Adaptive Multi-Node Incremental Relaying for Hybrid-ARQ in AF Relay Networks

Wan Choi, *Member, IEEE,* Dong In Kim, *Senior Member, IEEE,* and Byoung-Hoon Kim, *Member, IEEE*

***Abstract*—This paper proposes an adaptive multi-node incremental relaying technique in cooperative communications with amplify-and-forward (AF) relays. In order to reduce the excessive burden of MRC with all diversity paths at the destination node, the destination node decides if it combines signals over the first $N(< K)$ time slots/frames or over all of the K times slots, where K is the number of relay nodes. Our analytical and simulation results show that the proposed adaptive multi-node incremental relaying outperforms the conventional MRC in terms of outage probability in AF based cooperative communications since the proposed scheme effectively reduces the spectral efficiency loss. Our asymptotic analysis also shows that the proposed adaptive multi-node incremental relaying achieves full diversity order $K + 1$.**

***Index Terms*—Relay communications, hybrid-ARQ, amplify-and-forward.**

I. INTRODUCTION

BESIDES conventional diversity techniques using antenna arrays or RAKE receivers, recent studies have found that significant diversity gains can be achieved through cooperation among geographically distributed nodes or terminals, namely cooperative diversity [1], [2]. The concept of cooperative communications originated from a relay channel in information theory [3]. Initial studies on cooperative diversity mainly focused on a single relay configuration [1], [2], [4]–[6], but recent research pays attention to utilizing multiple relays since it can significantly increase the cooperative diversity gains [7]–[10]. Multiple relays are able to satisfy high QoS requirements not achievable by a single relay alone.

The received signals from relays and a source are desirable to be orthogonal to each other for diversity combining at the destination. Correspondingly, dedicated orthogonal channels are typically allocated to the source-to-destination (S-D) link and the relay-to-destination (R-D) links by time division or frequency division multiple access among source and relay nodes [5], [11]. Even though distributed space time coding [12]–[14] is able to provide cooperative diversity gains, it requires complicated coordination among source and relay nodes and suffers from synchronization problems [15]. So orthogonal channel allocation becomes a popular method for cooperative diversity despite spectral efficiency loss by orthogonal channel allocation.

In cooperative communications with multiple amplify-and-forward (AF) relays, all the diversity paths from relays are combined at a destination node since AF relays cannot decode the signal from a source and decide if they forward the signal to a destination node or not. Even though performance of diversity combining is generally proportional to the number of independent diversity paths, combining all the paths from relays might not be optimal in cooperative communications since more relays require more time-slotted orthogonal channels which causes a spectral efficiency loss due to transmit duty cycle. So minimizing the number of cooperating relays while satisfying the desired error rate performance at the destination node is important in cooperative diversity using multiple AF relays.

In this context, this paper proposes an adaptive multi-node incremental relaying scheme. The idea is motivated by [16] where the error detection was employed for diversity combining at RAKE receivers. When time-slotted orthogonal channels are adopted, the destination node first combines the signals over the earliest N time slots (signals from a S-D link and $N - 1$ R-D links), and checks if the combined SNR exceeds a pre-determined value. If the SNR test passes, the destination node broadcasts the success to relays to prevent remaining relays from transmitting the signals in their time slots/frames. Otherwise, the destination node waits until it gathers signals over all $K + 1$ time slots/frames and combines them. We show that the proposed adaptive multi-node incremental relaying improves the cooperative diversity performance in AF based cooperative relay communications. In terms of outage probability, the proposed scheme significantly outperforms the conventional MRC with $K + 1$ diversity paths since it can effectively reduce the spectral efficiency loss. It is also proved by an asymptotic analysis that the proposed adaptive multi-node incremental relaying achieves full diversity order $K + 1$.

II. ANALYSIS MODEL

A. A proposed adaptive multi-node incremental relaying

We consider an AF based cooperative communication model with K relay nodes as shown in Fig. 1. Since a repetition based half-duplex relaying protocol is employed, time-slotted orthogonal channels are allocated to source and relay nodes. In the fist time slot/frame, a source node broadcasts its

Manuscript received November 29, 2008; revised April 22, 2009 and October 17, 2009; accepted November 4, 2009. The associate editor coordinating the review of this letter and approving it for publication was C. Papadias.

W. Choi is with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, 305-701, Korea (e-mail: wchoi@ee.kaist.ac.kr).

D. I. Kim is with the School of Information and Communication Engineering, Sungkyunkwan University (SKKU), Suwon, Korea (e-mail: dikim@ece.skku.ac.kr).

B. H. Kim is with the Mobile Communication Technology Research Lab, LG Electronics, Inc., Anyang, Korea (e-mail: byoungk@lge.com).

Parts of this paper were presented in IEEE VTC 2009 Spring, Barcelona, Spain. This research was supported by the MKE (The Ministry of Knowledge Economy), Korea, under the ITRC (Information Technology Research Center) support program supervised by the NIPA (National IT Industry Promotion Agency) (NIPA-2009-(C1090-0902-0005)) and in part by the LG Electronics, Inc.

Digital Object Identifier 10.1109/TWC.2010.02.081583

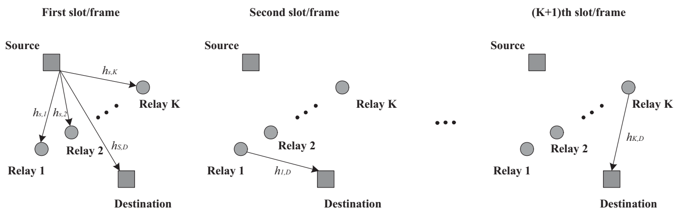

Fig. 1. AF based cooperative communications with time-slotted orthogonal channels.

signal to relays and the destination node. Then, the k th relay amplifies and forwards the received signal to a destination node in the $(k+1)$th time slot/frame as shown in Fig. 1. That is, $K+1$ time slots/frames are required to gather all the signals from relays. It should be noted that the source does not re-transmit its message except the first time slot/frame.

In conventional AF based cooperative communications with K relays, a destination combines all the signals from K relays. But combining all the paths from relays might not be optimal in cooperative communications as mentioned in Introduction. On the other hand, in our adaptive multi-node incremental relaying, the destination node first performs maximal ratio combining only with the signals in the earliest N slots/frames so that the destination does not wait until the last frame is received. Based on the threshold test for the output SNR of the combined signal over N time slots/frames, it switches to MRC with all the signals from K relays or terminate transmission from remaining time slots/frames. The SNR can be estimated by using estimated channel gains and average noise power at the destination. The detailed algorithm of the proposed technique is given in Fig. 2.

