Multiuser Performance of
$M$-ary Orthogonal Coded/Balanced
UWB Transmitted-Reference Systems

Dong In Kim, Senior Member, IEEE

Abstract—A tractable and compact closed-form expression on the channel-averaged signal-to-interference-plus-noise ratio (SINR) is derived for $M$-ary orthogonal coded/balanced transmitted-reference (BTR) systems, taking into consideration both inter-pulse interference (IPI) and multiple-access interference (MAI) in dense multipath ultra-wideband (UWB) channels. The UWB channel here is a realistic and standard one with lognormal channel gain distribution and double independent Poisson arrival distribution of cluster and ray. Hence, the analytical framework developed here can be applied to typical UWB channel models, especially considering the channel sparseness and cluster overlapping observed in realistic UWB channels. Based on the channel-averaged SINR, the effect of inter-pulse distance (between the reference and data pulses in BTR) on the multiuser performance is fully investigated. A proper selection of user-specific inter-pulse distances in multiuser scenario is then determined to maximize the user capacity for a given UWB channel.

Index Terms—Balanced transmitted-reference (BTR), ultra wideband (UWB), inter-pulse distance, inter-pulse interference (IPI), channel-averaged SINR, multiuser performance.

I. INTRODUCTION

RECENTLY there have been many proposals to implement a cost-effective transceiver for ultra-wideband (UWB) communications, which deals with dense multipath in indoor wireless channels [1]. First of all, the Rake receiver such as partial Rake was deemed as an effective means to collect the signal energies spread over multipath and combine them for the following signal detection [2]. In most cases, however, the number of multipaths combined should be sufficient enough to provide a certain signal-to-noise ratio (SNR) for reliable UWB communications, where the transmission of very narrow width pulses (i.e., sub-nanoseconds) does not contain enough energy per path per pulse. Meanwhile, if we increase the number of multipaths (or fingers) combined to yield enough SNR, it simply increases the power consumption of the receiver, which cannot be tolerated. Moreover, it places undue burden on the channel estimation, and its accuracy should be sacrificed due to the limited path energy per pulse, resulting in degraded performance.

This has motivated the use of a simple receiver structure known as transmitted-reference (TR) whose original form was proposed in [3] five decades ago. The first practical TR receiver for UWB systems has been reported in [4] where a pair of pulses are transmitted in each frame. Here, the preceding unmodulated pulse is used as a reference pulse for the following data pulse that sends information in terms of its polarity or position. In fact, it preserves a similar aspect to differential detection that suffers noise enhancement due to nonlinear cross-correlation operation on noise. Furthermore, additional transmission of the reference pulse that facilitates the channel estimation causes a loss in power efficiency of the UWB radio, which cannot again be tolerated. To overcome noise enhancement and power inefficiency, modified transceiver structures were proposed.

A recent transceiver design for UWB-TR systems one hand attempted to have a cleaner template (i.e., reference) [5], [6] to be used for the cross-correlation, so as to reduce the noise enhancement. On the other hand, differential TR at both frame level [7] and symbol level [8] was designed to overcome the power inefficiency, though the receiver complexity is undue increased because of the long delay line used. In the former, more general optimal and suboptimal TR receivers have been proposed in [9], [10] that require a prior estimation of the template, where a kind of symbol-rate matched filtering is performed before the cross-correlation. But, these approaches may not be feasible when the channel introduces per-path pulse distortion [11], making the prior estimation imprecise and causing a residual distortion to typical UWB radios. Besides, some of the proposals addressed the issue of inter-pulse interference (IPI) that arises when the separation between the pair of pulses is less than the delay spread [12]. To cancel out the IPI and also to increase the data rate with shorter inter-pulse distance, a dual pulse transmission scheme was proposed in [13]. Further, a balanced TR signaling scheme is designed that combines the dual pulse with $M$-ary orthogonal encoding [14], which aims to mitigate both the IPI and the multiple-access interference (MAI) by increasing the frame time without sacrificing the data rate. This increased frame time (or processing gain) can also be effective against another type of interference such as narrowband interference, whose detailed analysis can be found in [15].
More recent contribution [16] proposed an accurate analytical framework for TR performance analysis, but it assumed simple dense multipath channel without considering the sparseness of realistic UWB channels. This condition is crucial in analyzing the TR performance as a function of the integration time because the receiver collects only noise, but no signal energy, in the empty time bin. In this paper, using the balanced TR (BTR) signaling, the multiuser operation of UWB radios is investigated in realistic UWB channels following lognormal channel gain distribution and double independent Poisson arrival distribution of cluster and ray [17]. The analysis in [6] also considered the energy captured by the channel, but it needs to be evaluated by the simulation for the assumed UWB channels.

So far, a few papers addressed the multiuser performance in conjunction with TR receiver, mainly because the analysis becomes less tractable through a tedious evaluation of the MAI statistics after nonlinear operation on MAI. However, the prior works [18], [19] evaluated the multiuser performance in a semi-analytic way that needs a sufficient number of UWB channel realizations when taking the channel average. Since the author in [20] first analyzed the signal-to-interference-plus-noise ratio (SINR) in CDMA, there have been lots of further efforts to characterize the receiver statistics in more realistic channel and interference conditions [21]. The authors in [22] have first evaluated the channel-averaged SINR in UWB pure analytically (i.e., not semi-analytically), considering the sparseness of realistic UWB channels. This paper further generalizes the above channel-averaged SINR expression in tractable and compact closed-form, especially when the interpulse distance is shorter than the delay spread (i.e., the IPI is present). This condition is necessary and inevitable in implementing a practical TR receiver. It should be pointed out that the analytical framework being developed here for the BTR system can easily be modified for the analysis of other UWB-TR systems.

The rest of the paper is organized as follows. In Section II, M-ary orthogonal coded BTR system and indoor UWB channel model are described. A theoretical framework is developed in Section III to evaluate the channel-averaged SINR for the considered BTR system. In Section IV, the analytical results on the SINR are validated through simulations under various interference and channel conditions. Concluding remarks are given in Section V.

