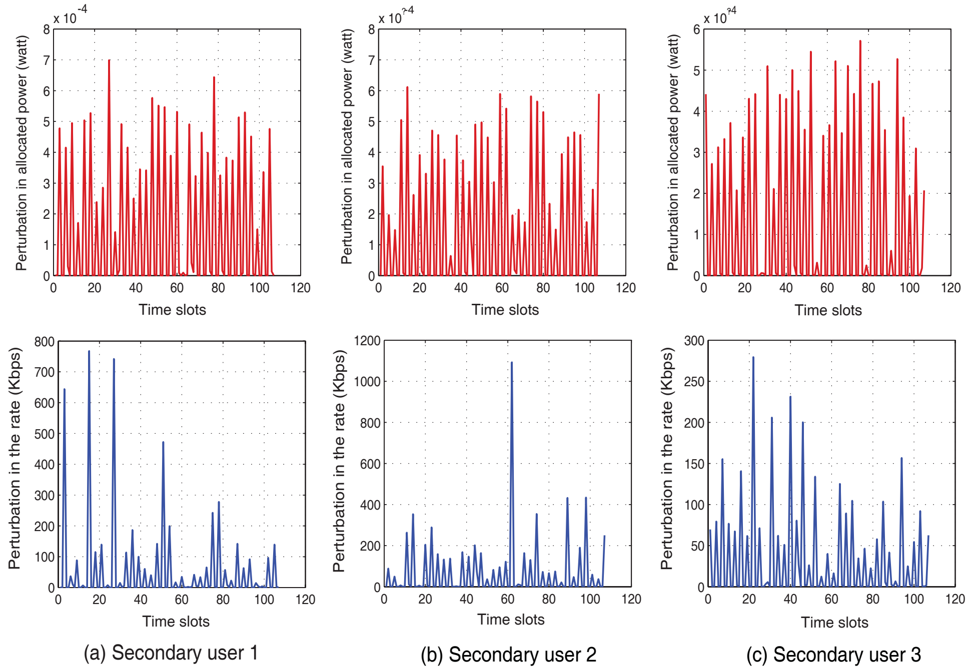

Fig. 2. Perturbation of secondary users' allocated powers and rates with a varying power allocation order.

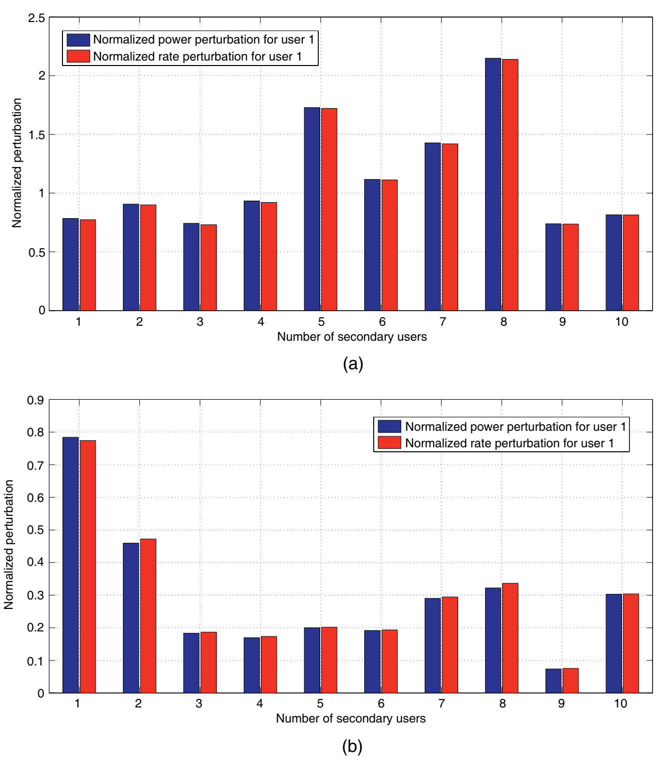

Fig. 4. Normalized perturbation of power and rate for secondary user one with (a) a fixed power allocation order, and (b) a varying power allocation order.