The proposed technique can be considered an extension of the incremental relaying proposed in [5] to multi-node configurations. Contrary to the conventional incremental relaying based on a single relay [5], the proposed scheme adaptively selects the number of relays for the first transmission and re-transmission. As an alternative to the proposed adaptive multi-node incremental relaying, an iterative incremental relaying which checks the outage event at each slot might yield better outage performance. However, the computational complexity and hence energy consumption will be much larger than the proposed scheme. The analysis of the iterative incremental relaying will be our future research topic.

B. Received signals and channel models

Assuming channels between nodes are static over K slots/frames for relaying, the received signals at the destination node and the kth relay node in the first time slot/frame is given, respectively, by

$$y_{s,d}[n] = h_{s,d}\sqrt{E_s}x[n] + z_d[n] \tag{1}$$

$$y_{s,k}[n] = h_{s,k}\sqrt{E_k}x[n] + z_k[n], \quad k = 1, \cdots, K \tag{2}$$

where subscripts s, d, and k denote source, destination, and the kth relay nodes, respectively. $h_{i,j}$ is an independent Rayleigh fading channel gain from node i to node j with $\mathbb{E}[|h_{i,j}|^2] = \sigma_{i,j}^2$ which reflects the geometry of the source, relay, and destination nodes. $z_i[n]$ is an additive white Gaussian noise

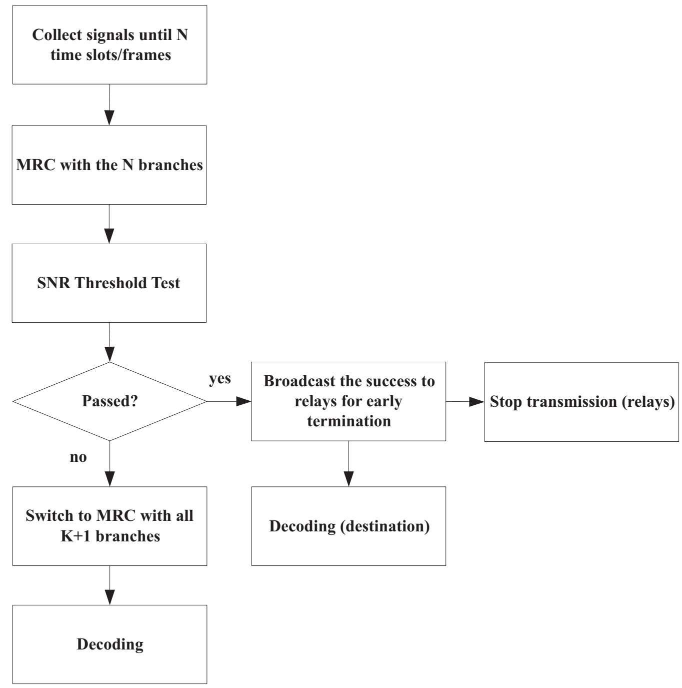

Fig. 2. The algorithm of proposed adaptive multi-node incremental relaying

(AWGN) at the node $i \sim \mathcal{CN}(0, N_0)$. In the kth slot/frame, the received signal at the destination node is given by

$$y_{k,d}[n+k] = h_{k,d}\alpha(h_{s,k}\sqrt{E_s}x[n] + z_k[n]) + z_{k,d}[n+k] \tag{3}$$

where $z_{k,d}[n]$ is an AWGN $\sim \mathcal{CN}(0, N_0)$ and α is the amplifier gain at each relay node. Since the available energy at each relay node is given by E_R, the amplifier gain α should satisfy

$$\alpha \leq \sqrt{\frac{E_R}{E_s|h_{s,k}|^2 + N_0}}. \tag{4}$$

C. Sufficient Statistic

The sufficient statistic of MRC only with the received signals in the first N slots/frames is given by [17]

$$\begin{aligned}\tilde{y}_N[n] = &\left(\frac{|h_{s,d}|^2 E_s}{N_0} + \sum_{k=1}^{N-1} \frac{|h_{s,k}|^2|h_{k,d}|^2\alpha^2 E_s}{N_0 + \alpha^2|h_{k,d}|^2 N_0}\right)x[n] \\ &+ \frac{h_{s,d}^*\sqrt{E_s}}{N_0}z_d[n] + \sum_{k=1}^{N-1}\frac{|h_{k,d}|^2\alpha^2\sqrt{E_s}h_{s,k}^*}{N_0 + \alpha^2|h_{k,d}|^2 N_0}z_k[n] \\ &+ \sum_{k=1}^{N-1}\frac{h_{s,k}^* h_{k,d}^*\alpha\sqrt{E_s}}{N_0 + \alpha^2|h_{k,d}|^2 N_0}z_{k,d}[n+k].\end{aligned} \tag{5}$$

It is known that SNR is maximized when equality holds in (4) but for analytical tractability we approximate α by $\alpha \approx \sqrt{\frac{E_R}{E_s|h_{s,k}|^2}}$. This approximation can be justified when the noise power is negligible compared to the signal power [17]. Also, it provides a lower bound on the outage probability. Then, the sufficient statistic after combining of signals in the first N time slots/frames is obtained by

$$\tilde{y}_N[n] = \left(\frac{|h_{s,d}|^2 E_s}{N_0} + \sum_{k=1}^{N-1} \frac{|h_{k,d}|^2 E_R}{N_k}\right) x[n] + \frac{h_{s,d}^* \sqrt{E_s}}{N_0} z_d[n] + \sum_{k=1}^{N-1} \left(\frac{|h_{k,d}|^2 E_R}{h_{s,k}\sqrt{E_s} N_k} z_k[n] + \frac{h_{s,k}^* h_{k,d}^* \sqrt{E_R}}{|h_{s,k}| N_k} z_{k,d}[n+k] \right) \quad (6)$$

where $N_k = N_0 + N_0 E_R |h_{k,d}|^2 / E_s |h_{s,k}|^2$. Then, SNR after combining is obtained by

$$\gamma_N = \tilde{\gamma}_0 + \sum_{k=1}^{N-1} \tilde{\gamma}_k \quad (7)$$

where

$$\tilde{\gamma}_0 = \frac{E_s |h_{s,d}|^2}{N_0},\ \tilde{\gamma}_k = \frac{\tilde{\gamma}_{s,k}\tilde{\gamma}_{k,d}}{\tilde{\gamma}_{s,k} + \tilde{\gamma}_{k,d}},$$
$$\tilde{\gamma}_{s,k} = \frac{E_s |h_{s,k}|^2}{N_0},\ \tilde{\gamma}_{k,d} = \frac{E_R |h_{k,d}|^2}{N_0}. \quad (8)$$

If we substitute $N-1$ with K in Eq. (7), the output SNR of MRC with all K relay paths and a direct path γ_{K+1} is obtained.