II. SYSTEM MODEL

A. M-ary orthogonal coded BTR signaling

The proposed signaling is designed first to cancel out the IPI between a pair of pulses of desired user received through a multipath channel and then to properly mitigate the MAI by differentiating the inter-pulse distances among different users with TH multiple access. To this end, a balanced TR signaling for the \( k \)th (\( k = 1, 2, \ldots, K \)) user is designed as

\[
s^{(k)}(t) = \sqrt{\frac{E_s}{2N_s}} \sum_{j=-\infty}^{+\infty} d^{(k)}_j \left[ p(t - jT_f - c^{(k)}_j T_c) + (-1)^j b^{(k)}_{j/N_s} \nu^{(k)}_{j/2} p(t - jT_f - c^{(k)}_j T_c - T_d^{(k)}) \right]
\]

where \( E_s \) is the symbol energy, \( N_s \) is the number of frames sent during one symbol time, \( p(t) \) is the unit-energy transmitted pulse of duration \( T_w \), and a pair of pulses are transmitted in the frame time \( T_f \), the first being used as the reference pulse while the second, which is transmitted \( T_d^{(k)} \) seconds later, as the data pulse carrying information. Here, the symbol energy is halved evenly between the reference and data pulses.

To shape the transmit signal spectrum according to the FCC spectral mask and achieve the TH multiple access, the TH code \( c^{(k)}_j \) is weighted by the modulation signals, e.g., \( c^{(k)}_j \in \{0, 1, \ldots, N_h^{(k)} - 1\} \). Here, the frame time \( T_f \) is assumed fixed for all users and meets the constraint \( T_f \geq T_d^{(k)} + T_{mds} + N_h^{(k)} T_c \) to avoid inter-frame interference (IFI), where \( T_c \) denotes the chip time with \( T_c \geq T_w \) and \( T_{mds} \) represents the channel delay spread. Moreover, if the multipath-delayed reference and data pulses across different users overlap within the inter-pulse distance \( T_d^{(k)} \), they will likely cause the worst-case MAI. Hence, it is required to differentiate the inter-pulse distances among users subject to the above constraint, in order to effectively reduce the MAI, as proposed in [18]

\[
T_d^{(k)} = T^{(1)}_d + n(k - 1)T_w
\]

\[
N_h^{(k)} = N_h^{(K)} + \lfloor n(K - k) \rfloor
\]

where \( n = 1/2, 1, 2, \ldots \) are assumed to adequately separate different user pulses.

The data pulse is weighted by the modulation signals, such as i) the alternating sign of \((-1)^j\) to allow for the IPI cancellation over two consecutive frames, ii) the bi-phase signal \( b^{(k)}_j \in \{-1, +1\} \) to send one-bit information (\(|x|\) is the integer part of \( x \)), and iii) the \( M \)-ary orthogonal sequence \( \nu^{(k)}_{N_s/2}, \nu^{(k)}_{N_s/2+1}, \ldots, \nu^{(k)}_{N_s/2+M-1} \) to carry additional \( \log_2 M \)-bit information at the \( k \)th signaling time where \( N_s = n(2M) \) for an integer \( n \). Note that if \( n > 1 \), the orthogonal sequence repeats \( n \) times over \( N_s \) frames per symbol, and it can be chosen from any row vector \( h_m \) of the Hadamard matrix \( H_M \) of size \( M \times M \), recursively generated with initial value \( H_1 = 1 \).

Fig. 1 illustrates the balanced TR signaling designed to send the \( (1 + \log_2 M) \)-bit information per symbol when \( M = 2 \) and \( N_s = 4 \), assuming the inter-pulse distance \( T_d^{(k)} = T_c \).

B. Indoor UWB channel model

A K-user asynchronous transmission is assumed along with multipath fading, which can be modeled as

\[
h^{(k)}(t - \tau^{(k)}) = \sum_{l=0}^{L-1} a^{(k)}_l \delta(t - \tau^{(k)} - lT_c)
\]

where \( L \) is the number of multipaths, \( \tau^{(k)} \) denotes the asynchronous delay associated with user \( k \), uniformly distributed over \([0, N_s T_f])\), and the discrete-time equivalent channel impulse response (CIR) \( h^{(k)}(t) \) is obtained from the IEEE 802.15.3a standard UWB channel model [17].
As a realistic UWB channel model, the discrete-time equivalent CIR is generated as follows:

\[ h^{(k)}(t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \alpha_{n,m} \delta(t - T_m - \tau_{n,m}) \]  

where \( \alpha_{n,m} \) is the channel gain coefficient of \( n \)th ray in \( m \)th cluster and \( \tau_{n,m} \) is the arrival time of \( n \)th ray relative to \( m \)th cluster’s arrival time \( T_m \). Here, the inter-cluster and inter-ray arrival times are independently exponentially distributed.

\[ \alpha_{n,m} = p_{n,m} \beta_{n,m} \]  

where \( p_{n,m} \) is a uniform random variable, denoted by \( 20 \log_{10}(\beta_{n,m}) \sim N(\mu_{n,m}, \sigma^2) \). The power profile is double exponentially decaying, given by

\[ E\{\beta_{n,m}^2\} = \Omega_0 \exp(-T_m/\Gamma) \exp(-\tau_{n,m}/\gamma) \]  

(5)

where \( \Omega_0 \) is the mean power of the first ray of the first cluster (i.e., \( m = n = 0 \), \( \Gamma \) and \( \gamma \) represent the power decay factors of the cluster and ray, respectively.

In the \( l \)th time bin \( [lT_c, (l+1)T_c) \), one or more multipath components (MPCs) can arrive because of cluster overlapping, or no arrival at all (channel sparseness). The channel gain coefficients of all the arrived MPCs within that time bin are linearly combined to yield a composite channel gain coefficient \( a_k^{(l)} \) in (4). It is to be noted that \( a_k^{(l)} \) could be zero if there is no MPC in the \( l \)th time bin, and the channel delay spread \((i.e., T_{mds} = LT_c)\) is a variable depending on the type of IEEE channel models such as CM1 to CM4, where \( \Omega_0 \) is set to meet \( \sum_{l=0}^{L-1} E\{a_k^{(l)^2}\} = 1 \) for all \( k \).

