Rate-Maximization Scheduling Schemes for Uplink OFDMA

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***Abstract*—In this paper, we propose and study several sum rate maximization algorithms for uplink orthogonal frequency division multiple access (OFDMA). For uplink scheduling without fairness consideration, we propose two Lagrangian duality optimization-based methods to maximize the weighted sum rate, which include a *cyclic dual-update algorithm* and a *per-stage dual-update algorithm*. For a low-complexity alternative, we design and analyze the transmit power and signal-to-noise ratio (SNR) product (PSP) based selective multiuser diversity (SMuD) schemes. Next, for fair scheduling, we propose rate maximization schemes under access proportional fairness (APF) and rate proportional fairness (RPF) constraints, respectively. The APF is achieved using normalized channel SNR (*n*-SNR) ranking-based SMuD for user selection per carrier, and the RPF is realized using dynamical carrier assignment based on the target rate ratios. Analytical throughput and fairness metrics are derived and verified via simulations. Numerical results illustrate the sum rate loss caused by rate fairness and access fairness constraints compared to the duality approach. Also, we show that unlike the downlink case, for uplink OFDMA the correlated frequency channels (carriers) cause significant ergodic sum rate degradation compared to the independent channels. These results provide new insight into the achievable uplink OFDMA performance with and without fairness constraints.**

***Index Terms*—OFDMA uplink, Lagrangian duality, water-filling power allocation, multiuser diversity, proportional fairness, throughput maximization.**

I. INTRODUCTION

THE orthogonal frequency division multiple access (OFDMA) is a promising candidate modulation and access scheme for 4G communication systems [1]–[6]. The downlink rate optimization for OFDMA has been well studied [1]–[4], [7]–[10]. In comparison, the uplink rate optimization problem is more complicated than the downlink optimization because of the K mobile users' different power constraints. Rate-maximization schemes for uplink OFDMA have been studied recently, see e.g., [5], [6], [11], [12] and references therein. In [11] two one-stage iterative carrier allocation and power allocation schemes were proposed, including water-filling (WF) and equal power allocation (EPA), termed as MaxRt+Wf and MaxRt+Eq in [11]. In [6] a weighted sum rate maximization scheme was proposed using the duality optimization tool and subject to the average transmit power constraint. The method in [11] is sub-optimal and the scheme in [6] generally requires to know the channel distribution information (CDI) besides channel state information (CSI), and an iterative average rate tracking method was designed to avoid using the CDI after convergence [6]. Besides rate maximization, fair resource allocation has been studied for OFDMA downlink in [3], [4], [8] for rate fairness and in [10], [13] for access fairness. The uplink fairness combined with sum rate maximization was recently considered in [5], where all the users' traffic must meet an instantaneous minimum rate constraint. In [12], a framework for minimizing the fractional packet loss cost function for uplink and downlink channels in IEEE 802.16 OFDMA networks was proposed, and a linear programming (LP) and several empirical proportional fairness algorithms were designed. In comparison, the proportional rate fairness and access fairness schemes based on low-complexity multiuser diversity strategy for uplink OFDMA have not been adequately addressed in the literature and deserve a more thorough investigation.

In this paper, we provide a comprehensive study on the OFDMA uplink sum rate-maximization problem under multiple users' individual power and target bit error rate (BER) constraints: Given that a total of K users and N subcarriers are available for allocation (a carrier is not shared between the users), how to achieve the maximum weighted or unweighted sum-rate taking into account (1) K users' individual power constraint; (2) K users' individual target BER constraint; and (3) with or without the proportional fairness constraints. Here, access proportional fairness (APF) and rate proportional fairness (RPF) are considered for unweighted sum-rate maximization. The APF requires that every user receives the same amount of channel assignment resource [10], [14]–[16] in terms of the average channel access probability (AAP). For OFDMA, we define the APF as that all the users have the equal numbers of assigned carriers in the time-average or statistical sense. On the other hand, the RPF refers to that every user k gets a rate proportional to its quality of service (QoS) weight factor α_k. A similar problem was studied for downlink in [3], [4], [8].

For uplink rate maximization without fairness consideration, we utilize the Lagrangian duality framework and design two primal-dual-update approaches to maximize the weighted sum rate, which include the cyclic dual update (CDU) and the per-

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stage dual update (PSDU) algorithms. Different from [6], [9] which considered the average transmission power constraint per user, we consider the instantaneous power constraint, which avoids requiring the knowledge of channel statistics but only that of the instantaneous CSI. The proposed PSDU duality scheme is based on a modified gradient search and can achieve a robust convergence, while the proposed CDU duality scheme can achieve a much faster convergence. Substantial performance improvement than the result in [11] is observed. As an even lower complexity alternative and to gain insight into the uplink performance, we apply the selective multiuser diversity (SMuD) strategy and design two transmission power and signal-to-noise-ratio (SNR) product (PSP) based ranking schemes which include a direct PSP scheme and a modified PSP scheme. Furthermore, for comparison purposes, we study the performance of the absolute channel SNR (*a*-SNR) ranking based SMuD [2], [10], [11] (originally designed for downlink systems) for uplink.

For uplink rate maximization with access fairness, we consider a long-term fairness [10] and propose a normalized-SNR (*n*-SNR) ranking based SMuD scheme which has a two-step procedure: (i) All the K users' channel gains on each subcarrier are ranked by their channel normalized SNRs (*n*-SNRs), and the carrier is assigned to the user with the largest *n*-SNR (namely, *n*-SNR SMuD); (ii) EPA or WF along the carriers under each user's transmit power constraint. For uplink optimization with rate fairness, we extend the concept in [3], [8] and design a low-complexity scheme which uses dynamical carrier assignment based on the target rate ratios for uplink, and iteratively allocate transmit power based on WF or EPA.

Analytical performance results for the proposed PSP-based and *n*-SNR-based SMuD schemes for OFDMA uplink scheduling are provided, including the sum and individual throughput and fairness metrics (such as AAP) for each user. Numerical results show good matching between simulation and analytical results. The rate gaps between the duality optimization approach and other methods are quantified. Our results also show that unlike the case of downlink OFDMA, the correlated subcarriers significantly degrade the ergodic rate for uplink OFDMA. These results provide new insights into the effects of various system and channel parameters, and will aid the design of OFDMA uplink with power, BER, and fairness constraints.

II. Signal Model

Consider the uplink OFDMA channels with K backlogged users and N available subchannels. At time t, the channel SNR of user k on carrier n is given by $\gamma^{\text{ch}}_{k,n}(t) = |H_{k,n}(t)|^2/\mathcal{N}_{k,n}$, where $H_{k,n}(t)$ and $\mathcal{N}_{k,n}$ are the complex channel gain and the bandwidth-limited noise power of user k, respectively, both on carrier n. Based on discrete Fourier transform (DFT) of the multipath channel gains, we know that the N carrier SNRs of user k $\{\gamma^{\text{ch}}_{k,n}\}_n$ have the same distribution, but they can be highly correlated in the case of a limited number of independent paths. We have $\bar{\gamma}^{\text{ch}}_{k,n} = \bar{\gamma}^{\text{ch}}_{k}$ for all n, where $\bar{\gamma}^{\text{ch}}_{k,n} = E[\gamma^{\text{ch}}_{k,n}]$ is the average value of $\gamma^{\text{ch}}_{k,n}$. Furthermore, we assume the channels between different users are independent but not necessarily identically distributed (i.n.d.).

Let $N_k(t)$ be the number of carriers assigned to user k, and so $\sum_{k=1}^{K} N_k(t) \le N$ holds. The maximum transmit power for user k is given by $P_{T,k}$, which is split among the $N_k(t)$ carriers that are assigned to user k for uplink transmission.

The base station (BS) implements the centralized scheduling assuming the knowledge of the available transmit power $\{P_{T,k}\}_{k=1}^{K}$ and the uplink channel SNRs $\gamma^{\text{ch}}_{k,n}$ of all k and n. The channel SNR information can be obtained using two different approaches: (i) The BS estimates the channel SNRs by using pilot signals from uplink transmitters; and (ii) Assuming the uplink and downlink channel reciprocity (which is valid, for example, in the time division duplex mode), the mobile subscribers (i.e., users) estimate the downlink channel SNRs and feed back such information to the BS.

The weighted sum rate maximization subject to (s.t.) the individual instantaneous power and target BER constraints can be defined as

$$\max_{\{P_{k,n}(t)\},\{\mathcal{S}_k(t)\}} \sum_{k=1}^{K} \sum_{n\in\mathcal{S}_k(t)} w_k C_{k,n}(t) \quad \text{s.t.} \sum_{n\in\mathcal{S}_k(t)} P_{k,n}(t) \le P_{T,k} \tag{1}$$

where w_k is the weight factor of user k' rate, $\mathcal{S}_k(t)$ denotes the set of carriers assigned to user k, and $\mathcal{S}_1(t), \ldots, \mathcal{S}_K(t)$ have no common elements based on the exclusive carrier assignment (ECA), which was a popular assumption employed in the literature [2]–[4], [6] for downlink and uplink OFDMA. $N_k(t) = |\mathcal{S}_k(t)|$ is the cardinality (the number of elements) of $\mathcal{S}_k(t)$. For example, if user 1 is assigned with channels $[1, 3, 7]$, then $\mathcal{S}_1(t) = \{1, 3, 7\}$ and $N_1(t) = 3$. In the case that multiple transceiver antennas and/or multiuser detection are included, the shared carrier assignment (SCA) will improve the sum rate than the ECA. On the other hand, for the case of single transmit and receive antennas without multiuser interference cancellation as considered in this paper, the ECA is an optimal strategy for downlink and uplink channels. $P_{k,n}(t)$ $(0 \le P_{k,n}(t) \le P_{T,k})$ and $C_{k,n}$ are the transmit power and the throughput of user k on carrier n, respectively. We have

$$C_{k,n}(t) = \frac{B_w}{N} \log(1 + P_{k,n}(t)\gamma_{k,n}(t)) \tag{2}$$

where $\log(x)$ is the natural logarithm, B_w is the total available bandwidth, and $\gamma_{k,n}(t) = \xi_k \gamma^{\text{ch}}_{k,n}(t)$. ξ_k is an SNR gap function related to the target BER $P_{e,k}$ and the modulation format used. When $\xi_k = 1$, (2) is the Shannon capacity, and when $\xi_k = -1.5/\log(5P_{e,k})$, (2) gives the throughput which satisfies the target BER $P_{e,k}$ assuming continuous-rate quadrature amplitude modulation (CR-QAM) [17]. Without loss of generality we assume $\frac{B_w}{N} = 1$ throughout the remainder of this paper.