III. Outage Probability Analysis

The outage event is defined as the case that the achievable rate is below a threshold level. Since all signals over $K+1$ time slots/frames are collected and combined if the combined SNR over the first N time slots/frames falls below a threshold $\gamma_{th,e}$, the outage probability of the proposed adaptive multi-node incremental relaying is given by

$$\mathrm{P}_{\text{out}} = \mathrm{Prob}\left[\{\gamma_N \geq \gamma_{th,e}\} \cap \left\{\frac{1}{N}\log_2(1+\gamma_N) < R\right\}\right] + \mathrm{Prob}\left[\{\gamma_N < \gamma_{th,e}\} \cap \left\{\frac{1}{K+1}\log_2(1+\gamma_{K+1}) < R\right\}\right] \quad (9)$$

where R is the required data rate. The first term corresponds to the outage event when the SNR test passes and the second term corresponds to the outage event when the SNR test with the signals of the first N time slots/frames fails. It should also be noted here that since the cooperative communication is performed over N and $K+1$ time slots/frames, the achievable data rate is normalized by N and $K+1$, respectively.

Using the SNR, the outage probability can be rewritten by

$$\begin{aligned} P_{\text{out}} &= \mathrm{Prob}\left[\gamma_{th,e} \leq \gamma_N < \gamma_{th,N}\right] + \mathrm{Prob}\left[\{\gamma_N < \gamma_{th,e}\} \cap \{\gamma_N + \gamma_{K+1-N} < \gamma_{th,K+1}\}\right] \\ &= \mathrm{Prob}\left[\gamma_{th,e} \leq \gamma_N < \gamma_{th,N}\right] + \int_0^{\gamma_{th,e}} \mathrm{Prob}\left[\gamma_{K+1-N} < \gamma_{th,K+1} - \gamma\right] f_{\gamma_N}(\gamma) d\gamma \end{aligned} \quad (10)$$

where $\gamma_{th,N} = 2^{RN} - 1$ and $\gamma_{th,K+1} = 2^{R(K+1)} - 1$.

The distributions of γ_N and γ_{K+1-N} are obtained from the moment generating functions (MGFs)[1] The MGF of $\tilde{\gamma}_k$ is given by [17]

$$\mathcal{M}_{\tilde{\gamma}_k}(s) = \left(\frac{4p(g(s)-q)\sqrt{g(s)^2}}{g(s)\sqrt{4p - g(s)^2}} \mathrm{acos}\left(\frac{g(s)}{2\sqrt{p}}\right) + qg(s) - 4p \right) / \left(g(s)^2 - 4p\right) \quad (11)$$

where $p = \frac{E_s E_R \sigma_{s,k}^2 \sigma_{k,d}^2}{N_0^2}$ and $q = \frac{E_s \sigma_{s,k}^2 + E_R \sigma_{k,d}^2}{N_0}$. Therefore, $\mathcal{M}_{\gamma_N}(s)$ and $\mathcal{M}_{\gamma_{K+1-N}}(s)$ are obtained, respectively, by

$$\mathcal{M}_{\gamma_N}(s) = \mathcal{M}_{\tilde{\gamma}_0}(s) \prod_{k=1}^{N-1} \mathcal{M}_{\tilde{\gamma}_k}(s),\ \mathcal{M}_{\gamma_{K+1-N}}(s) = \prod_{k=N}^{K} \mathcal{M}_{\tilde{\gamma}_k}(s) \quad (12)$$

where $\mathcal{M}_{\tilde{\gamma}_0}(s)$ is the MGF of $\tilde{\gamma}_0$ given by

$$\mathcal{M}_{\tilde{\gamma}_0}(s) = \frac{N_0/(E_s \sigma_{s,d}^2)}{-s + N_0/(E_s \sigma_{s,d}^2)}. \quad (13)$$

From the MGFs, the PDFs of γ_N and γ_{K+1-N} are obtained, respectively, by

$$f_{\gamma_N}(x) = \mathcal{L}^{-1}\left\{ \mathcal{M}_{\gamma_0}(s) \prod_{k=1}^{N-1} \mathcal{M}_{\gamma_k}(s) \right\} \quad \text{and}$$
$$f_{\gamma_{K+1-N}}(y) = \mathcal{L}^{-1}\left\{ \prod_{k=N+1}^{K+1} \mathcal{M}_{\gamma_k}(s) \right\} \quad (14)$$

where $\mathcal{L}^{-1}(\cdot)$ is the inverse Laplace transform. CDFs can be obtained by the numerical inversion of the Laplace transform and the trapezoidal summation in [17], [18].

Using the CDFs of γ_N and γ_{K+1-N}, the outage probability of the proposed adaptive multi-node incremental relaying in Eq. (10) is given by

$$P_{\text{out}} = F_{\gamma_N}(\gamma_{th,N}) - F_{\gamma_N}(\gamma_{th,e}) + \int_0^{\gamma_{th,e}} F_{\gamma_{K+1-N}}(\gamma_{th,K+1} - \gamma)\, \mathcal{L}^{-1}\left\{ \mathcal{M}_{\gamma_0}(s) \prod_{k=1}^{N-1} \mathcal{M}_{\gamma_k}(s) \right\} d\gamma. \quad (15)$$

Now we analyze the diversity order of the proposed adaptive multi-node incremental relaying by using the following lemmas. We assume that $E_S = E_R = E$ and different average channel gains for each link.