C. BTR receiver with IPI cancellation

The received and ideally filtered signal after passing through the UWB channel is of the form

\[ r(t) = \sqrt{\frac{E_s}{2N_s}} \sum_{j=-\infty}^{+\infty} d_j^{(1)} \left[ g_j^{(1)}(t - jT_f - c_j^{(1)}T_c) + (-1)^j b_{j/N_s}^{(1)} \nu_{j/2}^{(1)} g_j^{(1)}(t - jT_f - c_j^{(1)}T_c - T_{d}^{(1)}) \right] + i(t) + n(t) \]  

(6)

where the first term is desired signal, assuming perfect synchronization with \( \tau^{(1)} = 0 \), the second and third terms represent the MAI and additive filtered Gaussian noise with power spectral density of \( N_0/2 \) over \([-W, W]\), respectively, and

\[ i(t) = \sqrt{\frac{E_s}{2N_s}} \sum_{k=0}^{+\infty} \sum_{j=-\infty}^{+\infty} d_j^{(1)} \left[ g_j^{(1)}(t - \tau^{(k)} - jT_f - c_j^{(1)}T_c) + (-1)^j b_{j/N_s}^{(1)} \nu_{j/2}^{(1)} g_j^{(1)}(t - \tau^{(k)} - jT_f - c_j^{(1)}T_c - T_{d}^{(1)}) \right] + i(t) + n(t) \]  

(7)

for \( g_j^{(1)}(t - \tau^{(k)}) = \sum_{l=0}^{L-1} a_l^{(1)} p(t - \tau^{(k)} - lT_c) \).

Knowing that the BTR signal alternates the sign of the data pulses over two consecutive frames, the two adjacent frames are perfectly aligned, and they are linearly added and subtracted to produce the frame-averaged reference and data waveforms as

\[ \rho_{j+1}(t) = \frac{1}{2} \left[ d_{j-1}^{(1)} r(t - T_f - \Delta_j^{(1)}) + d_j^{(1)} r(t) \right] \]

\[ = \frac{E_s}{2N_s} g_j^{(1)}(t - jT_f - c_j^{(1)}T_c) + i_j(t) + n_j(t) \]  

(8)

\[ \rho_{j-1}(t) = \frac{1}{2} \left[ d_{j+1}^{(1)} r(t - T_f - \Delta_j^{(1)}) - d_j^{(1)} r(t) \right] \]

\[ = \frac{E_s}{2N_s} b_{j/N_s}^{(1)} \nu_{j/2}^{(1)} g_j^{(1)}(t - jT_f - c_j^{(1)}T_c - T_{d}^{(1)}) + i_{j-1}(t) + n_{j-1}(t) \]  

(9)

where \( \Delta_j^{(1)} = [c_j^{(1)} - c_{j-1}^{(1)}] T_c \) is a time-offset to align the two frames for \( j = \pm 1, \pm 3, \ldots \), and the \( j \)th frame interference and noise terms are given by

\[ i_j(t) = \frac{1}{2} \left[ d_{j-1}^{(1)} i(t - T_f - \Delta_j^{(1)}) + d_j^{(1)} i(t) \right] \]  

(10)

\[ n_j(t) = \frac{1}{2} \left[ d_{j-1}^{(1)} n(t - T_f - \Delta_j^{(1)}) + d_j^{(1)} n(t) \right] \]  

(11)

It is seen that the resulting waveforms \( [\rho_{j+1}(t), \rho_{j-1}(t)] \) completely isolate the reference and data signals of the first user, regardless of the inter-pulse distance \( T_{d}^{(1)} \). This leads to the IPI cancellation of the desired first user signal. Hence, a shorter distance much less than \( T_{mds} \) can be employed for an increased information rate, unlike conventional TR that requires \( T_{d}^{(1)} \geq T_{mds} \).

After the IPI cancellation, the two waveforms are applied to an analog cross-correlator that collects the multipath energies over an integration time \( T_{corr} \), giving rise to the \( j \)th frame decision statistics as

\[ \zeta_j = \int_0^{T_{corr}} \rho_{j+1}(t - T_{d}^{(1)}) \rho_{j-1}(t) dt \]  

(12)
Fig. 2. BTR receiver structure enabling the IPI cancellation and M-ary symbol/bi-phase bit detection, where the IPI cancellation (requiring $T_f$ long delay line) can be replaced with a short delay line of $T_f^{(1)}$ for multiuser operation.

for $j = i N_s + 1, i N_s + 3, \ldots, (i + 1) N_s - 1$, where $\Phi_j \triangleq [jT_f + c_j^{(1)}T_c + T_d^{(1)}, jT_f + c_j^{(1)}T_c + T_d^{(1)} + T_{corr}]$. Then, M-ary symbol decision variables are formed in digital domain as a linear sum, that yields the total energy collected over a symbol

$$\xi_m = \sum_{n=i N_s/2}^{(i+1)N_s/2-1} h_m, n \xi_{2n+1} = s_m + N_{1,m} + N_{2,m} + I_{1,m}$$

$$+ I_{2,m} + I_{3,m} \quad (13)$$

for $m = 0, 1, \ldots, M - 1$, where $h_m = (h_{m,0}, h_{m,1}, \ldots, h_{m,M-1})$ is the $n$th row vector of $H_M$ and $n = n$ modulo $M$. In the above, $s_m$ is the desired signal, $N_{1,m}$ and $N_{2,m}$ represent the noise-times-noise and noise-times-signal, while $I_{1,m}$, $I_{2,m}$ and $I_{3,m}$ the MAI-times-noise, MAI-times-MAI and MAI-times-signal, respectively.

Next, the decision variables $\{\xi_m \mid m = 0, 1, \ldots, M - 1\}$ are compared to detect additional $\log_2 M$-bit information via the index $q$ of the absolute maximum, corresponding to

$$|\xi_q| = \max_{0 \leq m \leq M - 1} \{|\xi_m|\} \quad (14)$$

and the primary one-bit information is detected by $\hat{b}^{(1)}_i = \text{sign}(\xi_q)$.

Fig. 2 shows the balanced TR receiver structure to achieve the IPI cancellation before nonlinear operation on noise and MAI and to produce $M$ decision variables in digital domain, so as to detect the $(1 + \log_2 M)$-bit information per symbol with a modest increase in receiver complexity.

III. ANALYSIS OF MULTIUSER PERFORMANCE

A tractable, closed-form expression is derived on both the noise and interference statistics after the nonlinear cross-correlation operation, leading to a well-defined first-order approximation of the channel-averaged signal-to-interference-plus-noise ratio (SINR) for the BTR system.