Depending on design objectives, the optimization in (1) may be furthermore subject to fairness constraints, such as APF or RPF. The APF requires that every user has the equal probability of accessing the carriers, which leads to $\overline{N}_1 = \cdots = \overline{N}_K = N/K$, i.e., every user has the same average number of carriers being allocated, where $\overline{N}_k = E[N_k]$ is the expected value of N_k, and $\overline{N}_k$ can be a non-integer.

The RPF considered in this paper specifies that for each transmit time interval (TTI) the rates of K users approximately follow the relation that

$$C_1(t) : \cdots : C_K(t) \simeq \alpha_1 : \cdots : \alpha_K \tag{3}$$

where $C_k(t)$ is the rate of user k and α_k is its pre-specified rate proportion factor [3], [4]. Note that this constraint is different from the minimum rate constraint per user studied in [5] for uplink OFDMA, where $C_k(t) > C_{\text{tar}}$ for all k, and C_{tar} is the minimum target rate per user.

We first propose the duality optimization schemes for weighted sum rate maximization. For unweighted sum rate maximization, we develop low-complexity PSP-based SMuD methods without fairness consideration, and two other schemes which consider access fairness and rate fairness, respectively. Some symbols frequently used in this paper are listed in Table II.

TABLE I
LIST OF FREQUENTLY USED SYMBOLS.

$P_{T,k}$	Available transmit power of user k
$P_{k,n}$	Allocated power to user k on carrier n
$\gamma_{k,n}$	Effective channel SNR of user k on carrier n
$\mathcal{S}_k$	Carrier set assigned to user k
C_k	Throughput of user k
α_k	Target rate ratio of user k
$\overline{N}_k$	Average number of subchannels assigned to user k
$\text{AAP}_{k,n}$	Average channel access probability of user k on carrier n
C_n^{PSP}	Throughput on carrier n based on the power-SNR-product algorithm.
$C_n^{n-\text{SNR}}$	Throughput on carrier n for the normalized SNR-ranking algorithm.
$\gamma_{k,n}^{\text{SC, PSP}}$	Conditional PSP of user k on carrier n when it is the largest using the PSP ranking.

III. Lagrangian Duality Based Optimization

We derive two weighted sum rate maximization algorithms using the duality-optimization technique, as given below. Based on the primal function in (1), we define the Lagrangian as

$$\mathcal{L}(\{\mathcal{S}_k(t)\}, \{P_{k,n}(t)\}) = \sum_{k=1}^{K} \sum_{n\in\mathcal{S}_k(t)} w_k C_{k,n}(t) - \sum_{k=1}^{K} \lambda_k(t) \left(\sum_{n\in\mathcal{S}_k(t)} P_{k,n}(t) - P_{T,k} \right) \tag{4}$$

Below, the time index t is dropped when no confusion arises. Define the duality function as

$$g(\{\lambda_k\}) = \max_{\{\mathcal{S}_k\},\{P_{k,n}\}} \mathcal{L}(\{\mathcal{S}_k\}, \{P_{k,n}\}). \tag{5}$$

The dual problem can be expressed as $\{\lambda_k\} = \text{argmin}_{\{\lambda_k\}} g(\{\lambda_k\})$, and $g(\{\lambda_k\})$ can be rewritten as $g(\{\lambda_k\}) = \sum_{n=1}^{N} g_n(\{\lambda_k\}) + \sum_{k=1}^{K} \lambda_k P_{T,k}$, where

$$g_n(\{\lambda_k\}) = \max_k \{w_k \log(1 + P_{k,n}\gamma_{k,n}) - \lambda_k P_{k,n}\}. \tag{6}$$

Based on (6), carrier n is allocated to user k^* if

$$k^* = \text{argmax}_k \{w_k \log(1 + P_{k,n}\gamma_{k,n}) - \lambda_k P_{k,n}\}. \tag{7}$$

To derive the Karush-Kuhn-Tucker (KKT) conditions, taking derivative of $g_n(\{\lambda_k\})$ with respect to $\{P_{k,n}\}_{n=1}^{N}$ and setting resulting equalities to zero, we obtain $\frac{w_k\gamma_{k,n}}{1+P_{k,n}\gamma_{k,n}} - \lambda_k = 0$, which leads to

$$P_{k,n} = (w_k/\lambda_k - 1/\gamma_{k,n})^+ \tag{8}$$

if $k = k^*$, where $(x)^+ = \max(0, x)$. Furthermore, $P_{k,n} = 0$ if $k \neq k^*$.

Finally, the WF-level related parameter λ_k is obtained by using the equality that $\sum_{n\in\mathcal{S}_k} P_{k,n} = \sum_{n\in\mathcal{S}_k} (w_k/\lambda_k - 1/\gamma_{k,n})^+ = P_{T,k}$, and we have

$$\lambda_k = \frac{w_k N_k}{P_{T,k} + \sum_{n\in\mathcal{S}_k}(1/\gamma_{k,n})} \quad \text{s.t.} \quad \lambda_k \leq \gamma_{k,n}. \tag{9}$$

Eqs. (7) – (9) present the rules for duality optimization. However, the method of dual update is not unique, and implementing it in an efficient manner is a nontrivial task. We design two Lagrangian duality-based weighted sum rate maximization algorithms, a *cyclic dual-variable update (CDU-duality) algorithm* and a *per-stage dual-variable update (PSDU-duality) algorithm*. Both algorithms require the knowledge of $\{P_{T,k}\}$ and $\{\gamma_{k,n}\}$ as input, and output the scheduling decisions including carrier assignment $\{\mathcal{S}_k\}$ and power allocation $\{P_{k,n}\}$.

A. Cyclic Dual-Variable Update Algorithm

The *CDU-duality* algorithm is given below.

(i) Initialize. Assume stage $s = 0$ and the sum rate $R_{\text{tot}}^{(s)} = 0$.

(ii) For $n = 1, \ldots, N$, do the following:

a) For $k = 1, \ldots, K$, assuming carrier n is allocated to user k, find the allocated power $P_{k,n}$ and the WF level λ_k using (8) and (9), for all k.

b) Perform user-wise WF power allocation for all k (as will be described next).

c) Carrier n is assigned to user k^* according to (7). Update the carrier set $\mathcal{S}_{k^*}$ and remove carrier n from the user who originally possessed this carrier.

(iii) Find the sum rate $R_{\text{tot}}^{(s)}$ at stage s according to the carrier set $\mathcal{S}_k$ and $P_{T,k}$ for all k. Compute the sum rate difference $\Delta R_{\text{tot}}^{(s)} = R_{\text{tot}}^{(s)} - R_{\text{tot}}^{(s-1)}$.

(iv) If $\Delta R_{\text{tot}}^{(s)} < \epsilon_e$, where ϵ_e is a pre-specified small positive scalar close to 0, then stop the multi-stage iteration and output $\mathcal{S}_k$ and $P_{k,n}$ for all k and n. Otherwise, increase s by one, and go to step (ii) and repeat. ■

To implement the user-wise WF in Step (ii.b), a very low-complexity exact WF algorithm is designed, as shown below (**user-wise-WF** Algorithm), which should be implemented individually for all K users. Assume that user k is assigned with carrier set $\mathcal{S}_k$ (with cardinality N_k).

1) Initialize. Let $N_{k,\text{eff}} = N_k$.

2) Rank $\{\gamma_{k,n}\}_{n\in\mathcal{S}_k}$ in a descending order resulting in $\{\gamma_{k,(n)}\}$, where $\gamma_{k,(1)} \geq \cdots \geq \gamma_{k,(N_k)}$ holds.
3) Find λ_k using (9) with N_k being replaced by $N_{k,\text{eff}}$ therein.
4) Check if $\lambda_k \leq \gamma_{k,(N_{k,\text{eff}})}$ holds. If true, go to step 5). Otherwise, update set $\mathcal{S}_k$ by removing $\gamma_{k,(N_{k,\text{eff}})}$ from the set $\{\gamma_{k,(1)}, \ldots, \gamma_{k,(N_{k,\text{eff}})}\}$, set $N_{k,\text{eff}} = N_{k,\text{eff}} - 1$, and go to step 2).
5) The λ_k and $N_{k,\text{eff}}$ are now obtained. The allocated power to carrier n of user k, $P_{k,n}$, is given by (8), for $n = 1, \ldots, N_{k,\text{eff}}$. ■

B. Per-Stage Dual-Variable Update Algorithm

The *PSDU-duality* algorithm is given below.

(i) Assume stage $s = 0$ and the sum rate $R_{\text{tot}}^{(s)} = 0$. Initialize $\{\lambda_k^{(0)}\}_k$.

(ii) For $n = 1, \ldots, N$, do the following:

a) For $k = 1, \ldots, K$, find the allocated power $P_{k,n}$ using

$$P_{k,n} = \min\left\{\left(w_k/\lambda_k^{(s)} - 1/\gamma_{k,n}\right)^+, P_{T,k}\right\}. \quad (10)$$

Slightly different from (8), the min operator is used in (10) to avoid very large value of $P_{k,n}$ which could occur due to the possible abnormal value of $\lambda_k^{(s)}$ in the iteration.

b) Carrier n is assigned to user k^* according to (7). Update carrier set $\mathcal{S}_{k^*}$.

(iii) Update dual variables $\{\lambda_k\}$ by using a sub-gradient search (for all k)

$$\lambda_k^{(s+1)} = \max\{\lambda_k^{(s)} - \beta_k \Delta P_k, \epsilon_k\} \quad (11)$$

where $\Delta P_k = P_{T,k} - \sum_{n\in\mathcal{S}_k} P_{k,n}$ and $0 < \beta_k < 1$ is a scalar which controls the convergence speed and stability. In (11), to avoid numerical instability when $\lambda_k^{(s+1)}$ is set to zero, $0 < \epsilon_k \ll 1$ is introduced to avoid such a problem.

(iv) Find the sum rate $R_{\text{tot}}^{(s)}$ at stage s according to the carrier set $\mathcal{S}_k$ and $P_{T,k}$ for all k. This can be calculated using the proposed *user-wise WF* algorithm.