***Lemma** 1 (A lower bound on P_{out}): For different average channel gains, the average SNR of the S-D link, the kth S-R link, and the kth R-D link are defined, respectively, as $\rho_{s,d} = \frac{\sigma_{s,d}^2 E}{N_0} \triangleq \rho_0$, $\rho_{s,k} = \frac{\sigma_{s,k}^2 E}{N_0}$, $\rho_{k,d} = \frac{\sigma_{k,d}^2 E}{N_0}$. Then, the outage probability in Eq. (15) is lower bounded on*

$$P_{\text{out}} \geq \sum_{i=0}^{N-1} c_i \rho_i \left(e^{-\frac{\gamma_{th,e}}{\rho_i}} - e^{-\frac{\gamma_{th,N}}{\rho_i}} \right) + \sum_{j=N}^{K} \sum_{i=0}^{N-1} c_i c_j \rho_j \left\{ \rho_i \left(1 - e^{-\frac{\gamma_{th,e}}{\rho_i}}\right) - \frac{\rho_i \rho_j}{\rho_j - \rho_i} e^{-\frac{\gamma_{th,K+1}}{\rho_j}} \left(1 - e^{-\frac{\rho_j - \rho_i}{\rho_i \rho_j}\gamma_{th,e}}\right) \right\} \quad (16)$$

[1]The moment generating function of a random variable γ is defined by $\mathcal{M}_\gamma(s) = \int_0^\infty f_\gamma(\gamma) e^{s\gamma} d\gamma$ where $f_\gamma(\gamma)$ is the PDF of the random variable γ.

where

$$c_i = \begin{cases} \prod_{k=0,k\neq i}^{N-1} \frac{1}{\rho_i-\rho_k} & \text{for } 0 \leq i \leq N-1 \\ \prod_{k=N,k\neq i}^{K} \frac{1}{\rho_i-\rho_k} & \text{for } N \leq i \leq K \end{cases} \quad \textit{and}$$

$$\frac{1}{\rho_k} = \frac{1}{\rho_{s,k}} + \frac{1}{\rho_{k,d}} \quad \textit{for } k \in \{1, \cdots, K\}.$$

Proof: See Appendix A. ■

***Lemma** 2 (An upper bound on P_{out}): For different average channel gains, the average SNR of the S-D link, the kth S-R link, and the kth R-D link are defined, respectively, as* $\rho_{s,d} = \frac{\sigma_{s,d}^2 E}{N_0} \triangleq \rho_0$, $\rho_{s,k} = \frac{\sigma_{s,k}^2 E}{N_0}$, $\rho_{k,d} = \frac{\sigma_{k,d}^2 E}{N_0}$. *Then, the outage probability in Eq. (15) is upper bounded on*

$$\begin{aligned} P_{\text{out}} &\leq c_0\rho_0\left(e^{-\frac{\gamma_{th,e}}{\rho_0}} - e^{-\frac{\gamma_{th,N}}{\rho_0}}\right) + \sum_{i=1}^{N-1} \frac{c_i\rho_i}{2}\left(e^{-\frac{2\gamma_{th,e}}{\rho_0}} - e^{-\frac{2\gamma_{th,N}}{\rho_0}}\right) \\ &+ \sum_{j=N}^{K} \frac{c_0 c_j \rho_j}{2}\left\{\rho_0\left(1 - e^{-\frac{\gamma_{th,e}}{\rho_0}}\right)\right. \\ &\left. - e^{-\frac{2\gamma_{th,K+1}}{\rho_j}} \frac{\rho_0\rho_j}{\rho_j - 2\rho_0}\left(1 - e^{-\frac{(\rho_j-2\rho_0)\gamma_{th,e}}{\rho_0\rho_j}}\right)\right\} \\ &- \sum_{j=N}^{K}\sum_{i=1}^{N-1} \frac{c_i c_j \rho_j}{2} e^{-\frac{2\gamma_{th,K+1}}{\rho_j}} \frac{\rho_i\rho_j}{2(\rho_j-\rho_i)}\left(1 - e^{-\frac{2(\rho_j-\rho_i)\gamma_{th,e}}{\rho_i\rho_j}}\right) \end{aligned} \quad (17)$$

where

$$c_0 = \prod_{k=1}^{N-1} \frac{2\rho_0}{2\rho_0 - \rho_k} \quad \textit{and}$$

$$c_i = \begin{cases} \frac{2\rho_i}{\rho_i - 2\rho_0} \prod_{k=1,k\neq i}^{N-1} \frac{\rho_i}{\rho_i-\rho_k} & \text{for } 1 \leq i \leq N-1 \\ \prod_{k=N,k\neq i}^{K} \frac{2\rho_i}{\rho_i-\rho_k} & \text{for } N \leq i \leq K. \end{cases}$$

Proof: See Appendix B. ■

From Lemma 1 and Lemma 2, we can derive the following theorem and conclude that the diversity order of the proposed adaptive multi-node incremental relaying is $K+1$.

***Theorem** 1 (Diversity order): The diversity order of the proposed adaptive multi-node incremental relaying with K AF relays is $K+1$ when the threshold test is perfect, i.e., $\gamma_{th,e} = \gamma_{th,N}$* [2].

Proof: See Appendix C. ■

IV. Numerical Results

In this section, outage probabilities of the proposed adaptive multi-node incremental relaying are compared with those of the conventional MRC in various environments. We assume that the available energy of each node is the same as $E_s = E_r = E$ as in the analysis part. The required spectral efficiency R is set to be 0.5 bps/Hz.

Fig. 3 shows the outage probabilities of the proposed adaptive multi-node incremental relaying with various N when $K = 10$ and $\gamma_{th,e} = \gamma_{th,N}$. The average channel gains $\sigma_{s,d}^2 = -10$ dB, $\{\sigma_{s,k}^2\}_{k=1}^{10} = \{-5, -4.8, -4.6, -4.4, -4.2, -4.0, -3.8, -3.6, -3.4, -3.2\}$ dB, and $\{\sigma_{k,d}^2\}_{k=1}^{10} = \{-3.1, -3.3, -3.5, -3.7, -3.9, -4.1, -4.3, -4.5, -4.7, -4.9\}$ dB. In Fig. 3, the proposed technique significantly outperforms the conventional MRC with order $K+1$(=11) in terms of outage probability owing to the early termination of transmission based on the threshold test. The adaptive multi-node incremental relaying with $N = 5$ shows the best performance. It is also confirmed that the analytical results slightly lower bound the simulation results since the noise power is neglected but the gaps are marginal. Based on the slope of outage probability curves, we can also verify that the diversity order of the proposed adaptive multi-node incremental relaying is $K+1$ as predicted in Theorem 1.