A. Desired signal and noise terms

Assuming that $h_m$ was sent for the information carrying sequence $[x_{i N_s/2}^{(1)} x_{i N_s/2+1}^{(1)} \ldots x_{i N_s/2+M-1}^{(1)}]$ in (1), the desired signal term $s_m$ in (13) is found to be

$$s_m = \frac{E_s}{4} b^{(1)}_i \delta[m - q] \int_0^{T_{corr}} [g^{(1)}(t)]^2 dt$$

$$= \frac{E_s}{4} b^{(1)}_i \delta[m - q] \sum_{l=0}^{L_p-1} a^{(1)}_l^2 \quad (15)$$

where $\delta[n] = 1$ if $n = 0$ and zero otherwise, and the integration time $T_{corr}$ is assumed an integer multiple of the chip time, denoted by $L_p T_c$ for $0 < L_p \leq L$.

Next, the noise-times-noise term $N_{1,m}$ is of the form

$$N_{1,m} = \sum_{n=i N_s/2}^{(i+1)N_s/2-1} h_m, n \int_{\Phi_j} n_j(t - T_d^{(1)} - T_{corr}) dt \quad (16)$$

for $j = 2n+1$, whose variance can be evaluated as (see [16] for details)

$$\sigma^2_{N_{1,m}} = E_N \{ N_{1,m}^2 \} = \frac{1}{16} N_s N_o^2 WT_{corr} \quad (17)$$

provided $T_d^{(1)} \geq T_c$ and $WT_{corr} \gg 1$ for the one-sided receiver bandwidth $W$.

Similarly, the noise-times-signal term $N_{2,m}$ is written as

$$N_{2,m} = \sqrt{\frac{E_s}{2 N_s}} \sum_{n=i N_s/2}^{(i+1)N_s/2-1} h_m, n \int_{\Phi_j} g^{(1)}(t - j T_f - c_j^{(1)} T_c - T_d^{(1)}) \left[ n_j(t) + b^{(1)}_i h_{q,n} n_j(t - T_d^{(1)}) \right] dt \quad (18)$$
which yields the variance
\[
\sigma_{N_2,m}^2 = \frac{1}{8} \varepsilon_s N_0 \sum_{l=0}^{L_p-1} \mathbb{E}\left\{ a_l^{(1)} \right\}^2
\] (19)
where the factor \( \varsigma \) accounts for the noise filtering at the receiver frontend and is evaluated as
\[
\varsigma = \int_0^{T_c} \int_0^{T_c} p(t) p(s) 2W \text{sinc}[2W(t - s)] \, dt \, ds
\]
with \( \lim_{W \to \infty} \varsigma = 1 \) (normalized).

### B. Interference terms

Consider the \( j \)-th frame time interval \([jT_f, (j+1)T_f)\) at the receiver where the \( \text{desired} \) first user's reference time is given by \( \tau^{(1)}_1 = jT_f + c_{j,j'}T_c \), assuming \( \tau^{(1)} = 0 \). Then, the \( k \)-th user's reference time \( \tau^{(k)} = \tau^{(1)} + j'T_f + c_{j,j'}T_c \) can be related to \( \tau^{(1)} \) as
\[
\tau^{(k)} = \tau^{(1)} + \phi_{j,j'}^{(k)}
\]
where \( \phi_{j,j'}^{(k)} = \left[ c_{j,j'}^k - c_{j,j'}^{k-1} \right] T_c + \delta^{(k)} \) and \( \delta^{(k)} = \tau^{(k)} \pmod{T_f} \) is uniformly distributed over \([0, T_f)\) for \( \tau^{(k)} = j(k)T_f + \delta^{(k)} \) and \( j' = j - j^\prime \).

With this delay model, the frame-averaged interference terms in (10) can be formulated as
\[
i_{j,\pm}(t + \tau^{(1)}) = \frac{1}{2} \sqrt{\frac{E_s}{2N_0}} \sum_{k=2}^{K} \left[ \pm u_{j,j'}^{(k)} g_{j,j'}(t - \theta_{j,j'}) + u_{j,j'}^{(k-1)} g_{j,j'}(t + \tau^{(k-1)}) + v_{j,j'}^{(k-1)} g_{j,j'}(t - \theta_{j,j'} - \tau^{(k-1)}) + \left[ \pm \alpha_{j,j'}^{(k)} u_{j,j'}^{(k)} g_{j,j'}(t - \theta_{j,j'}) - T_d^{(k)}) + \alpha_{j,j'}^{(k-1)} u_{j,j'}^{(k-1)} g_{j,j'}(t - \theta_{j,j'} - \tau^{(k-1)}) - T_d^{(k)}) + \alpha_{j,j'}^{(k-1)} v_{j,j'}^{(k-1)} g_{j,j'}(t - \theta_{j,j'} - \tau^{(k-1)}) - T_d^{(k)}) \right] \right]
\] (20)
where \( u_{j,j'}^{(k)} = d_{j,j'}^{(k)} d_{j,j'}^{(1)} \), \( v_{j,j'}^{(k)} = d_{j,j'}^{(k)} d_{j,j'}^{(1)} \), and \( \alpha_{j,j'}^{(k)} = (-1)^{j'} j_{j,j'}^{(k)} \). Note that, the receiver searches for the \( j \)-th frame signal of the first user in the presence of possible overlap (or collision) with the previous \( j' - 2 \) or \( j' - 1 \) or current \( j' \) frame signals of the \( k \)-th user \((k = 2, 3, \ldots, K)\), depending on the relative delays \( \{\theta_{j,j'}^{(k)}\} \).

First, the MAI-times-noise term \( I_{1,m} \) in (13) is expressed as
\[
I_{1,m} = \sum_{n=1}^{\left(\frac{i+1}{2}\right)} h_{m,n} \int_{\Phi_j} \left[ i_{j,\pm}(t - T_d^{(1)}) n_{j,-}(t) + i_{j,-}(t) n_{j,+}(t - T_d^{(1)}) \right] dt
\] (21)
for \( j = 2n+1 \). In Appendix A, by considering the probability distribution of potential interferers at their relative delays \( \{\phi_{j,j'}^{(k)}\} \) from the desired signal, the variance of \( I_{1,m} \), denoted by \( \sigma_{I_{1,m}}^2 \), is evaluated as
\[
\sigma_{I_{1,m}}^2 = \frac{E_s N_0}{8T_f} \sum_{k=2}^{K} \int_{L_pT_c}^{L_pT_c} \int_{L_pT_c}^{L_pT_c} \mathbb{E}\left\{ g_{j,j'}^{(k)}(t + x) \times g_{j,j'}^{(k)}(s + x) \right\} 2W \text{sinc}[2W(t - s)] \, dt \, ds \, dx.
\]
(22)
Further, if \( x \) is assumed to take on \( x = nT_c \) for \( n = -L_p, -L_p + 1, \ldots, L - 1 \), then
\[
\int_{L_pT_c}^{L_pT_c} \int_{L_pT_c}^{L_pT_c} \mathbb{E}\left\{ g_{j,j'}^{(k)}(t + x) g_{j,j'}^{(k)}(s + x) \right\} 2W \text{sinc}[2W(t - s)] \, dt \, ds = \varsigma \sum_{l=0}^{L_p-1} \mathbb{E}\left\{ a_{l+n}^{(k)} \right\}^2
\]
where \( a_l^{(k)} = 0 \) for \( l < 0 \) and \( l \geq L \).