(v) If $|R_{\text{tot}}^{(s)} - R_{\text{tot}}^{(s-1)}| < \epsilon_e$, then stop the multi-stage iteration and output $\mathcal{S}_k$ and $P_{k,n}$ for all k and n. Otherwise, increase s by one, and go to step (ii) and repeat. ■

The algorithm above is termed as the *improved* or *robust* PSDU algorithm. On the other hand, if we use (8) instead of (10), and $\lambda_k^{(s+1)} = (\lambda_k^{(s)} - \beta_k \Delta P_k)^+$ instead of (11) in the PSDU algorithm, we term it as the *conventional* PSDU algorithm.

C. Discussions

Some comments are in order: The proposed CDU algorithm updates $\{\lambda_k\}$ per carrier iteration (up to N such updates per stage), and when $s > 1$, the algorithm updates all $\{\lambda_k\}$ and power allocation on N carriers cyclically per stage. Each update is under all K users' power constraints and based on the user-wise WF. This speeds up the convergence and enhances the robustness of the algorithm. Furthermore, it does not require initial stage setting of $\{\lambda_k\}$.

The proposed PSDU algorithm updates $\{\lambda_k\}$ once per stage of iteration (for all N carriers). This is theoretically more strict than the CDU algorithm. However, it is susceptible to numerical instability due to sub-gradient search and often suffers from slow convergence, especially when K (the number of dual variables) becomes large .

To enhance the numerical stability of the PSDU sub-gradient search-based algorithm, first, (10) and (11) are used which are different from the conventional PSDU algorithm where the min operator and the ϵ_k were not used. The new design significantly improves the robustness of the iteration. Second, at the end of each stage s (in step (iv) of the PSDU algorithm), the proposed *user-wise WF* algorithm is used which gives accurate evaluation of the true rate of each user k. On the other hand, $\lambda_k^{(s)}$ should not be used for rate calculation at each stage, because it is only the transient value unless the convergence is achieved. To our knowledge, this technique was not discussed in the literature. Third, each iteration keeps the duality objective function non-decreasing, and the output weighted sum rate is asymptotically optimal when the ratio N/K becomes large [18].

The user-wise WF algorithm for user k requires the complexity of $O(N_k \log_2(N_k))$ for sorting the N_k channel SNRs. The total operation complexity for all K users per carrier (required for Steps (ii.a)-(ii.c) of the CDU algorithm) is about $O(N \log_2(N/K))$ by approximating N_k as K/N for all k. For all N carriers the total complexity is approximated by $O(N^2 \log_2(N/K))$. For an S_1 stage iteration the maximum complexity of the CDU method is about $O(S_1 \times N^2 \log_2(N/K))$.

Compared to the sub-optimal methods in [11] which has the complexity $O(N^2 \log_2(N/K))$, the complexity of the proposed CDU duality algorithm is only increased by a factor of S_1. We show by simulation that only a very small S_1 is typically required to achieve convergence.

In comparison, the complexity of the PSDU method is $O(NKS_2)$, where S_2 is the number of iterations required for convergence. Typically $S_2 \gg S_1$. When $S_1/S_2 < \frac{K}{N\log_2(N/K)}$ holds, the CDU method will have a lower complexity than the PSDU method. This occurs, for example, when K is comparable to N.

IV. Power-SNR-Product based Scheduling

The duality approach requires a few stages of iterations to converge. For an alternative with an even lower complexity, we consider designing the SMuD based methods without multistage iteration. For downlink scheduling, it is known that the rate-maximization strategy is to assign the carrier to the user with best channel SNR in that carrier [2], [19], termed as the *a*-SNR SMuD scheme in [10]. This method relies on the fact that the assigned power comes from a common pool at the BS or the access point for downlink, but it cannot be readily extended to the uplink design due to the K users' individual power constraint. To understand whether it is feasible to deploy the SMuD strategy to uplink to achieve a near-optimal performance, we propose two methods based on transmission power-SNR-product based ranking for carrier competition. The relevant performance analysis also reveals that unlike the downlink case, correlated carriers degrade the ergodic sum rate for uplink OFDMA, which is a new observation to our knowledge. The proposed PSP algorithms are used for sum-rate maximization for uplink OFDMA, and they have a significantly lower complexity than the duality-based schemes.

A. Direct PSP-ranking based SMuD

Under the K individual power constraints the optimization problem can be posed as

$$\max_{\{\mathcal{S}_k\},\{P_n\}} \sum_{k=1}^{K} \sum_{n\in\mathcal{S}_k} C_n^{\text{PSP}} \quad \text{s.t.} \quad \sum_{n\in\mathcal{S}_k} P_n \le P_{T,k} \tag{12}$$

where C_n^{PSP} is the throughput for the selected user k^* on carrier n, and

$$k^* = \text{argmax}_k \{\gamma_{k,n}^{\text{PSP}}\}_{k=1}^{K} \tag{13}$$

where $\gamma_{k,n}^{\text{PSP}} = \gamma_{k,n} P_{T,k}$ is the PSP for user k on carrier n. The direct PSP algorithm is given below.

1) For $n = 1, \ldots, N$, find the PSP $\gamma_{k,n}^{\text{PSP}}$ for all k, and assign carrier n to user k^* based on (13).
2) Based on carrier set $\mathcal{S}_k$ and $P_{T,k}$ for all k, implement either EPA or WF power allocation for each user.

Based on user-wise waterfilling, the WF solution is given by

$$P^*_{n|n\in\mathcal{S}_k} = (1/\lambda_k - 1/\gamma_{k^*,n})^+ \tag{14}$$

where

$$\lambda_k = N_k \Bigg/ \left[P_{T,k} + \sum_{n\in\mathcal{S}_k} \frac{1}{\gamma_{k^*,n}}\right] \text{ s.t. } \quad \lambda_k \le \gamma_{k^*,n}. \tag{15}$$

The efficient implementation of (15) can follow the procedure given in the user-wise waterfilling algorithm in Section III. On the other hand, for EPA one needs to implement power allocation based on $P^*_{n|n\in\mathcal{S}_k} = P_{T,k}/N_k$. Below, we provide a performance analysis for $\overline{N}_k$ and individual rate C_k for the direct-PSP EPA method. Such a result provides insight into the sub-optimality of uplink SMuD schemes compared to the downlink case.

1) AAP and Fairness Metrics: Define the largest PSP on carrier n as $\gamma_n^{\text{SC,PSP}} = \max\{\gamma_{1,n}^{\text{PSP}}, \ldots, \gamma_{K,n}^{\text{PSP}}\}$. The probability density function (PDF) of $\gamma_n^{\text{SC,PSP}}$ is derived as

$$f_{\gamma_n^{\text{SC,PSP}}}(x) = \sum_{k=1}^{K} f_{\gamma_{k,n}^{\text{SC,PSP}}}(x) \tag{16}$$

where $f_{\gamma_{k,n}^{\text{SC,PSP}}}(x)$ is the PDF of $\gamma_{k,n}^{\text{PSP}}$ conditioned on that user k has the largest PSP on carrier n, and

$$f_{\gamma_{k,n}^{\text{SC,PSP}}}(x) = \frac{1}{\bar{\gamma}_{k,n}^{\text{PSP}}} \exp(-x/\bar{\gamma}_{k,n}^{\text{PSP}}) \times \prod_{k'=1,k'\ne k}^{K} \left(1 - e^{-x/\bar{\gamma}_{k',n}^{\text{PSP}}}\right) \tag{17}$$

where $\bar{\gamma}_{k',n}^{\text{PSP}} = \bar{\gamma}_{k',n} P_{T,k'}$ by definition.

Let $\text{AAP}_{k,n}^{\text{PSP}}$ denote the probability that carrier n is allocated to user k in the PSP scheme. It follows that

$$\text{AAP}_{k,n}^{\text{PSP}} = \int_0^\infty f_{\gamma_{k,n}^{\text{SC,PSP}}}(x)dx. \tag{18}$$

A closed-form expression for $\text{AAP}_{k,n}^{\text{PSP}}$ is given by (48) in the Appendix. Thus, the average number of carriers for user k is given by

$$\overline{N}_k^{\text{PSP}} = \sum_{n=1}^{N} \text{AAP}_{k,n}^{\text{PSP}} = N \sum_{\tau\in J_{K-1}} \frac{\text{sign}(\tau)}{(|\tau|\bar{\gamma}_{k,n}^{\text{PSP}} + 1)}, \tag{19}$$

where J_{K-1} is the expansion set of $\left\{1/\bar{\gamma}_{k',n}^{\text{PSP}}\right\}_{k'\ne k}^{K}$ defined in the Appendix.

2) Rate for direct-PSP method with EPA: We analyze the sum and individual rates of the proposed direct-PSP EPA scheme below. The rate of user k is derived as

$$C_{k,\text{ EPA}}^{\text{PSP}} = E_t\left[\sum_{n\in\mathcal{S}_k} \log(1 + \gamma_{k,n}^{\text{SC,PSP}}/N_k)\right] \tag{20}$$

where $E_t[\cdot]$ denotes the expectation with respect to time. Assume that all the users' channels on all the carriers are stationary and ergodic. The time average may be replaced by the ensemble average over the PDFs of the channel SNRs. However, N_k is a random variable here which makes the analytical evaluation of (20) difficult.

For the case that the SNRs are independent among the N carriers of each user, as a tight approximation, we use the average value $\overline{N}_k$ instead of N_k in (20) to evaluate rate $C_{k,\text{ EPA}}^{\text{PSP}}$ and obtain

$$C_{k,\text{ EPA}}^{\text{PSP}} \simeq \sum_{n=1}^{N} \int_0^\infty \log(1 + y/\overline{N}_k^{\text{PSP}}) \cdot f_{\gamma_{k,n}^{\text{SC,PSP}}}(y)dy \tag{21}$$

where $f_{\gamma_{k,n}^{\text{SC,PSP}}}(y)$ was given by (17) and (47). When the carrier SNRs are not independent (i.e., correlated carriers), (21) provides a rate upper bound.

After some manipulations we obtain a closed-form expression for (21) as

$$C_{k,\text{ EPA}}^{\text{PSP}} \simeq N \sum_{\tau\in J_{K-1}} \frac{\text{sign}(\tau)\exp(A)E_1(A)}{(|\tau|\bar{\gamma}_{k,n}P_{T,k} + 1)} \tag{22}$$

where $A = \overline{N}_k^{\text{PSP}}(|\tau| + 1/(\bar{\gamma}_{k,n}P_{T,k}))$, and

$$E_1(A) = \int_1^\infty \frac{\exp(-yA)}{y}dy \tag{23}$$

is the exponential-integral function [14], [15], which has a closed-form expression. In deriving (22) we exploited the equality that $\int_1^\infty \log(y)\exp(-yA)dy = E_1(A)/A$.