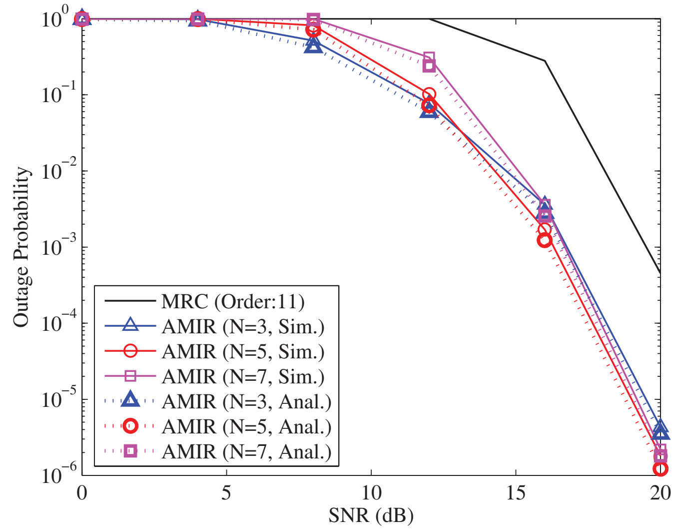

Fig. 3. Outage probabilities when $K = 10$ and $\gamma_{th,e} = \gamma_{th,N}$. The proposed adaptive multi-node incremental relaying significantly outperforms conventional MRC with order $K+1(=11)$.

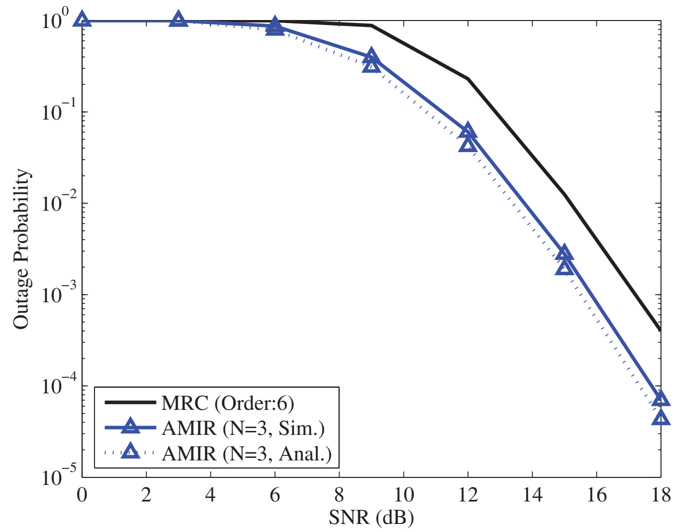

Fig. 4. Outage probabilities when $K = 5$ and $\gamma_{th,e} = \gamma_{th,N}$. The proposed adaptive multi-node incremental relaying outperforms conventional MRC but the performance gap is reduced compared to $K = 10$.

Fig. 4 shows the outage probabilities with the optimal selection of N when $K = 5$ and $\gamma_{th,e} = \gamma_{th,N}$. The average channel gains $\sigma_{s,d}^2 = -20$ dB, $\{\sigma_{s,k}^2\}_{k=1}^{5} = \{-5, -4.8, -4.6, -4.4, -4.2\}$ dB, and $\{\sigma_{k,d}^2\}_{k=1}^{5} = \{-4.1, -4.3, -4.5, -4.7, -4.9\}$ dB. The proposed adaptive multi-node incremental relaying scheme still outperforms the conventional MRC but the performance gap reduces as the number

[2]The SNR threshold $\gamma_{th,e}$ is typically set to be equal or similarly to $\gamma_{th,N}$ in order to minimize outage probability.

of relays decreases compared to the previous figure. It should also be noted that the gaps between the analytical results and simulation results are relatively larger than those when $K = 10$ since the negligence of the noise power is less effective as K becomes smaller.

V. Conclusions

Multiple relays are exploited to satisfy high QoS requirements not achievable by a single relay alone in cooperative communications. If amplify-and-forward (AF) relays are adopted, each relay amplifies and forwards the received signal from the source without any decoding. Consequently, the destination node combines the signals from all relays and a source node if there is no relay selection protocol which causes significant signaling overhead. The proposed adaptive multi-node incremental relaying effectively reduces the burden of MRC with all the diversity paths in AF based cooperative communications and improves the outage probability compared to conventional MRC with all diversity paths. Our analytical and simulation results have shown that the improvement by the proposed adaptive multi-node incremental relaying becomes larger as the number of relays increases. Our asymptotic analysis has proved that the proposed adaptive multi-node incremental relaying achieves full diversity order $K + 1$.

Appendix A
Proof of Lemma 1

The SNR after combining the signals over the first N time slots/frames in (7) is upper bounded on

$$\gamma_N \le \gamma_0 + \sum_{k=1}^{N-1} \min\{\gamma_{s,k},\ \gamma_{k,d}\} \tag{A.1}$$

from $\gamma_k = \frac{\gamma_{s,k}\gamma_{k,d}}{\gamma_{s,k}+\gamma_{k,d}} \le \min\{\gamma_{s,k},\ \gamma_{k,d}\}$. Since $\gamma_{s,d}$, $\gamma_{s,k}$ and $\gamma_{k,d}$ are exponential random variables ($\gamma_{s,d} \sim \exp(1/\rho_{s,d})$, $\gamma_{s,k} \sim \exp(1/\rho_{s,k})$, $\gamma_{k,d} \sim \exp(1/\rho_{k,d})$), $\min\{\gamma_{s,k},\ \gamma_{k,d}\}$ is an exponentially distributed random variable $\sim \exp\left(\frac{1}{\rho_{s,k}} + \frac{1}{\rho_{k,d}}\right)$ and its MGF is given by

$$\mathcal{M}_{\gamma_{\min}}(s) = \frac{\rho_k^{-1}}{-s + \rho_k^{-1}} \tag{A.2}$$

where $\frac{1}{\rho_k} = \frac{1}{\rho_{s,k}} + \frac{1}{\rho_{k,d}}$. Therefore, the PDF of the upper bound on γ_N in (A.1) is obtained by