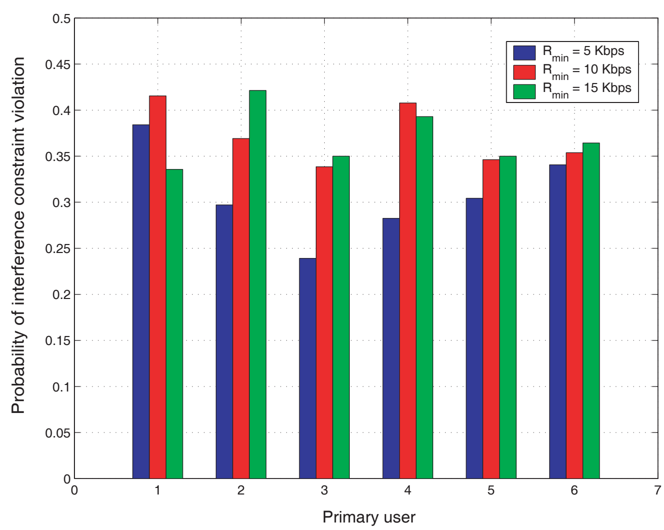

Fig. 5. Probability of interference constraint violation for different values of R_{min} (for $\Gamma = 20\sigma^2$).

fixed power allocation order, perturbation of allocated powers and rates are higher for secondary user 1. The reason is that power allocation always starts with secondary user 1; therefore, this user does not have any a priori knowledge of other secondary users' allocated powers.

Fig. 4 shows normalized perturbation of power and rate versus the number of secondary users for $K = 2$. However, it can be seen that Fig. 4 does not show any specific trend. The reason is that when we change the number of secondary users, channel gains are also changing in the primary and secondary networks. Hence, we cannot only observe the effect of increasing the number of secondary users in the system. However, this figure is useful as it shows normalized perturbation of power and rate are almost the same even for a small number of users (e.g., $M \leq 10$). This behavior is related to Lemma 4.1 which provides perturbation of rates in terms of perturbation of allocated power.

Fig. 5 shows the probability of interference constraint violation for different values of R_{min} and $\Gamma = 20\sigma^2$ when instantaneous cross channel gains are replaced by time-

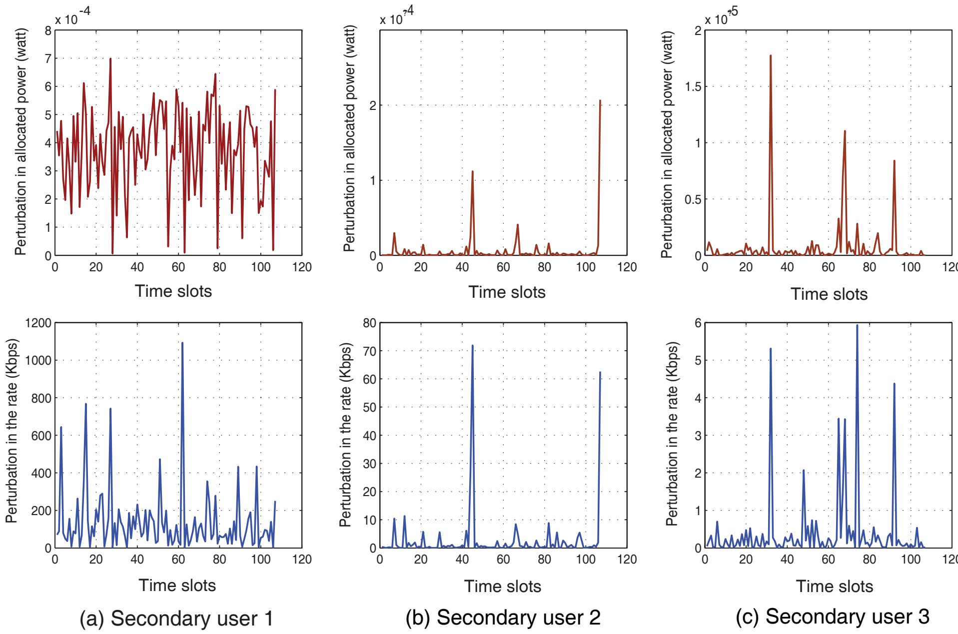

Fig. 3. Perturbation of secondary users' allocated powers and rates with a fixed power allocation order.

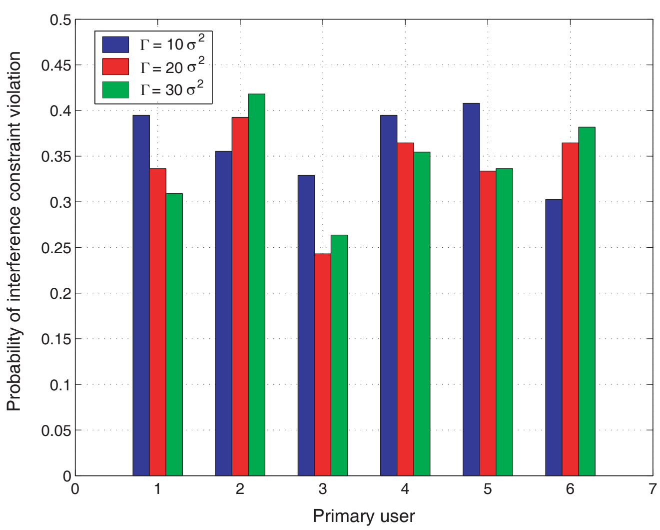

Fig. 6. Probability of interference constraint violation for different values of Γ (for $R_{min} = 10Kbps$).