Hence, the variance \( \sigma_{I_{1,m}}^2 \) in (22) can be piece-wise approximated to
\[
\sigma_{I_{1,m}}^2 \approx \frac{E_s N_0}{8N_f} \sum_{k=2}^{K} \sum_{n=-L_p}^{L_p-1} \mathbb{E}\left\{ I_{n,l}^{(k)}(n) \right\}
\] (23)
where the piece-wise probability mass is
\[
\frac{1}{N_f} = \frac{1}{T_f} \int_{nT_c}^{(n+1)T_c} \int \, dx \quad \text{for} \quad n = -L_p, -L_p + 1, \ldots, L - 1.
\]
Next, the MAI-times-MAI term \( I_{2,m} \) has the expression
\[
I_{2,m} = \sum_{n=1}^{\left(\frac{i+1}{2}\right)} h_{m,n} \int_{\Phi_j} \left[ i_{j,\pm}(t - T_d^{(1)}) i_{j,-}(t) \right] dt.
\] (24)
Specifically, \( I_{2,m} = I_{2,m,A} + I_{2,m,B} \) can be classified as
\[
I_{2,m,A} = \sum_{n=1}^{\left(\frac{i+1}{2}\right)} h_{m,n} \int_{\Phi_j} \sum_{k=2}^{K} \left[ i_{j,j'}^{(k)}(t) + (t - T_d^{(1)}) \right] dt
\] (25)
\[
I_{2,m,B} = \sum_{n=1}^{\left(\frac{i+1}{2}\right)} h_{m,n} \int_{\Phi_j} \sum_{k=2}^{K} \sum_{k' \neq k} \left[ i_{j,j'}^{(k')} + (t - T_d^{(1)}) \right] dt
\] (26)
where \( i_{j,\pm}(t) = \sum_{k=2}^{K} i_{j,j'}^{(k)}(t) = \sum_{k' \neq k} i_{j,j'}^{(k')}(t) \) in (20) with \( j' = j - j'' \) replaced by \( j'' = j - j^{(k'')} \) for the latter. Here, \( I_{2,m,A} \) represents the interferences resulting from correlating the signals of the same interferer, referred to as self-MAI-times-MAI, while \( I_{2,m,B} \) accounts for the interferences resulting from correlating the signals of different interferers, referred to as cross-MAI-times-MAI.

Similarly, using the probability distribution of potential interferers at their relative delays, along with the balance property of the BTR signaling as detailed in Appendix B,
the variance $\sigma^2_{2,m} = \sigma^2_{2,m,A} + \sigma^2_{2,m,B}$ can be evaluated as

$$
\sigma^2_{2,m} \approx \frac{E_s}{64N_T f} \sum_{k=2}^{K} \int_{-L_p T_c}^{T_{mds}} \left[ F_k(x, x + T_d^{(1)} - T_d^{(k)}) \left( F_k(x, x + h_1 T_c + h_k T_c + T_d^{(1)}) + F_k(x, x + h_1 T_c + h_k T_c + T_d^{(k)}) \right) dx + \varepsilon \left( T_d^{(k)} < T_{mds} \right) \right] dx dy
$$

(27)

where $h_k$ denotes a random variable taking the values of $\{-N_h^{(k)} + 1, -N_h^{(k)} + 2, \ldots, N_h^{(k)} - 1\}$ with probability $\left[ N_h^{(k)} - |h_k| \right] / \left[ N_h^{(k)} \right]^2$, $\varepsilon \left( T_d^{(k)} < T_{mds} \right)$ represents the self-MAI-times-MAI caused by the IPI when $T_d^{(1)} < T_{mds}$ and otherwise zero,\(^1\) and

$$
F_k(x, x + \alpha) = \mathbb{E} \left\{ \left[ \int_0^{L_p T_c} g(k)(t + x)g(k)(t + x + \alpha) dt \right]^2 \right\}
$$

$$
F_{k,k'}(x, y) = \mathbb{E} \left\{ \left[ \int_0^{L_p T_c} g(k)(t + x)g(k')(t + y) dt \right]^2 \right\}
$$

Further, if $x$ takes on $x = nT_c$ for $n = -L_p, -L_p + 1, \ldots, L - 1$, it can be shown that

$$
F_k(x, x + T_d^{(1)} - T_d^{(k)}) = \sum_{i=0}^{L_p-1} \mathbb{E} \left\{ a_{i+n}^{(k)} \right\} \mathbb{E} \left\{ \left( a_{i+n+1-n_k}^{(k)} \right)^2 \right\}
$$

$$
F_k(x, x + h_1 T_c + h_k T_c + T_d^{(1)} - T_d^{(k)}) = \sum_{h_1=-N_h^{(k)}+1}^{N_h^{(k)}-1} \sum_{h_k=-N_h^{(k)}+1}^{N_h^{(k)}-1} \frac{N_h^{(1)} - |h_1|}{N_h^{(1)}} \frac{N_h^{(k)} - |h_k|}{N_h^{(k)}^2} \sum_{i=0}^{L_p-1} \mathbb{E} \left\{ a_{i+n}^{(k)} \right\} \mathbb{E} \left\{ \left( a_{i+n+1-n_k}^{(k)} \right)^2 \right\}
$$

$$
F_{k,k'}(x, y) = \sum_{i=0}^{L_p-1} \mathbb{E} \left\{ a_{i+n}^{(k)} \right\} \mathbb{E} \left\{ \left( a_{i+m}^{(k')} \right)^2 \right\}
$$

where $T_d^{(k)} = n_k T_c$ is assumed and $y = nT_c + \phi$ ($m = -L_p, -L_p + 1, \ldots, L - 1$) to account for the chip-asynchronous delay $\phi$ between different users and $k'$, uniformly distributed over $[0, T_c)$, and the mean-squared chip-pulse autocorrelation $\kappa$ is evaluated as

$$
\kappa = \mathbb{E} \left\{ \left[ \int_0^{T_c} p(t)p(t + \phi) dt \right]^2 \right\}
$$