The sum rate for all K users is obtained as

$$C_{\text{tot, EPA}}^{\text{PSP}} = \sum_{k=1}^{K} C_{k,\text{ EPA}}^{\text{PSP}} \simeq \sum_{k=1}^{K} \sum_{\tau\in J_{K-1}} N \cdot \frac{\text{sign}(\tau)\exp(A)E_1(A)}{(|\tau|\bar{\gamma}_{k,n}P_{T,k} + 1)}. \tag{24}$$

Note that when all the carriers of the same user have the same statistics, $\bar{\gamma}_{k,n} = \bar{\gamma}_k$ holds.

B. Modified PSP Method

The direct-PSP method considers the transmission power and the channel SNRs in carrier competition, but it does not include the effect of N_k, the number of assigned carriers.

It is observed from (20) that to maximize the ergodic sum rate (assuming EPA) a modified-PSP for user k on carrier n should be used, which is defined as $\gamma_{k,n}^{\text{m-PSP}} = \gamma_{k,n} P_{T,k}/N_k^{\text{opt}}$, where N_k^{opt} is the optimal number of carriers that should be assigned to user k. Based on the maximum output $\gamma_{k,n}^{\text{m-PSP}}$ on each carrier n, an optimal multiuser diversity scheme similar to the downlink case can be designed. Unfortunately, N_k^{opt} is a variable dependent on the instantaneous CSI, and is unknown to the scheduler. Thus, it is interesting and meaningful to use an approximation of N_k^{opt} to develop a modified-PSP SMuD scheme and check its achievable performance.

Assume EPA. Let $C_n^{\text{SC,m-PSP}}$ denote the throughput for the selected user on carrier n. It is given by $C_n^{\text{SC,m-PSP}} = \log(1 + \gamma_n^{\text{SC,m-PSP}})$. Let k^* be the index of the user selected based on the modified-PSP ranking, then

$$\begin{aligned} k^* &= \text{argmax}_k \{\gamma_{k,n}^{\text{m-PSP}}\}_{k=1}^{K} \\ &= \text{argmax}_k \{\gamma_{k,n} P_{T,k}/N_k^{\text{opt}}\}_{k=1}^{K}. \end{aligned} \tag{25}$$

The dilemma for solving (25) arises from the fact that before N_k^{opt} is known to user k or the scheduler, the optimal carrier competition cannot be implemented based on SMuD. Yet, the factor $P_{T,k}/N_k^{\text{opt}}$ should be used in the multiuser diversity scheme. To circumvent this difficulty, we introduce a tight approximation that user k will use power $P_{T,k}/\overline{N}_k^{\text{opt}}$ on each subchannel, instead of $P_{T,k}/N_k$, to compete for all carriers, where $\overline{N}_k^{\text{opt}}$ is the average value of N_k^{opt}, and it may not be an integer. This approximation causes the sub-optimality of the proposed m-PSP scheme, and shows why for uplink scheduling per carrier SMuD schemes (without iteration) are sub-optimal.

We define an approximate modified PSP as (for user k on carrier n) $\gamma_{k,n}^{\text{m-PSP}} \simeq \gamma_{k,n} P_{T,k}/\overline{N}_k^{\text{opt}}$. Therefore, (25) is approximated by

$$k^* = \text{argmax}_k \{\gamma_{k,n} P_{T,k}/\overline{N}_k^{\text{opt}}\}_{k=1}^{K}. \tag{26}$$

The modified PSP SMuD algorithm is described below.

1) Determine $\overline{N}_k^{\text{opt}}$ in a statistical sense. This procedure is implemented by solving a nested optimization problem iteratively.
2) Assume EPA. For $n = 1, \ldots, N$, find the modified PSP $\gamma_{k,n}^{\text{m-PSP}}$ for all k, and assign the carriers based on (26).
3) Based on carrier set $\mathcal{S}_k$ and $P_{T,k}$ for all k, re-implement either EPA or WF power allocation for each user.

The user-wise WF algorithm for the modified-PSP scheme can use a procedure similar to that for the direct PSP SMuD scheme, and is omitted here for brevity.

To implement this algorithm, $\overline{N}_k^{\text{opt}}$ and AAPs for all k have to be found. This procedure is given below.

1) Method to Search $\overline{N}_k^{\text{opt}}$: The PDF of modified-PSP $\gamma_n^{\text{SC,m-PSP}}$ on carrier n is derived as

$$f_{\gamma_n^{\text{SC,m-PSP}}}(x) = \sum_{k=1}^{K} f_{\gamma_{k,n}^{\text{SC,m-PSP}}}(x, \overline{N}_k) \tag{27}$$

where $f_{\gamma_{k,n}^{\text{SC,m-PSP}}}(x, \overline{N}_k)$ is the PDF of $\gamma_{k,n}^{\text{SC,m-PSP}}$ conditioned on that user k has the largest m-PSP, and

$$f_{\gamma_{k,n}^{\text{SC,m-PSP}}}(x, \overline{N}_k) = \frac{1}{\bar{\gamma}_{k,n}^{\text{m-PSP}}} \exp(-x/\bar{\gamma}_{k,n}^{\text{m-PSP}}) \cdot \prod_{k'=1, k' \neq k}^{K} \left(1 - e^{-x/\bar{\gamma}_{k',n}^{\text{m-PSP}}}\right) \tag{28}$$

where $\bar{\gamma}_{k',n}^{\text{m-PSP}} = \bar{\gamma}_{k',n} P_{T,k'}/\overline{N}_{k'}$ by definition. The AAP for user k (identical on all N subchannels) is given by

$$\text{AAP}_{k,n}^{\text{m-PSP}}(\overline{N}_k) = \int_0^{\infty} f_{\gamma_{k,n}^{\text{SC,m-PSP}}}(x, \overline{N}_k) dx. \tag{29}$$

A closed-form expression for $\text{AAP}_{k,n}^{\text{m-PSP}}$ can be derived, omitted here due to the space limitation. We obtain $\overline{N}_k = \sum_{n=1}^{N} \text{AAP}_{k,n}^{\text{m-PSP}}$. The solution of $\overline{N}_k^{\text{opt}}$ should satisfy the following equality,

$$\begin{aligned} \overline{N}_k^{\text{opt}} &= \sum_{n=1}^{N} \int_0^{\infty} f_{\gamma_{k,n}^{\text{SC,m-PSP}}}(x, \overline{N}_k^{\text{opt}}) dx \\ &= \sum_{n=1}^{N} \text{AAP}_{k,n}^{\text{m-PSP}}(\overline{N}_k^{\text{opt}}). \end{aligned} \tag{30}$$

It is difficult to solve (30) directly. An iterative procedure to search for $\overline{N}_k^{\text{opt}}$ is designed below. Assume we will use a total of L_a stages of iterations, and let $\overline{N}_k^{(i)}$ be the ith stage solution for $\overline{N}_k^{\text{opt}}$.

1) Let $i = 0$ and $\overline{N}_k^{(0)} = N/K$.
2) Increase i by one. Define the m-PSP for user k on carrier n at stage i as $\gamma_{k,n}^{(i)} = \gamma_{k,n}/\overline{N}_k^{(i-1)} P_{T,k}^{(i)}$, where $P_{T,k}^{(i)}$ is a nominal available transmit power of user k at stage i.
3) Find a new estimate of $\overline{N}_k$ as $\overline{N}_k^{(i)}$ using the following relation
$$\overline{N}_k^{(i)} = \sum_{n=1}^{N} \int_0^{\infty} f_{\gamma_{k,n}^{\text{SC,m-PSP}}}(x, \overline{N}_k^{(i-1)}) dx. \tag{31}$$
4) If $i < L_a$, go to step 2); otherwise, stop. $\overline{N}_k^{(L_a)}$ gives the solution for $\overline{N}_k^{\text{opt}}$, for all k.

Here, (31) is sensitive to the value of $\overline{N}_k^{(i-1)}$. For numerical stability purposes we set $P_{T,k}^{(i)} < P_{T,k}$ until the last stage, and increase $P_{T,k}^{(i)}$ gradually at each stage. In the simulation, we set $P_{T,k}^{(i)} = P_{T,k}(i/L_a)^4$, and the factor $(i/L_a)^4$ in step 2) is a conservative factor used in the effective transmit PSP. The proposed algorithm converges with very small residual errors.

2) Rate for Modified-PSP with EPA: We use $\overline{N}_k^{\text{opt}}$ instead of N_k^{opt} and obtain an approximate analytical rate formula for user k as

$$C_{k,\text{ EPA}}^{\text{m-PSP}} \simeq \sum_{n=1}^{N} \int_0^{\infty} \log(1+y) \cdot f_{\gamma_{k,n}^{\text{SC,m-PSP}}}(y, \overline{N}_k^{\text{opt}}) dy. \tag{32}$$

After some manipulations we obtain a closed-form expression as

$$C_{k,\text{ EPA}}^{\text{m-PSP}} \simeq N \sum_{\tau \in J_{K-1}} \frac{\text{sign}(\tau) \exp(A) E_1(A)}{(|\tau| \bar{\gamma}_{k,n} P_{T,k}/\overline{N}_k^{\text{opt}} + 1)} \tag{33}$$

where $A = \left(|\tau| + \overline{N}_k^{\text{opt}}/(\bar{\gamma}_{k,n} P_{T,k})\right)$, $E_1(A)$ was defined in (23), and J_{K-1} is the expansion set of $\left\{1/\bar{\gamma}_{k',n}^{\text{m-PSP}}\right\}_{k' \neq k}^{K}$.

The sum rate is obtained as

$$C^{\text{m-PSP}}_{\text{tot, EPA}} = \sum_{k=1}^{K} C^{\text{m-PSP}}_{k,\text{ EPA}} \simeq \sum_{k=1}^{K} \sum_{\tau \in J_{K-1}} \frac{N \text{sign}(\tau) \exp(A) E_1(A)}{(|\tau| \bar{\gamma}_{k,n} P_{T,k} / \overline{N}_k^{\text{opt}} + 1)}. \quad (34)$$

The proposed m-PSP scheme is near-optimal for the case of EPA and independent carriers in the sense that a near-optimal carrier allocation is achieved (though with a tight approximation) based on SMuD, and EPA often has a negligible performance loss compared to the optimal WF solution. It significantly outperforms the *a*-SNR SMuD scheme which is directly based on downlink strategy.