$$\begin{aligned} f_{\gamma_{N,ub}}(\gamma) &= \mathcal{L}^{-1}\left\{\frac{\rho_0^{-1}}{-s+\rho_0^{-1}} \prod_{k=1}^{N-1} \frac{\rho_k^{-1}}{-s+\rho_k^{-1}}\right\} \\ &= \mathcal{L}^{-1}\left\{\frac{c_0}{-s+\rho_0^{-1}} + \frac{c_1}{-s+\rho_1^{-1}} + \cdots + \frac{c_{N-1}}{-s+\rho_{N-1}^{-1}}\right\} \end{aligned} \tag{A.3}$$

where $\rho_0 = \rho_{s,d}$ and the coefficients of the partial fractions are given by

$$c_i = \rho_i^{-1} \prod_{k=0,k\neq i}^{N-1} \frac{\rho_k^{-1}}{-\rho_i^{-1} + \rho_k^{-1}}, \quad i = 0, 1, \cdots, N-1. \tag{A.4}$$

The PDF and CDF of $\gamma_{N,ub}$ reduce, respectively, to

$$f_{\gamma_{N,ub}}(\gamma) = \sum_{i=0}^{N-1} c_i e^{-\frac{\gamma}{\rho_i}} \text{ and } F_{\gamma_{N,ub}}(\gamma) = \sum_{i=0}^{N-1} c_i\rho_i\left(1 - e^{-\frac{\gamma}{\rho_i}}\right) \tag{A.5}$$

from the following property of inverse Laplace transform:

$$\mathcal{L}^{-1}\left\{\left(\frac{1}{s+a}\right)^n\right\} = \frac{1}{(n-1)!}\gamma^{n-1}\exp(-a\gamma);\ \gamma \ge 0,\ n \ge 1. \tag{A.6}$$

Similarly, the PDF and CDF of $\gamma_{K+1-N,ub}$ are obtained, respectively, by

$$\begin{aligned} f_{\gamma_{K+1-N,ub}}(\gamma) &= \sum_{i=N}^{K} c_i e^{-\frac{\gamma}{\rho_i}}, \\ F_{\gamma_{K+1-N,ub}}(\gamma) &= \sum_{i=N}^{K} c_i\rho_i\left(1 - e^{-\frac{\gamma}{\rho_i}}\right). \end{aligned} \tag{A.7}$$

Substituting $F_{\gamma_N}(\gamma)$, $f_{\gamma_N}(\gamma)$, and $F_{\gamma_{K+1-N}}(\gamma)$ with $F_{\gamma_{N,ub}}(\gamma)$, $f_{\gamma_{N,ub}}(\gamma)$, and $F_{\gamma_{K+1-N,ub}}(\gamma)$, respectively, the outage probability in (15) is lower bounded on

$$\begin{aligned} P_{\text{out}} \ge& \sum_{i=0}^{N-1} c_i\rho_i\left(e^{-\frac{\gamma_{th,e}}{\rho_i}} - e^{-\frac{\gamma_{th,N}}{\rho_i}}\right) \\ &+ \sum_{j=N}^{K}\sum_{i=0}^{N-1} c_ic_j\rho_j\Bigg\{\rho_i\left(1 - e^{-\frac{\gamma_{th,e}}{\rho_i}}\right) \\ &- e^{-\frac{\gamma_{th,K+1}}{\rho_j}}\frac{\rho_i\rho_j}{\rho_j - \rho_i}\left(1 - e^{-\frac{\rho_j-\rho_i}{\rho_i\rho_j}\gamma_{th,e}}\right)\Bigg\}. \end{aligned} \tag{A.8}$$

Appendix B
Proof of Lemma 2

The SNR after combining the signals over the first N time slots/frames in (7) is lower bounded on

$$\gamma_N \ge \gamma_0 + \sum_{k=1}^{N-1} \frac{1}{2}\min\{\gamma_{s,k},\ \gamma_{k,d}\} \tag{B.1}$$

from $\gamma_k = \frac{\gamma_{s,k}\gamma_{k,d}}{\gamma_{s,k}+\gamma_{k,d}} \ge \frac{1}{2}\min\{\gamma_{s,k},\ \gamma_{k,d}\}$. It is known that $\frac{1}{2}\min\{\gamma_{s,k},\ \gamma_{k,d}\}$ is an exponentially distributed random variable $\sim \exp\left(\frac{2}{\rho_{s,k}} + \frac{2}{\rho_{k,d}}\right)$ and the PDF of the lower bound on γ_N in (B.1) is obtained by

$$\begin{aligned} f_{\gamma_{N,lb}}(\gamma) &= \mathcal{L}^{-1}\left\{\frac{\rho_0^{-1}}{-s+\rho_0^{-1}} \prod_{k=1}^{N-1} \frac{2\rho_k^{-1}}{-s+2\rho_k^{-1}}\right\} \\ &= \mathcal{L}^{-1}\left\{\frac{c_0}{-s+\rho_0^{-1}} + \frac{c_1}{-s+2\rho_1^{-1}} + \cdots + \frac{c_{N-1}}{-s+2\rho_{N-1}^{-1}}\right\} \end{aligned} \tag{B.2}$$

where the coefficients of the partial fractions are given by

$$\begin{aligned} c_0 &= \rho_0^{-1} \prod_{k=1}^{N-1} \frac{2\rho_k^{-1}}{-\rho_0^{-1} + 2\rho_k^{-1}}, \\ c_i &= \frac{2\rho_0^{-1}\rho_i^{-1}}{-2\rho_i^{-1}+\rho_0^{-1}} \prod_{k=1,k\neq i}^{N-1} \frac{\rho_k^{-1}}{-\rho_i^{-1}+\rho_k^{-1}} \text{ for } i = 1, \cdots, N-1. \end{aligned} \tag{B.3}$$

The PDF and CDF of $\gamma_{N,lb}$ are obtained, respectively, by

$$f_{\gamma_{N,lb}}(\gamma) = c_0 e^{-\frac{\gamma}{\rho_0}} + \sum_{i=1}^{N-1} c_i e^{-\frac{2\gamma}{\rho_i}} \quad \text{and}$$

$$F_{\gamma_{N,lb}}(\gamma) = c_0\rho_0\left(1 - e^{-\frac{\gamma}{\rho_0}}\right) + \sum_{i=1}^{N-1} c_i \frac{\rho_i}{2}\left(1 - e^{-\frac{2\gamma}{\rho_i}}\right). \tag{B.4}$$