(30)

The above piece-wise approximation is then applied to obtain

$$
\sigma^2_{2,m} \approx \frac{E_s}{16N_T f} \sum_{k=2}^{K} \sum_{i=0}^{L_p-1} \mathbb{E} \left\{ a_{i+n}^{(k)} \right\} \mathbb{E} \left\{ \left( a_{i+n+1-n_k}^{(k)} \right)^2 \right\}
$$

(39)

\(^1\)The contribution by the IPI when $T_d^{(1)} < T_{mds}$ as detailed in Appendix B and results section is limited and can be ignored.

Finally, the MAI-times-signal term $I_{3,m}$ is written as

$$
I_{3,m} = \sqrt{\frac{E_s}{2N_T f}} \sum_{n=-N/2}^{N/2} h_m.n \int g(t - jT_f - c_j^{(1)}T_c) \left[ i_j(t) + b_j^{(1)}h_q.n i_j(t - T_d^{(1)}) \right] dt,
$$

which yields the variance in Appendix C as

$$
\sigma^2_{3,m} = \frac{E_s}{4N_T f} \sum_{k=2}^{K} \int_{-L_p T_c}^{T_{mds}} F_{k,k'}(x, y) dx.
$$

(31)

Also, it turns out that

$$
F_{k,0}(x) = \sum_{i=0}^{L_p-1} \mathbb{E} \left\{ a_{i+n}^{(1)} \right\} \mathbb{E} \left\{ \left( a_{i+n+1}^{(k)} \right)^2 \right\} + \mathbb{E} \left\{ \left( a_{i+n+1}^{(k)} \right)^2 \right\}
$$

(32)

for $x = nT_c + \phi$ ($n = -L_p, -L_p + 1, \ldots, L - 1$).

Applying the piece-wise approximation, (31) can be evaluated as

$$
\sigma^2_{3,m} \approx \frac{E_s}{16N_T f} \sum_{k=2}^{K} \sum_{i=0}^{L_p-1} \mathbb{E} \left\{ a_{i+n}^{(k)} \right\} \mathbb{E} \left\{ \left( a_{i+n+1}^{(k)} \right)^2 \right\}
$$

(33)

$$
\varphi_{i,j}^{(k)}(n) \triangleq 4 \sum_{i=0}^{L_p-1} \mathbb{E} \left\{ a_{i+n}^{(k)} \right\} \mathbb{E} \left\{ \left( a_{i+n+1}^{(k)} \right)^2 \right\} + \mathbb{E} \left\{ \left( a_{i+n+1}^{(k)} \right)^2 \right\}
$$

C. Channel-averaged SINR

In deriving an exact channel-averaged SINR, we should be able to take the joint expectation as $\text{SINR}(\{}a^{(k)}\text{)} = \mathbb{E}_{\{a^{(k)}\}} \left\{ \text{SINR}(\{}a^{(k)}\text{)} \right\}$ for $a^{(k)} = \{a^{(k)}\}_{i=0}^{L_p-1}, k = 1, 2, \ldots, K$, which requires the joint probability density function of sum of lognormal distributions (due to the channel gains $\{a^{(k)}\}$). The joint distribution is a well-known mathematical problem that seems to be intractable. For this reason, the first-order approximation is made to the desired signal $s_m (m = q)$ in (15) as its mean $\mathbb{E}_{\{a^{(k)}\}} \{s_m\}$, while the MAI is accounted for in terms of the second-order moment due to time asynchronism, i.e., $\mathbb{E}_{\{a^{(k)}\}} \{\text{MAI} (\{}a^{(k)}\text{)} \} = \mathbb{E}_{\{a^{(k)}\}} \{\text{MAI} (\{}a^{(k)}\text{)} \} \{a^{(1)}\}$, given the channel.
gain coefficients of the desired first user that are being estimated.

Hence, the first-order SINR approximation for $\xi_m$ in (13) is formulated as

$$\operatorname{SINR} \triangleq \frac{\gamma}{\gamma_R} = \frac{E_s}{2N_0} \sum_{l=0}^{L_p-1} \mathbb{E}\left\{ a_l(1)^2 \right\}$$

(34)

If the instantaneous received signal-to-noise ratio (SNR) in average sense is defined by [16]

$$\gamma_R \triangleq \frac{E_s}{2N_0} \sum_{l=0}^{L_p-1} \mathbb{E}\left\{ a_l(1)^2 \right\},$$

then the channel-averaged SINR can be derived as

$$\operatorname{SINR}(L_p, \{n_k\}) \Rightarrow \frac{\gamma^2}{\gamma_R} \left[ \gamma_R + \frac{N_0}{4} W L_p T_c + \sum_{k=2}^{K} \sum_{m=-L_p}^{L_p-1} \sum_{l=0}^{L_p-1} \mathbb{E}\left\{ a_l(1)^2 \right\} \right]^{-1}$$

(35)

where $(L_p, \{n_k\})$ explicitly denotes the dependence of SINR on $L_p$ and $\{n_k\}$, and

$$\varphi_{c}(n) = \frac{1}{[\sum_{l=0}^{L_p-1} \mathbb{E}\left\{ a_l(1)^2 \right\}]^{m}} \varphi_{c}(n)$$

with $m = 1$ if $e = 1$ and otherwise, $m = 2$.

From [17, eqs. (18) and (19)], the average path energy can be evaluated in closed-form as

$$E\left\{ a_l(1)^2 \right\} = \frac{\Omega_0}{\varphi_{c}(n)} \left[ \frac{l T_c}{\gamma} + \frac{T_c}{\gamma} \right]$$

(36)

where $P_c = \gamma T_c$ and $P_c = \lambda T_c$ ($\Lambda$ and $\lambda$ denote the cluster and ray arrival rates, respectively), and $\eta = \exp\left(\frac{L_p}{m} - \frac{l T_c}{T_c}\right)$.