The proposed direct-PSP and modified-PSP methods have the same complexity $O(K \times N)$ for online implementation, except that the modified-PSP method needs an offline optimization stage to find the $\overline{N}_k^{\text{opt}}$, which has a complexity of evaluating $L_a K$ single integrals.

C. Discussions

1) Comparison to the a-SNR Ranking Scheme: For downlink OFDMA with continuous rate modulation, it is known that the *a*-SNR ranking carrier competition with WF power allocation achieves the maximum sum rate [2], [10], [19]. When the *a*-SNR method is used for uplink, it is interesting to know its analytical throughput for each user. In the past, only simulation results were available.

Let γ_n^{sc} be the selected SNR on carrier n based on the *a*-SNR SMuD. Its PDF is derived as $f_{\gamma_n^{\text{sc}}}(x) = \sum_{k=1}^{K} f_{\gamma_{k,n}^{\text{sc}}}(x)$, where $f_{\gamma_{k,n}^{\text{sc}}}(x)$ is the PDF of $\gamma_{k,n}$ conditioned on that user k has the largest SNR on carrier n, and

$$f_{\gamma_{k,n}^{\text{sc}}}(x) = \frac{1}{\bar{\gamma}_{k,n}} \exp(-x/\bar{\gamma}_{k,n}) \cdot \prod_{k'=1, k' \neq k}^{K} \left(1 - e^{-x/\bar{\gamma}_{k',n}}\right). \quad (35)$$

The AAP for user k on carrier n is given by $\text{AAP}_{k,n} = \int_0^\infty f_{\gamma_{k,n}^{\text{sc}}}(x) dx$. A closed-form expression for $\text{AAP}_{k,n}$ can be derived, omitted here for brevity. It follows that $\overline{N}_k = N \cdot \text{AAP}_{k,n}$.

We analyze the sum and individual rates of the uplink EPA *a*-SNR OFDMA scheme below. The rate of user k is given by $C_{k,\text{ EPA}} = E[\sum_{n \in \mathcal{S}_k} \log(1 + \gamma_{k,n}^{\text{sc}} P_{T,k}/N_k)]$. Assume the carrier SNRs are independent. As a tight approximation, we use the average value $\overline{N}_k$ instead of N_k to evaluate rate $C_{k,\text{ EPA}}$, and obtain

$$C_{k,\text{ EPA}} \simeq N \int_0^\infty \log(1 + P_{T,k} y / \overline{N}_k) \cdot f_{\gamma_{k,n}^{\text{sc}}}(y) dy \quad (36)$$

where $f_{\gamma_{k,n}^{\text{sc}}}(y)$ was given by (35). A closed-form expression for (36) can be derived, omitted for brevity.

2) Effect of Correlated Channels: We discuss the effect of frequency channel (carrier) correlation on the ergodic rates of downlink and uplink OFDMA systems. To facilitate the discussion, we assume the EPA inside each carrier set $\{\mathcal{S}_k\}$. In the case of a limited number of resolvable paths in time domain, the N channels experienced by user k, $\{\gamma_{k,n}\}_{n=1}^N$, can be highly correlated, and the frequency diversity gain is reduced. The overall channel for the same user becomes more volatile. This does not affect $\overline{N}_k$, the average number of carriers assigned to user k, but it affects the distribution of N_k, namely, its variance becomes larger with channel correlation. Given a fixed transmission power $P_{T,k}$ for user k, as N_k deviates more from its mean $\overline{N}_k^{\text{opt}}$, the average sum throughput decreases. In other words, by affecting the power allocation to each carrier, the correlation decreases the uplink sum rate.

For the downlink system, the sum rate formula for EPA is given by

$$C_{\text{tot}} = \sum_{k=1}^{K} \sum_{n \in \mathcal{S}_k} \log(1 + \gamma_{k,n} P_T / N) \quad (37)$$

where P_T is the available transmission power at the BS. We observe that fading correlation still affects carrier allocation N_k for every user, but it has no impact on power allocation to each carrier, because P_T comes from a single pool at BS. By taking ensemble average of (37) over all channel SNRs, we observe that the ergodic sum rate is not affected by channel correlation. Numerical results in [10] have verified this observation for downlink OFDMA. Some additional relevant simulation results are given in Section VII of this paper.

V. Rate Maximization with Access Fairness

The access fairness is of interest for cellular communications, such as in CDMA [16] and TDMA systems [15]. For OFDMA based networks, to achieve access fairness, we propose the following long-term fairness metric

$$\text{AAP}_{1,n} : \cdots : \text{AAP}_{K,n} = 1 : \cdots : 1, \quad (\forall\, n). \quad (38)$$

To achieve this, we propose a normalized-SNR ranking based SMuD scheme to achieve the maximum sum rate with APF. The purpose of the *n*-SNR ranking is to ensure a uniform AAP for all the users on each carrier. Let $\tilde{\gamma}_{k,n} = \gamma_{k,n}/\bar{\gamma}_{k,n}$ be the normalized SNR of the kth user on carrier n. The largest *n*-SNR is denoted by $\tilde{\gamma}_n^{\text{sc}} = \max\{\tilde{\gamma}_{1,n}, \ldots, \tilde{\gamma}_{K,n}\}$. In this scheme, the user with the largest *n*-SNR is allocated with carrier n. After all carriers are assigned, the transmission powers of K users are allocated using either EPA or user-wise WF.

The AAP for user k is given by $\text{AAP}_{k,n}^{\text{n-SNR}} = \int_0^\infty f_{\tilde{\gamma}_{k,n}^{\text{sc}}}(x) dx$, where $f_{\tilde{\gamma}_{k,n}^{\text{sc}}}(x)$ is the PDF of $\tilde{\gamma}_{k,n}$ conditioned on that user k has the largest *n*-SNR on carrier n, and $f_{\tilde{\gamma}_{k,n}^{\text{sc}}}(x) = e^{-x}(1 - e^{-x})^{K-1}$. Note that the PDF of $\tilde{\gamma}_n^{\text{sc}}$ is given by $f_{\tilde{\gamma}_n^{\text{sc}}}(x) = \sum_{k=1}^{K} f_{\tilde{\gamma}_{k,n}^{\text{sc}}}(x) = K e^{-x}(1 - e^{-x})^{K-1}$. We can readily show that $\text{AAP}_{k,n}^{\text{n-SNR}} = 1/K$, and $\overline{N}_k = \sum_{n=1}^{N} \text{AAP}_{k,n}^{\text{n-SNR}} = N/K$, for all k. Assuming additional constraint (38) and equal weights ($w_k = 1$ for all k), we modify the sum rate optimization problem in (1) as

$$\max_{\{\mathcal{S}_k\}, \{P_n\}} \sum_{n=1}^{N} C_n^{\text{n-SNR}} \quad \text{s.t.} \quad \sum_{n \in \mathcal{S}_k} P_n \leq P_{T,k} \quad (39)$$

where P_n and $C_n^{\text{n-SNR}}$ are the allocated power and the throughput of the selected user on carrier n, respectively, and

$$C_n^{\text{n-SNR}} = \log(1 + P_n \gamma_n^{\text{SC,n-SNR}}) \quad (40)$$

where $\gamma_n^{\text{SC,n-SNR}} = \gamma_{k^*,n}$, and k^* is the index of user being selected, i.e., $k^* = \text{argmax}_k\{\tilde{\gamma}_{k,n}\}_{k=1}^K$.

Below, we provide performance analysis for the EPA n-SNR scheme. Under the EPA, $P_n = P_{T,k}/N_k$ for $n \in \mathcal{S}_k$. The instantaneous sum rate is given by $C_{\text{tot, EPA}}^{\text{n-SNR}} = \sum_{n=1}^N C_n^{\text{n-SNR}}$, where $C_n^{\text{n-SNR}} = \log(1+\gamma_n^{\text{SC,n-SNR}}P_{T,k}/N_k)$ when $n \in \mathcal{S}_k$ holds. As a tight approximation, we assume that the power allocated to carrier n (when $n \in \mathcal{S}_k$) is approximately $P_{T,k}/\overline{N}_k$ and obtain

$$C_{\text{tot, EPA}}^{\text{n-SNR}} \simeq \sum_{k=1}^{K}\sum_{n\in\mathcal{S}_k} \log(1+\gamma_n^{\text{SC,n-SNR}}P_{T,k}/\overline{N}_k). \quad (41)$$

Thus, the rate of user k $(k = 1,\ldots,K)$ is approximated by

$$\begin{aligned} C_{k,\text{ EPA}}^{\text{n-SNR}} &= E_t[C_{k,\text{ EPA}}^{\text{n-SNR}}] \\ &\simeq \sum_{n=1}^{N}\int_0^\infty \log(1+yP_{T,k}/\overline{N}_k)\cdot f_{\gamma_{k,n}^{\text{SC,n-SNR}}}(y)dy \quad (42)\end{aligned}$$

where $\overline{N}_k = N/K$ and $f_{\gamma_{k,n}^{\text{SC,n-SNR}}}(y)$ is the PDF of $\gamma_{k,n}$ conditioned on that user k is selected with the n-SNR ranking. Using a result in [15], [20], for i.n.d. Rayleigh-faded users we have

$$\begin{aligned} f_{\gamma_{k,n}^{\text{SC,n-SNR}}}(x) &= \frac{1}{\bar{\gamma}_{k,n}}\exp\left(-\frac{x}{\bar{\gamma}_{k,n}}\right) \\ &\quad\cdot\left[1-\exp\left(-\frac{x}{\bar{\gamma}_{k,n}}\right)\right]^{K-1}. \quad (43)\end{aligned}$$

After some manipulations we obtain

$$C_{k,\text{ EPA}}^{\text{n-SNR}} = N\sum_{l=0}^{K-1}\binom{K-1}{l}\frac{(-1)^l}{(l+1)}\cdot\exp(A_{k,l})E_1(A_{k,l}) \quad (44)$$

where $A_{k,l} = \frac{(l+1)N}{KP_{T,k}\bar{\gamma}_{k,n}}$. A closed-form sum-rate formula is then given by

$$\begin{aligned} C_{\text{tot, EPA}}^{\text{n-SNR}} &= \sum_{k=1}^{K} C_{k,\text{ EPA}}^{\text{n-SNR}} \\ &\simeq N\sum_{k=1}^{K}\sum_{l=0}^{K-1}\binom{K-1}{l}\frac{(-1)^l}{(l+1)} \\ &\quad\cdot\exp(A_{k,l})E_1(A_{k,l}). \quad (45)\end{aligned}$$

VI. Rate Maximization with Rate Fairness

For the optimization problem in (1) assuming constraint (3) and equal weights ($w_k = 1$ for all k), we design the following sub-optimal approach using SMuD and multicarrier diversity, by extending the downlink RPF strategy [3], [8] to the uplink.