Similarly, the PDF and CDF of $\gamma_{K+1-N,lb}$ are obtained, respectively, by

$$f_{\gamma_{K+1-N,lb}}(\gamma) = \sum_{i=N}^{K} c_i e^{-\frac{2\gamma}{\rho_i}} \quad \text{and}$$

$$F_{\gamma_{K+1-N,lb}}(\gamma) = \sum_{i=N}^{K} c_i \frac{\rho_i}{2}\left(1 - e^{-\frac{2\gamma}{\rho_i}}\right) \tag{B.5}$$

where $c_i = 2\rho_i^{-1}\prod_{k=N,k\neq i}^{K}\frac{\rho_k^{-1}}{-\rho_i^{-1}+\rho_k^{-1}}$ for $i = N, \cdots, K$.

Then, the outage probability is upper bounded on

$$\begin{aligned}P_{\text{out}} \leq\ & c_0\rho_0\left(e^{-\frac{\gamma_{th,e}}{\rho_0}} - e^{-\frac{\gamma_{th,N}}{\rho_0}}\right) \\ &+ \sum_{i=1}^{N-1} c_i\frac{\rho_i}{2}\left(e^{-\frac{2\gamma_{th,e}}{\rho_0}} - e^{-\frac{2\gamma_{th,N}}{\rho_0}}\right) \\ &+ \sum_{j=N}^{K}\frac{c_0 c_j\rho_j}{2}\left\{\rho_0\left(1 - e^{-\frac{\gamma_{th,e}}{\rho_0}}\right)\right. \\ &\left. - e^{-\frac{2\gamma_{th,K+1}}{\rho_j}}\frac{\rho_0\rho_j}{\rho_j - 2\rho_0}\left(1 - e^{-\frac{(\rho_j-2\rho_0)\gamma_{th,e}}{\rho_0\rho_j}}\right)\right\} \\ &- \sum_{j=N}^{K}\sum_{i=1}^{N-1}\frac{c_i c_j\rho_j}{2}e^{-\frac{2\gamma_{th,K+1}}{\rho_j}} \\ &\cdot\frac{\rho_i\rho_j}{2(\rho_j-\rho_i)}\left(1 - e^{-\frac{2(\rho_j-\rho_i)\gamma_{th,e}}{\rho_i\rho_j}}\right). \end{aligned} \tag{B.6}$$

Appendix C
Proof of Theorem 1

When E is sufficiently large ($\rho_i \to \infty$, $\forall i$) and $\gamma_{th,e} = \gamma_{th,N}$, the first term in the lower bound in (16) becomes zero and hence,

$$\begin{aligned}\lim_{\rho_i\to\infty\ \forall i} P_{\text{out}} \geq\ & \sum_{i=0}^{N-1} c_i\rho_i\underbrace{\left(e^{-\frac{\gamma_{th,e}}{\rho_i}} - e^{-\frac{\gamma_{th,N}}{\rho_i}}\right)}_{=0} \\ &+ \sum_{j=N}^{K}\sum_{i=0}^{N-1} c_i c_j\rho_j\left[\rho_i\left(1 - e^{-\frac{\gamma_{th,e}}{\rho_i}}\right)\right. \\ &\left. - e^{-\frac{\gamma_{th,K+1}}{\rho_j}}\frac{\rho_i\rho_j}{\rho_j-\rho_i}\left(1 - e^{-\frac{\rho_j-\rho_i}{\rho_i\rho_j}\gamma_{th,e}}\right)\right] \\ =\ & \sum_{j=N}^{K}\sum_{i=0}^{N-1} c_i c_j\rho_j\left[\underbrace{\rho_i\left(1 - e^{-\frac{\gamma_{th,e}}{\rho_i}}\right)}_{=\gamma_{th,e}+O\left(\frac{1}{E}\right)}\right. \\ &\left. - e^{-\frac{\gamma_{th,K+1}}{\rho_j}}\underbrace{\frac{\rho_i\rho_j}{\rho_j-\rho_i}\left(1 - e^{-\frac{\rho_j-\rho_i}{\rho_i\rho_j}\gamma_{th,e}}\right)}_{=\gamma_{th,e}+O\left(\frac{1}{E}\right)}\right] \end{aligned} \tag{C.1}$$

$$= \sum_{j=N}^{K}\sum_{i=0}^{N-1} c_i c_j\left(\gamma_{th,e} + O\left(\frac{1}{E}\right)\right)\rho_j\left(1 - e^{-\frac{\gamma_{th,K+1}}{\rho_j}}\right) \tag{C.2}$$

$$= \sum_{j=N}^{K} c_j\left(\gamma_{th,K+1} + O\left(\frac{1}{E}\right)\right)\sum_{i=0}^{N-1} c_i\left(\gamma_{th,e} + O\left(\frac{1}{E}\right)\right) \tag{C.3}$$

where the notation $g(x) = O(f(x))$ or $(g(x) \sim f(x))$ denotes $\lim_{x\to\infty \text{ or } 0}\frac{g(x)}{f(x)} = c$, $0 < |c| < \infty$ and both (C.2) and (C.3) come from the fact that $\exp(-x) = 1 - x + O(x^2)$ when x is sufficiently small.

In Lemma 1, the coefficients c_i is given by $c_i = \prod_{k=0,k\neq i}^{N-1}\frac{1}{\rho_i-\rho_k} = O\left(\frac{1}{E^{N-1}}\right)$ for $i \in \{0, \cdots, N-1\}$ and $\sum_{i=0}^{N-1} c_i = 0$ from (A.3). Therefore, the term in (C.3) becomes

$$\begin{aligned}\sum_{i=0}^{N-1} c_i\left(\gamma_{th,e} + O\left(\frac{1}{E}\right)\right) &= \gamma_{th,e}\sum_{i=0}^{N-1} c_i + \sum_{i=0}^{N-1} c_i O\left(\frac{1}{E}\right) \\ &= O\left(\frac{1}{E^N}\right). \end{aligned} \tag{C.4}$$

Similarly, the first term in (C.3) becomes

$$\sum_{j=N}^{K} c_j\left(\gamma_{th,K+1} + O\left(\frac{1}{E}\right)\right) = O\left(\frac{1}{E^{K-N+1}}\right). \tag{C.5}$$