Also, by defining $E_c \triangleq \sum_{l=0}^{L_p-1} \mathbb{E}\left\{ a_l(1)^2 \right\}$, the mean power of the first ray of the first channel $\Omega_0$ can be theoretically determined as

$$\Omega_0 = \frac{1}{E_0} \sum_{l=0}^{L_p-1} \mathbb{E}\left\{ a_l(1)^2 \right\}$$

(37)

so as to make $E_c = 1$ (normalized).

Therefore, the channel-averaged SINR, $\operatorname{SINR}(L_p, \{n_k\})$ as derived in (35) can be computed without resorting to the tedious simulations via UWB channel realizations, and also enables us to optimize the integration time (via $L_p$) and the inter-pulse distances (via $\{n_k\}$), in order to maximize the SINR in average sense. Especially, the CM1 to CM4 UWB channel models are characterized by the key parameters such as $(\Gamma, \gamma)$ and $(\Lambda, \lambda)$ in evaluating the average path energy in (36).

Note that if the decision variables $\{\xi_m\}$ in (13) can be modeled as Gaussian,$^2$ we may use a simple formula to evaluate the symbol-error rate (SER) [23]

$$P_s = 1 - \Pr \left[ \bigcap_{m \neq q} |\xi_m| < \xi_q \bigg| b^{(1)}_q = 1, h_q \right]$$

$$= 1 - \int_0^{\infty} \left[ 1 - 2 Q(x) \right]^{M-1} \times \phi \left[ x - \sqrt{\text{SINR}(L_p, \{n_k\})} \right] dx$$

(37)

where $Q(x) = \int_x^\infty \phi(y) dy$ for $\phi(y) = 1/\sqrt{2\pi} \exp(-y^2/2)$.

IV. RESULTS

For channel-averaged analysis and simulation purposes, the IEEE 802.15.3a channel models [17] have been used. The results will be presented for CM1 and CM3, where the channel sparseness and cluster overlapping are accounted for, which represent a line-of-sight (LOS) and non-line-of-sight (NLOS) channels. The continuous-time channel impulse response is sampled at 6GHz and the maximum channel delay spread $T_{mds}$ is around $L = 200$ and $L = 400$ time bins (chips) for the LOS and NLOS channels, respectively. The received pulse is assumed to be the second-order derivative of Gaussian pulse with pulse width of 0.167ns (equal to the chip time $T_c$). To mitigate the MAI, the pulse-inter distance $T_{d}$ and the maximum-time-hopping value $N_{h}$ are set according to (2) and (3) with $n = 1$ (except Fig. 10). In any case, the basic frame time $T_{F} = T_{d} + T_{mds} + N_{h} T_{c} = 460 T_{c}$ is set to avoid the IFI and is fixed for all users. But the frame time is adjusted according to $(1 + \log_2 M)T_{F}$ for $M$-ary BTR below, while keeping the same data rate and average transmit power (strictly imposed by the FCC spectral mask). Note that, in Figs. 4 and 7 the frame time is set to $T_{F} = 460 T_{c}$ even with $M = 2$, doubling the data rate.

First, the channel-averaged noise and interference statistics are validated by the normalized variances, such as $\sigma^2 = \left[ \mathbb{E}\left\{ s_m \right\} \right]^2$ in (34), when the number of users are

---

$^2$It is observed in [10] and Fig. 7 that the integration time largely affects the decision statistics after cross-correlation operation.
Fig. 4. Normalized variance $\hat{\sigma}^2$ versus $K$ for $M = 2$-ary BTR system when $E_b/N_o = 16$dB, $T_{d,\text{min}} = 100T_c$ and $L_p = 30$.

Fig. 5. Channel-averaged SINR versus $K$ for $M = 2$-ary BTR system when $T_{d,\text{min}} = 107T_c, 100T_c, 200T_c$, $L_p = 200$ ($E_b/N_o = 22$dB) and $L_p = 30$ ($E_b/N_o = 16$dB).

Fig. 6. Channel-averaged SINR versus $K$ for $M = 4$-ary BTR system when $T_{d,\text{min}} = 10T_c, 100T_c, 200T_c$, $L_p = 200$ ($E_b/N_o = 22$dB) and $L_p = 30$ ($E_b/N_o = 16$dB).

Fig. 7. Channel-averaged SER versus $E_b/N_o$ for $M = 2$-ary BTR system when $K = 2, 5, 10$, $T_{d,\text{min}} = 100T_c$ and $L_p = 50, 100, 200$.

$K = [2, 5, 10, 50]$, inter-pulse distance $T_{d}^{(1)} = T_{d,\text{min}} = 100T_c < T_{\text{mds}}$ (i.e., the IPI is present), $N_s = 8$ and $L = 200$ for CM1, assuming the receive bandwidth $W = 2/T_c$ in (17). Fig. 3 shows the normalized variances when $M = 2$-ary orthogonal encoding is used with bi-phase modulation in (1), carrying $(1 + \log_2 M) = 2$ bits per symbol, per-bit SNR $E_b/N_o = 22$dB, $L_p = 200$. To further validate the SINR analysis here, the case of per-bit SNR $E_b/N_o = 16$dB is plotted in Fig. 4, where the noise-times-noise is dominant and a shorter integration time $L_p = 30$ is used. It is seen that the analytical results closely match the simulation results, where the MAI-times-MAI has been approximated, thereby exhibiting a slight discrepancy but it is still within acceptable range.

Next, to investigate the effect of IPI on the channel-averaged SINR, Figs. 5 and 6 plot the SINR versus $K$ for $M = 2$ and $M = 4$, respectively, when the inter-pulse distance $T_{d,\text{min}}$ varies from $10T_c, 100T_c$ to $200T_c \cong T_{\text{mds}}$ for CM1. First, the IPI is not the critical factor that determines the multiuser performance in terms of the SINR, and the variations in SINR is dominated by the MAI as $K$ increases, when $E_b/N_o = 22$dB. Second, the selection of $T_{d,\text{min}} = 100T_c$ is deemed appropriate because there is no actual penalty in the achieved SINR, compared to the IPI-free case when $T_{d,\text{min}} = T_{\text{mds}}$. Third, the multiuser performance predicted by the channel-averaged SINR via (37) closely follows the SER based multiuser performance via simulation in Fig. 7, provided the integration time is at least half the channel delay spread, namely $L_p \geq 100$ for CM1 ($L = 200$). A mixed case of CM3 (desired user) and CM1 (interfering users) is also considered in Fig. 8, which reveals that the SINR analysis still holds even when there exists a power mismatch for the mixed LOS and NLOS users, as long as the integration time is sufficient (i.e., $L_p = 200, 300, 400$ for $L = 400$ in CM3).