This algorithm requires knowledge of $\{\alpha_k\}$, $\{P_{T,k}\}$ and $\{\gamma_{k,n}\}$ as input, and outputs the scheduling decision including carrier assignment $\{\mathcal{S}_k\}$ and power allocation $\{P_{k,n}\}$. The pseudo code for carrier and power allocation at each TTI is given below.

(i) Let set $\mathcal{S}_k$ be empty for all k and $\tilde{\mathcal{S}} = \{1,2,\ldots,N\}$, where $\tilde{\mathcal{S}}$ is the set of all unallocated carriers.

(ii) For $k = 1,\ldots,K$, do the following:
User k gets carrier n^* if $n^* = \text{argmax}_n\gamma_{k,n}$, where $n \in \tilde{\mathcal{S}}$. Add element n^* to set $\mathcal{S}_k$ and remove it from $\tilde{\mathcal{S}}$.

(iii) For the remaining $N-K$ unallocated carriers, while $\tilde{\mathcal{S}}$ is non-empty, do the following,

(1) Compute C_k and C_{tot} for all k. $C_k = \sum_{n\in\mathcal{S}_k}\log(1+\gamma_{k,n}P_{k,n})$ and $C_{\text{tot}} = \sum_{k=1}^K C_k$, where $P_{k,n}$ is transmission power of user k on carrier n, and is computed using either EPA or WF.

(2) Compute normalized rate ratio $\tilde{r}_k = \frac{C_k}{C_{\text{tot}}\alpha_k}$. Find the minimum element in $\{\tilde{r}_k\}_{k=1}^K$, denoted as $\tilde{r}_{k_{\min}}$. User $k_{\min}$ is the least satisfied in proportional rate.

(3) User $k_{\min}$ is assigned with carrier n^* by $n^* = \text{argmax}_n\gamma_{k,n}$, where $n \in \tilde{\mathcal{S}}$.
Add element n^* to set $\mathcal{S}_k$ and remove it from $\tilde{\mathcal{S}}$.

This method has a complexity of $O(N\times K)$. The rate fairness is attained in steps (iii.2)–(iii.3) by comparing normalized rate ratios $\tilde{r}_1,\ldots,\tilde{r}_K$ and assign more carriers to users whose proportional rates are the least met. Furthermore, the rate maximization is achieved by multiuser selective carrier allocation and WF power allocation.

The Jain's fairness metric [21] for the rate fairness scheme will be used to evaluate the performance of the proposed method. The fairness metric for time t is

$$\text{FA}(t) = \left[\sum_{k=1}^{K} C_k(t)\right]^2 \Bigg/ \left[K\sum_{k=1}^{K} C_k^2(t)\right].$$

The average fairness metric is given by $\overline{\text{FA}} = E[\text{FA}(t)]$. It is known that $\overline{\text{FA}} \in (1/K, 1)$, where $\overline{\text{FA}} = 1/K$ corresponds to the most unfair case and $\overline{\text{FA}} = 1$ gives the uniform rate fairness.

VII. Numerical Results

We assume two cases of user statistics: (1) independent and identically distributed (i.i.d.) users, where all K users have equal transmission powers (with $P_{T,k} = 1$) for all k; equal channel average SNRs (ASNRs), and the same target BERs given by $P_e = 10^{-4}$; and (2) i.n.d. users, where the K users have non-equal transmission powers with $P_{T,k+1} = P_{T,k}10^{\delta/10}$ for $k = 1,\ldots,K-1$ with $\delta = 1$, and we assume that $(1/K)\sum_{k=1}^K P_{T,k} = 1$ holds. Next, the channel ASNRs decrease successively by 2 dB from the strongest user to the weakest one, i.e., $\bar{\gamma}_{k+1} = \bar{\gamma}_k 10^{-2/10}$, for $k = 1,\ldots,K-1$. The cases of i.i.d. and i.n.d. users will be compared based on the same user-average channel ASNR. For i.n.d. users, following the same order of users we assume the target BERs are given by 10^{-3} for the first $K/2$ users, and by 10^{-5} for the remaining $K/2$ users, which reflect the non-equal BER constraints for different applications.

Three cases of carrier generation are considered: (1) N i.i.d. carriers for each user. This case occurs when each user's time domain channel has abundant resolvable multipath; (2) Carriers are generated based on DFT of L resolvable paths which have a uniform power delay profile (PDP); and (3) The L resolvable paths have an exponentially decayed PDP, and the path power decreases by 3 dB successively from the first path to the last one.

We used 1000 independent time slots to obtain simulation results unless otherwise stated. All the rate results are plotted after converting nats/s to bits/s (i.e., the log to the $\log_2$ scale) and have been normalized by factor B_w/N, which is the bandwidth per carrier.

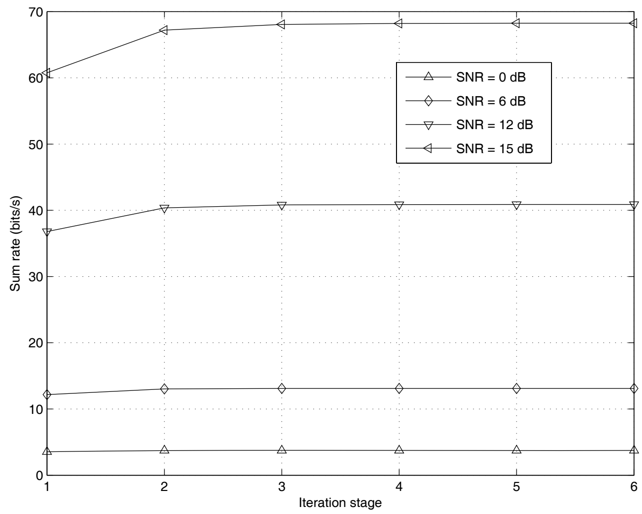

Fig. 1. Sum rate vs. iteration stage for the CDU duality algorithm for i.n.d. users, when $K = 12$, $N = 64$, and $L = 16$ paths with an exponentially decayed PDP.

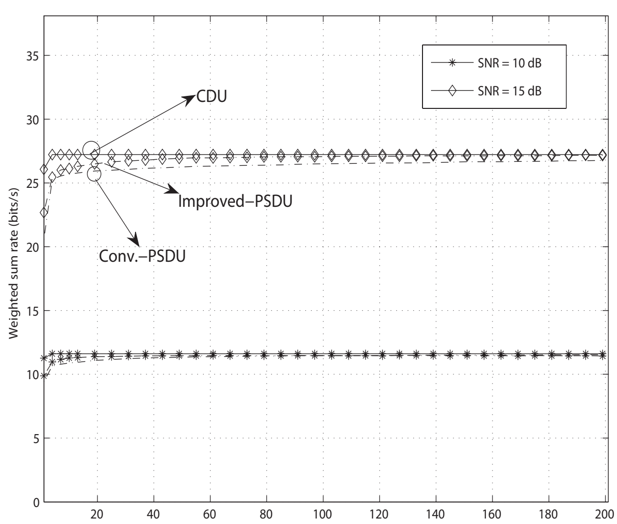

Fig. 2. Weighted sum rate vs. iteration stage for the CDU, conventional- and improved-PSDU duality algorithms for i.n.d. users when $K = 4$, $N = 32$, and $L = 4$ paths with an exponentially decayed PDP.

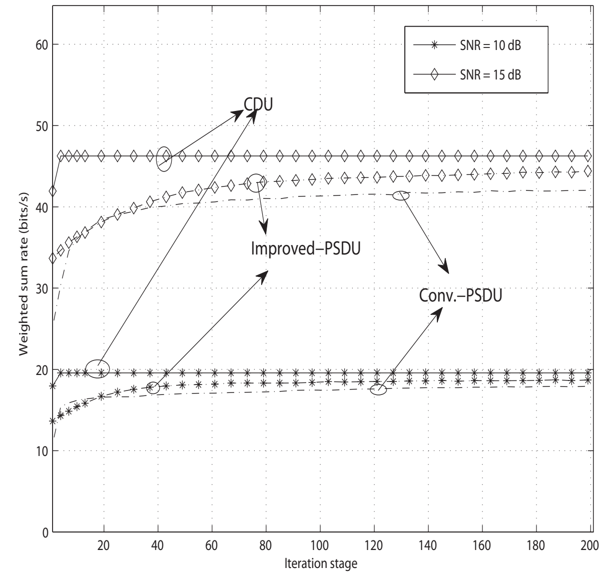

Fig. 3. Weighted sum rate vs. iteration stage for the CDU, conventional- and improved-PSDU duality algorithms for i.n.d. users when $K = 12$, $N = 32$, and $L = 4$ paths with an exponentially decayed PDP.

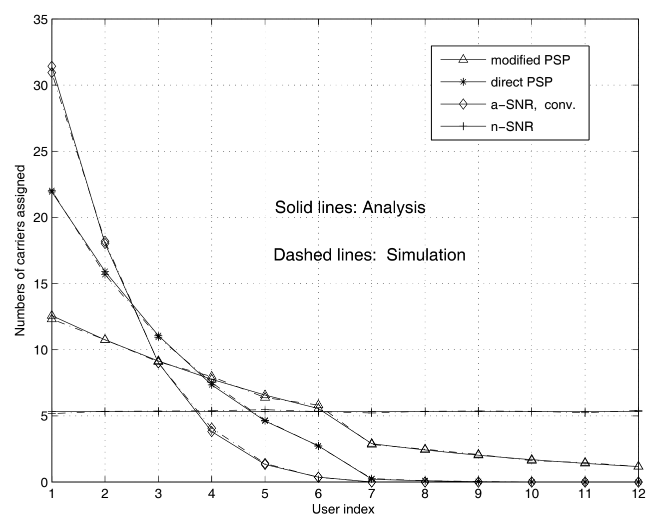

Fig. 4. $\overline{N}_k$ vs. user index k for i.n.d. users when $K = 12$, $N = 64$, and $L = 16$ paths with an exponentially decayed PDP.