Therefore, substituting (C.4) and (C.5) with (C.3), the order of the lower bound on outage probability in the high SNR region is obtained by

$$\lim_{E\to\infty} P_{\text{out}} \geq O\left(\frac{1}{E^{K+1}}\right) \tag{C.6}$$

On the other hand, when E is sufficiently large, the upper bound on the outage probability in Lemma 2 reduces to

$$\begin{aligned}&\lim_{\rho_i\to\infty,\ \forall i} P_{\text{out}} \\ &\leq c_0\rho_0\underbrace{\left(e^{-\frac{\gamma_{th,e}}{\rho_0}} - e^{-\frac{\gamma_{th,N}}{\rho_0}}\right)}_{\to 0} + \sum_{i=1}^{N-1} c_i\frac{\rho_i}{2}\underbrace{\left(e^{-\frac{2\gamma_{th,e}}{\rho_0}} - e^{-\frac{2\gamma_{th,N}}{\rho_0}}\right)}_{\to 0} \\ &\quad + \sum_{j=N}^{K}\frac{c_0 c_j\rho_j}{2}\left[\underbrace{\rho_0\left(1 - e^{-\frac{\gamma_{th,e}}{\rho_0}}\right)}_{=\gamma_{th,e}+O\left(\frac{1}{E}\right)}\right. \\ &\qquad \left. - e^{-\frac{2\gamma_{th,K+1}}{\rho_j}}\underbrace{\frac{\rho_0\rho_j}{\rho_j - 2\rho_0}\left(1 - e^{-\frac{(\rho_j-2\rho_0)\gamma_{th,e}}{\rho_0\rho_j}}\right)}_{=\gamma_{th,e}+O\left(\frac{1}{E}\right)}\right] \\ &\quad - \sum_{j=N}^{K}\sum_{i=1}^{N-1}\frac{c_i c_j\rho_j}{2}e^{-\frac{2\gamma_{th,K+1}}{\rho_j}}\underbrace{\frac{\rho_i\rho_j}{2(\rho_j-\rho_i)}\left(1 - e^{-\frac{2(\rho_j-\rho_i)\gamma_{th,e}}{\rho_i\rho_j}}\right)}_{=\gamma_{th,e}+O\left(\frac{1}{E}\right)} \\ &= \sum_{j=N}^{K}\frac{c_j}{2}\left(\gamma_{th,e} + O\left(\frac{1}{E}\right)\right) \\ &\quad \cdot\underbrace{\left[c_0\left(2\gamma_{th,K+1} + O\left(\frac{1}{E}\right)\right) - \rho_j e^{-\frac{2\gamma_{th,K+1}}{\rho_j}}\sum_{i=1}^{N-1} c_i\right]}_{=c_0\left(O\left(\frac{1}{E}\right)+\rho_j+\rho_j O\left(\frac{1}{E^2}\right)\right)} \end{aligned} \tag{C.7}$$

$$= \sum_{j=N}^{K} \frac{c_0 c_j}{2} \left(\gamma_{th,e} + O\left(\frac{1}{E}\right)\right) O\left(\frac{1}{E}\right) \tag{C.8}$$

where the last equality comes from the facts that $\sum_{i=0}^{N-1} c_i = 0$ and $\rho_j = O(E)$.

The coefficients c_j are given in Lemma 2 and $\sum_{j=N}^{K} c_j = 0$ so that the order of the upper bound on outage probability in the high SNR region is obtained by

$$\begin{aligned} \lim_{E\to\infty} P_{\text{out}} &\le \sum_{j=N}^{K} c_0 c_j O\left(\frac{1}{E^2}\right) \\ &= O\left(\frac{1}{E^{N-1}}\right) O\left(\frac{1}{E^{K-N}}\right) O\left(\frac{1}{E^2}\right) \\ &= O\left(\frac{1}{E^{K+1}}\right) \end{aligned} \tag{C.9}$$

From (C.6) and (C.9), it can be verified that the outage probability decays as fast as $\left(\frac{1}{E}\right)^{K+1}$ in the high SNR region and hence the diversity order of the proposed adaptive multi-node incremental relaying is $K+1$.

Acknowledgement

The authors would like to thank to Mr. JunPyo Hong for his help in simulations. The authors also appreciate the helpful comments of anonymous reviewers which significantly improved the quality of this paper.

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—part I: system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927-1938, Nov. 2003.

[2] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74-80, Oct. 2004.

[3] T. M. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572-584, Sept. 1979.

[4] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.

[5] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[6] T. E. Hunter, S. Sanayei, and A. Nosratinia, "Outage analysis of coded cooperation," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 375-391, Feb. 2006.

[7] A. Bletsas, A. Khisiti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659-672, Mar. 2006.

[8] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450-3460, Sept. 2007.

[9] D. Chen and J. N. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1785-1794, July 2006.

[10] T. W. Ban, W. Choi, B. C. Jung, and D. K. Sung, "A cooperative phase steering scheme in multi-relay node environments," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 72-77, Jan. 2009.

[11] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1099-1107, Aug. 2004.

[12] P. A. Anghel, G. Leus, and M. Kaveh, "Distributed space-time cooperative systems with regenerative relays," *IEEE Trans. Wireless Commun.*, vol. 5, no. 11, pp. 3130-3141, Nov. 2006.

[13] M. Janani, A. Hedayat, T. E. Hunter, and A. Nosratinia, "Coded cooperation in wireless communications: space-time transmission and iterative decoding," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 362-371, Feb. 2004.

[14] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524-3536, Dec. 2006.

[15] R. Mudumbai, G. Barriac, and U. Madhow, "On the feasibility of distributed beamforming in wireless networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1754-1763, May 2007.

[16] D. I. Kim, "Adaptive selection/maximal ratio combining based on error detection of multidimensional multicode DS-CDMA," *IEEE Trans. Commun.*, vol. 52, no. 3, pp. 446-456, Mar. 2004.

[17] P. A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1416-1421, Sept. 2004.

[18] Y. C. Ko, M. S. Alouini, and M. K. Simon, "Outage probability of diversity systems over generalized fading channels," *IEEE Trans. Commun.*, vol. 48, no. 11, pp. 1783-1787, Nov. 2000.