averaged channel gains. Fig. 6 indicates the same probability for different values of Γ and $R_{min} = 10$ Kbps. Note that increasing R_{min} or decreasing Γ reduces the chance of finding a feasible solution in each time slot. Furthermore, it can be seen that there is no clear trend for the probability of interference constraint violation in terms of R_{min} and Γ. The reason is that allocated powers depend on R_{min} and Γ through constraints of optimization problem (2). In other words, the probability of interference constraint violation can be written as

$$\mathbb{P}\left(\sum_{j=1}^{M} P_j^*(R_{min},\Gamma)\,|g_{jk}|^2 > \Gamma\right). \tag{20}$$

Hence, changing Γ and R_{min} affects the aforementioned probability in a complicated way. For this reason we do not see a clear trend in Fig. 5 and Fig. 6. However, these figures provide a useful intuition regarding the chance of interference constraint violation for a rapidly changing environment with parameters presented herein.

V. Conclusion

In this paper, a primary ad-hoc network working in parallel with a secondary ad-hoc network has been considered. A distributed power allocation scheme has been presented for secondary users and its performance has been analyzed when expected channel gains are substituted for instantaneous channel gains. In this way, it is not necessary to exchange instantaneous channel information; however, users' allocated power will be perturbed. Hence, there is a trade-off between the amount of information to be exchanged among network nodes and accuracy of the power allocation scheme. Using some results from the stability analysis of optimization problems, it has been shown how perturbation of the knowledge of cross channel gains affects power allocation and the network performance. Moreover, in the asymptotic case (i.e., $M \to \infty$), an upper bound in terms of the number of secondary links has been obtained for perturbation of secondary users' allocated powers and transmission rates as well as interference caused by the secondary transmitters to a primary receiver.

Acknowledgment

This work by E. Hossain was supported by Discovery Grant 249500-2009 from the Natural Sciences and Engineering Research Council (NSERC) of Canada. This research by D. I. Kim was supported by the MKE (Ministry of Knowledge Economy), Korea, under the ITRC (Information Technology Research Center) support program supervised by the NIPA (National IT Industry Promotion Agency) (NIPA-2010-C1090-1011-0005). The authors are thankful to the reviewers for their constructive comments and suggestions.

REFERENCES

[1] F. F. Digham, "Joint power and channel allocation for cognitive radios," in *Proc. IEEE Wireless Communications and Networking Conf. (WCNC)*, pp. 882–887, Las Vegas, USA, Apr. 2008.

[2] Y. Wu and D. H. K. Tsang, "Distributed multichannel power allocation algorithm for spectrum sharing cognitive radio networks," in *Proc. IEEE Wireless Communications and Networking Conf. (WCNC)*, pp. 1436–1441, Las Vegas, USA, Apr. 2008.

[3] D. I. Kim, L. B. Le, and E. Hossain, "Joint rate and power allocation for cognitive radios in dynamic spectrum access environment," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5517–5527, Dec. 2008.

[4] L. B. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5306–5315, Dec. 2008.

[5] F. Wang, M. Krunz, and S. Cui, "Spectrum sharing in cognitive radio networks," in *Proc. IEEE Conf. Computer Communications (INFOCOM)*, pp. 1885–1893, Phoenix, USA, Apr. 2008.

[6] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[7] R. G. Bland, D. Goldfarb, and M. J. Todd, "The ellipsoid method: a survey," *Operations Research*, vol. 29, no. 6, pp. 1039–1091, 1981.

[8] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave N-person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

[9] M. J. Cánovas, A. Hantoute, M. A. López, and J. Parra, "Lipschitz modulus in convex semi-infinite optimization via d.c. functions," *ESAIM: Control, Optimization and Calculus of Variations*. DOI: 10.1051/cocv:2008052, 2008.

[10] S. H. Sung, "Strong laws for weighted sums of i.i.d. random variables," *Statistics and Probability Lett.*, vol. 52, no. 4, pp. 413–419, 2001.

[11] R. T. Rockafellar, *Convex Analysis*. Princeton University Press, 1970.