In Fig. 1, the sum rate of the proposed CDU duality method for uplink OFDMA vs. iteration stage is presented, over a highly-correlated frequency channels with $L = 16$ time domain paths which have an exponentially decayed PDP, and $K = 12$ and $N = 64$. The users have decreasing channel SNRs (by 2 dB successively) and non-identical transmit powers. The result shows that the CDU algorithm converges very fast. The iterations converge within no more than two stages for low to medium SNRs, and in 3 stages for high SNR.

In Figs. 2 and 3, we compare the convergence behavior of the CDU, conventional- and improved-PSDU duality schemes, where the rate curves were averaged over 200 independent TTIs. We assume $L = 4$, $N = 32$, $K = 4$ in Fig. 2 and $K = 12$ in Fig. 3. The step-size factors for the conventional- and improved-PSDU algorithms are, respectively, set to $\beta_k = 0.01$ and $\beta_k = 0.05$. The conventional PSDU algorithm is less stable than the improved PSDU algorithm and so a smaller β_k was chosen than the latter in this example. To initialize dual variables $\{\lambda_k\}$ we empirically set $\lambda_k^{(0)} = w_k/[P_{T,k} + \min_n\{1/\gamma_{k,n}\}]$ for all k. The weight factor $\mathbf{w}$ is chosen as the normalized version of vector $[K, K-1, \ldots, 1]$ such that $\sum_{k=1}^{K} w_k = K$ holds. Result shows that the improved-PSDU algorithm converges much faster than the conventional PSDU algorithm, and its sum rate loss compared to the CDU algorithm is smaller than the latter. Comparisons between Figs. 2 and 3 show that for a small K (e.g., $K = 4$), the rate gap between the CDU and the improved PSDU schemes is negligible after convergence. However, for a larger K ($K = 12$), the rate advantage of the CDU scheme than the

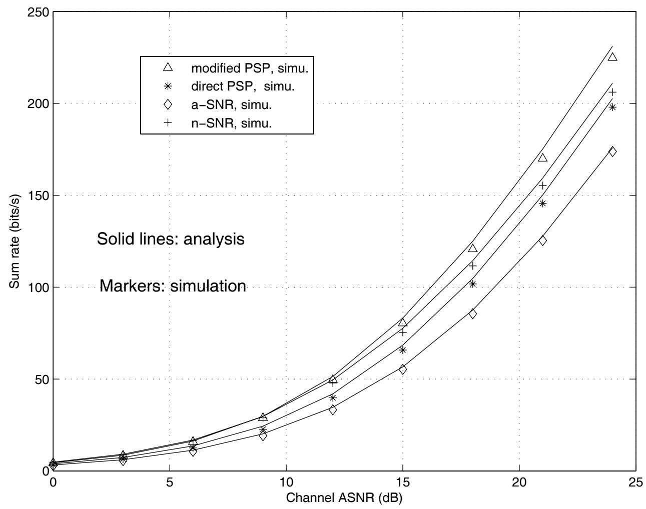

Fig. 5. Sum rate vs. channel ASNR using the PSP, n-SNR and conventional a-SNR schemes (all with EPA), assuming independent carriers and i.n.d. users, $N = 64$ and $K = 10$.

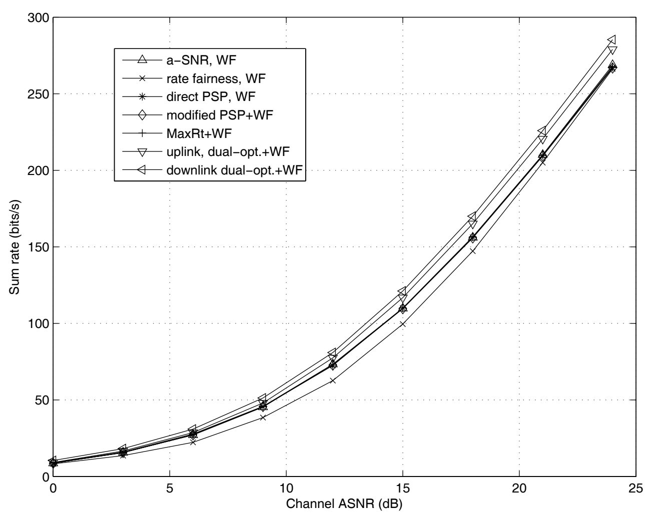

Fig. 6. Sum rate vs. channel ASNR using the proposed duality optimization, PSP, n-SNR, and rate fairness schemes (all with WF), assuming correlated carriers with exponentially decayed multipath PDP with $L = 16$, and i.i.d. users, $N = 64$ and $K = 10$.

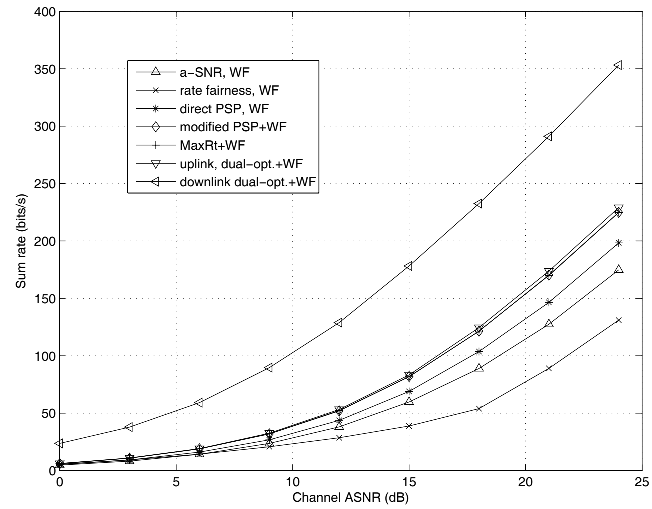

Fig. 7. Sum rate vs. channel ASNR using the proposed duality optimization, PSP, n-SNR, and rate fairness schemes (all with WF), assuming independent carriers and i.n.d users, $N = 64$ and $K = 10$.

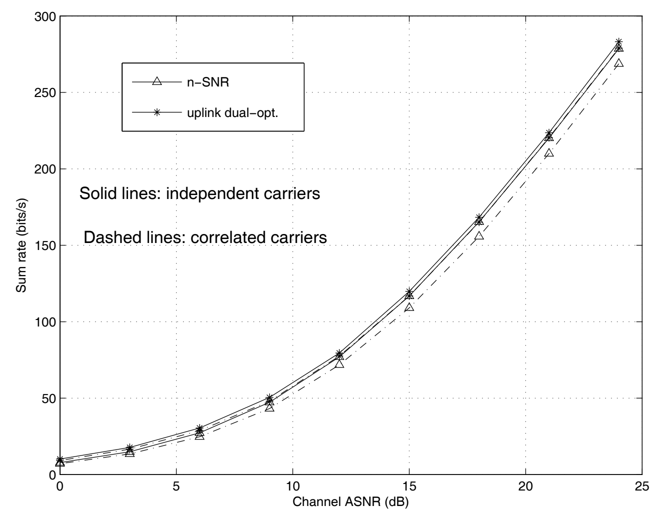

Fig. 8. Sum rate vs. channel ASNR for correlated (with a uniform PDP and $L = 16$) and independent carrier channels, respectively, assuming i.i.d. users, $N = 64$, and $K = 10$.

PSDU schemes becomes significant. A much larger number of stage S_2 is required for the PSDU methods to converge to the near-optimal solution when K and/or the SNR increases. In comparison, the proposed CDU duality approach demonstrates an excellent convergence property and stable performance and is very attractive for such cases. Nevertheless, for numerical examples not shown here, we also observed that the PSDU could achieve a rate very close to (or even slightly higher than) the CDU scheme after convergence, for example, when K/N is very small and the users are i.i.d.

From here on we study only unweighted sum rate performance of the rate-maximization and fairness schemes. Furthermore, among the duality optimization schemes only the CDU duality scheme is considered. Assuming the same system and channel parameters as for Fig. 1, we present in Fig. 4 the simulated and analytical $\overline{N}_k$, the average number of assigned carriers to each user k, for the proposed n-SNR, PSP ranking, and conventional a-SNR schemes. In simulation, we set iteration stage $L_a = 12$ for the m-PSP scheme in the offline optimization part. The results confirm the validity and accuracy of our analysis on $\overline{N}_k$ of all the considered schemes, even for the case of correlated carriers. The proposed n-SNR scheme achieves perfect access fairness, but the PSP schemes and the a-SNR scheme give less channel access proportions to users who have smaller channels SNRs (as user index k increases).

In Fig. 5 we test the accuracy of our rate analysis on the proposed PSP and n-SNR schemes and the conventional a-SNR scheme (all with EPA), assuming independent carriers with $K = 10$, $N = 64$, but non-identical user statistics. The result shows that the analytical result fits simulation well. The modified PSP scheme gives better performance than both the direct PSP and the a-SNR schemes. The n-SNR scheme has a sum rate close to that of the modified PSP, while maintaining access fairness.

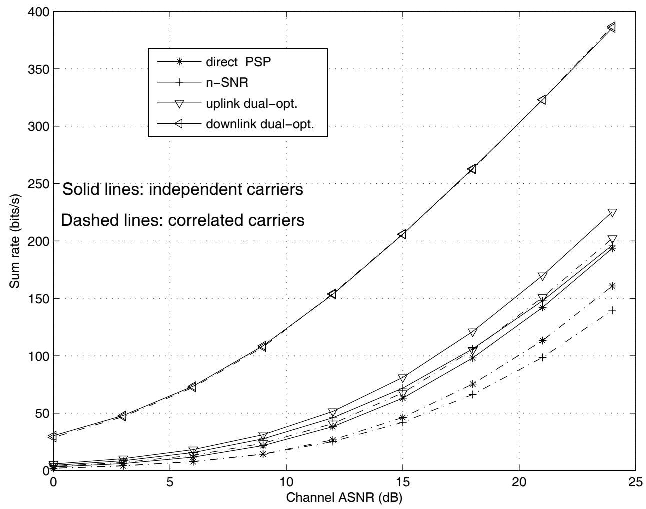

Fig. 9. Sum rate vs. channel ASNR with correlated (with an exponentially decayed PDP and $L = 16$) and independent carrier channels, respectively, assuming non-i.i.d. users, $N = 64$, and $K = 10$.

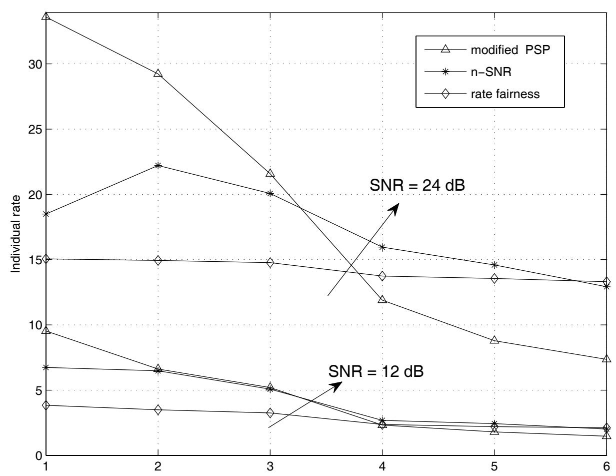

Fig. 10. Individual rate vs. user index k for several OFDMA schemes, assuming i.n.d. users with $L = 8$ paths which have an exponentially decayed PDP, $N = 32$ and $K = 6$.

Next, we compare the sum rates of the proposed schemes, including the duality optimization, PSP, n-SNR, and rate fairness schemes. For comparison purposes, the sum rate of the duality optimization method in downlink OFDMA is given, where the total transmit power at the BS is equal to the sum transmit power of all the K uplink transmitters. Assuming the i.i.d users, but correlated subchannels generated from $L = 16$ paths with a unform PDP, the sum rate result is presented in Fig. 6, which shows that there is an obvious performance gap between uplink and downlink channels. This is mainly caused by channel correlation. Without correlation the gap will be reduced but is still non-zero. The proposed modified PSP scheme gives a similar rate performance to the MaxRt+WF scheme [11] but has a lower complexity for online implementation.

In Fig. 7 we consider the i.i.d carriers for the same user but non-i.i.d. users. The result shows that the gap between the uplink and downlink optimal receivers becomes much larger

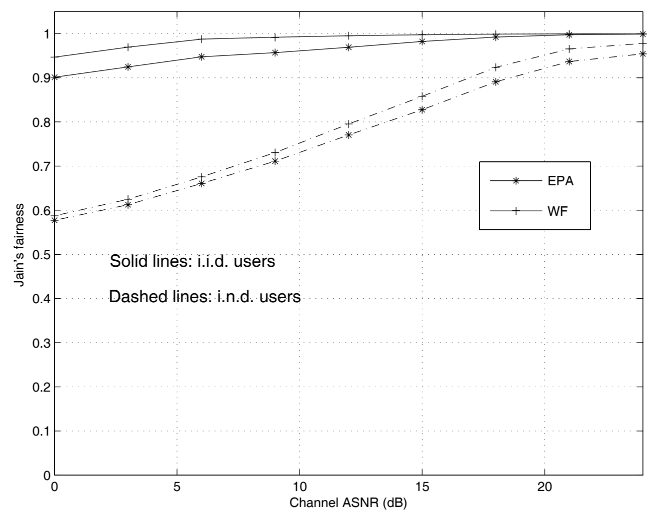

Fig. 11. Fairness index vs. channel ASNR for the rate fairness scheme with EPA and WF for i.i.d users (furthermore with a uniform PDP) and i.n.d. users (furthermore with an exponential PDP), respectively, when $N = 64$ and $K = 10$.

in this case due to non-identical power and channel statistics and different BER constraints of K users. The modified PSP performs better than the direct PSP and conventional a-SNR schemes, while the rate fairness scheme has the lowest sum rate. For both Figs. 6 and 7 we assume $\alpha_k = 1/K$ for all k for the rate fairness scheme. The case of non-equal α_k was not considered in this example. This result reveals that the rate fairness constraint can cause significant penalty in sum rate, especially for the case of i.n.d. users.

In Fig. 8 (for i.i.d. users) and Fig. 9 (for non-i.i.d. users), we study the effect of correlated carriers on the proposed schemes, and the optimal downlink performance is shown for comparison in Fig. 9. Results show that with correlated carriers the ergodic sum rate performance of all uplink receivers degrade, while the rate of duality-based downlink receiver is not affected. This has been explained in Section IV.

Comparisons between Figs. 8 and 9 show that for the i.i.d. users the rate degradation is smaller compared to the non-i.i.d. users, because for the latter case the power allocation among the users is more severely affected by correlated carriers.

In Fig. 10, the individual rate C_k vs. user index k for the proposed rate fairness, modified PSP, and n-SNR schemes are given assuming $N = 32$ and $K = 6$. We also assume non-i.i.d. users, WF power allocation, and $\alpha_k = 1/K$ for all k. The results show that the rate fairness scheme makes the rate distribution fairer among the users compared to other schemes. The n-SNR scheme gives near-perfect access fairness but not rate fairness, though it still gives better rate fairness than the PSP scheme.

In Fig. 11, the Jain's fairness metric (averaged over 400 TTIs) for the rate fairness scheme is given assuming $N = 64$ and $K = 10$. The results show that the fairness metric is higher for i.i.d users than i.n.d. users, and the WF gives a slightly better fairness than the EPA scheme. Furthermore, as the SNR increases, the fairness improves for all i.i.d users and i.n.d. users.

VIII. Conclusions

In this paper, we have proposed several schemes to maximize the sum rate of OFDMA uplink with and without fairness considerations. Based on the Lagrangian dual optimization tool, two new dual update algorithms have been designed to maximize the weighted sum rate. Especially, the proposed CDU-duality scheme was shown to be robust and converge very fast. For a low-complexity alternative, we have also proposed power-SNR-product based selective multiuser diversity schemes which extend the downlink SMuD strategy to the uplink and achieve a good sum rate performance. Furthermore, to maintain access fairness and rate fairness, respectively, we have developed low-complexity carrier and power allocation schemes based on the SMuD strategy. Accurate approximations (in closed-form formulas) for the sum and individual rates and fairness metrics (such as AAP) for the PSP and n-SNR SMuD schemes have been derived. The effects of N and K, non-equal target BERs, transmission powers, and channel SNRs have been studied, and the analytical results have been verified by simulations. We have shown that unlike the downlink case, correlated carriers degrade the uplink performance, and an intuitive explanation was given. These results provide practical schemes and new insights into the effects of various system and channel parameters on the achievable performance of uplink OFDMA under various constraints.

Appendix A
Closed-form expression for AAP for direct-PSP method

We derive a closed-form expression for $\mathrm{AAP}_{k,n}^{\mathrm{PSP}}$. Define an expansion that

$$\prod_{l=1}^{N}(1-\exp(-x/\bar{\gamma}_l)) = \sum_{\tau\in J_N\left(\left\{\frac{1}{\bar{\gamma}_l}\right\}_{l=1}^{N}\right)} e^{-x|\tau|}\mathrm{sign}(\tau)$$

where $J_N\left(\left\{\frac{1}{\bar{\gamma}_l}\right\}_{l=1}^{N}\right) = \{0, T_1, \ldots, T_N\}$. Here, $T_k = \left\{(-1)^k \sum_{l_1,l_2,\ldots,l_k} \frac{1}{\bar{\gamma}_{l_k}}\right\}$, and $\sum_{l_1,l_2,\ldots,l_k}$ refers to all the possible $\binom{N}{k}$ combinations of choosing k elements out of the set $\left\{\frac{1}{\bar{\gamma}_l}\right\}_{l=1}^{N}$. For example, $T_1\left(\left\{\frac{1}{\bar{\gamma}_l}\right\}_{l=1}^{N}\right) = \left\{-\frac{1}{\bar{\gamma}_1}, \ldots, -\frac{1}{\bar{\gamma}_N}\right\}$, and $T_2\left(\left\{\frac{1}{\bar{\gamma}_l}\right\}_{l=1}^{N}\right) = \left\{\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}, \frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_3}, \ldots, \frac{1}{\bar{\gamma}_{N-1}}+\frac{1}{\bar{\gamma}_N}\right\}$. We define

$$\prod_{k'=1,k'\neq k}^{K}\left(1-e^{-x/\bar{\gamma}_{k',n}^{\mathrm{PSP}}}\right) = \sum_{\tau\in J_{K-1}\left(\left\{1/\bar{\gamma}_{k',n}^{\mathrm{PSP}}\right\}_{k'\neq k}^{K}\right)} \mathrm{sign}(\tau)\exp(-x|\tau|) \quad (46)$$

where $J_{K-1}\left(\left\{1/\bar{\gamma}_{k',n}^{\mathrm{PSP}}\right\}_{k'\neq k}^{K}\right)$ is the expansion set [20] of $\left\{1/\bar{\gamma}_{k',n}^{\mathrm{PSP}}\right\}_{k'\neq k}^{K}$ and contains 2^{K-1} elements, and J_{K-1} is a shorthand for $J_{K-1}\left(\left\{1/\bar{\gamma}_{k',n}^{\mathrm{PSP}}\right\}_{k'\neq k}^{K}\right)$. Thus, we have

$$f_{\gamma_{k,n}^{\mathrm{SC,PSP}}}(x) = \sum_{\tau\in J_{K-1}} \frac{\mathrm{sign}(\tau)}{\bar{\gamma}_{k,n}^{\mathrm{PSP}}} \exp\left(-x\left[|\tau|+\frac{1}{\bar{\gamma}_{k,n}^{\mathrm{PSP}}}\right]\right). \quad (47)$$

Using (18) and (47), we can readily show that

$$\mathrm{AAP}_{k,n}^{\mathrm{PSP}} = \sum_{\tau\in J_{K-1}} \frac{\mathrm{sign}(\tau)}{(|\tau|\bar{\gamma}_{k,n}^{\mathrm{PSP}}+1)}. \quad (48)$$

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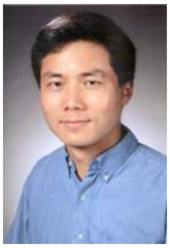

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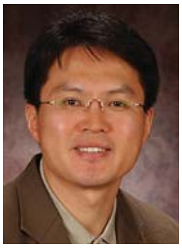

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