To find an optimum integration time via $L_p$, the channel-averaged SINR is plotted versus $L_p$ when $K = 10$, $E_b/N_o = 16$dB, and $T_{d,\text{min}} = 100T_c$ for $M = 2, 4$ in Fig. 9. It is observed that the SINR reaches its maximum around $L_p = [30, 40]$, where an optimum $L_p$ appears to slightly increase with $M$. This is because the noise and interference statistics tend to be averaged with increased frame time. The analytical results seem to be accurate enough to predict the multiuser performance when $L_p$ varies from 10, 20 up to 100.
with varying $L$ as explicitly formulated in (35).

Channel-averaged

Fig. 8. Channel-averaged SER versus $K$ for $M = 2$-ary BTR system when $T_{d,\min} = 100T_c$ assuming a mixed scenario of LOS/NLOS users, along with varying $L_p = 200, 300, 400$ ($E_b/N_o = 22$dB) for $L = 400$ in CM3 (desired NLOS user).

as explicitly formulated in (35).

Finally, the effect of inter-pulse distances via $\{n_k\}$ ($T_d^{(k)} = n_kT_c$) on the SINR is investigated in Fig. 10 when $K = 50$, $n_1 = 100$ ($T_{d,\min} = 100T_c$) and $n = n_k - n_{k-1} = 1/2, 1, 2, 3, 4, 5$ in (2) with $T_w = T_c$ for $k = 2, 3, \ldots, K$. It is noticed that, as long as the user-specific inter-pulse distances are assigned (i.e., $n \geq 1/2$), a larger separation does not help to improve the multiuser performance, further the smallest separation (i.e., $n = 1/2$) is preferred for an increased TH shift so as to accommodate more users, subject to the above IFI-free constraint. The analyses are also shown to be within acceptable range when $\{n_k\}$ vary largely, as explicitly formulated in (35).

V. CONCLUSION

This paper has developed an analytical framework to evaluate the channel-averaged SINR, which can be used to predict the multiuser performance for the $M$-ary BTR system, considering realistic UWB channels. Further, this framework can easily be modified to analyze the multiuser performance for other UWB-TR systems, in terms of the SINR as an useful performance measure. First of all, the IPI does not largely affect the multiuser performance as long as the inter-pulse distance is set to be more than half the channel delay spread, regardless of the IPI cancellation.\(^3\) Hence, the two-frame averaging designed for the IPI cancellation could be removed for feasible implementation, in which case the IPI cancellation in Fig. 2 can be replaced by a short delay line of $T_d^{(1)}$, like conventional TR. Second, the $M$-ary BTR system is able to increase the frame time with increased $M$ (without reducing the data rate and increasing the average transmit power), which yields an increased processing gain (PG) (by the factor of $1 + \log_2 M$) against the MAI. This feature is very important because the frame averaging\(^4\) (that helps to reduce the MAI) can be replaced by $M$-ary BTR receiver with a modest increase in receiver complexity. Third, the user-specific inter-pulse distances do not largely affect the multiuser performance as long as the incremental distance is more than half the chip time, which in turn helps to implement the shorter delay line in multiuser environment.

APPENDIX A: EVALUATION OF $\sigma^2_{I_{1,m}}$ IN (22)

The second-order moment $\sigma^2_{I_{1,m}}$ in (21) can be formulated as

\[
\sigma^2_{I_{1,m}} = \frac{N_r}{4} \sum_{i=\frac{M}{2}}^{(i+1)\frac{M}{2} - 1} \int_{\Phi_j} \int_{\Phi_1} \left[ E \{ i_j(t - T_d^{(1)}) \} \times i_j(t) i_{j-s}(-s) \right] \\
\times 2W\text{sinc}[2W(t - s)] dt ds \tag{38}
\]

for $j = 2n + 1$, where the BTR signal has no IFI (i.e., $T_f \geq T_d^{(1)} + T_{mds} + N_h^{(1)}T_c$), due to which all cross-terms over the frame index $j$ (i.e., $n$) disappear, and the cross-noise terms

\(^3\)This is because only the IPI resulting from a desired user signal is cancelled, which becomes insignificant as $K$ increases.

\(^4\)The increased PG is not effective against noise, so digital TR schemes could be adopted to have a cleaner template against the noise, which can also be implemented in conjunction with the $M$-ary BTR system, so as to further reduce the MAI.
within the frame are also averaged out as

$$E\{n_j(t)n_{j'}(s-T_d^{(1)})\} = \frac{1}{4} \left\{ R_n(t-s+T_d^{(1)}) + d_{j-1}^{(1)}d_{j'}^{(1)} \left[ R_n(t-s+T_d^{(1)}-T_f-\Delta_j^{(1)}) - R_n(t-s+T_d^{(1)}) \right] - R_n(t-s+T_d^{(1)}+T_f+\Delta_j^{(1)}) - R_n(t-s+T_d^{(1)}) \right\} \equiv 0$$

for the noise autocorrelation function $R_n(\tau) = N_wW\text{sinc}(2W\tau)$. In the above, the first and last terms in brackets cancel out because of the balance property of the BTR signaling, and the second and third terms are ignored because $|t-s| \leq T_{corr} \leq T_{mds}, T_f-T_d^{(1)}+\Delta_j^{(1)} \geq T_{mds}+T_c$, and $R_n(\tau) \equiv 0$ for $|\tau| \geq T_c$.

Now, (38) can be rewritten as

$$\sigma_{I_{2,m}}^2 = \frac{N_o}{4} \sum_{n=N_s/2}^{(i+1)N_s/2-1} \int_0^{T_{corr}} \int_0^{T_{corr}} \left[ E\left\{i_{j}(t+s+\bar{\tau})i_{j'}(s+\bar{\tau}(1))\right\} + E\left\{i_{j'}(t+s+\bar{\tau})i_{j}(s+\bar{\tau}(1))\right\} \right] \times 2W\text{sinc}(2W(t-s)) \, dt \, ds.$$  \hspace{1cm} (39)

Then, given the integration range $[0, T_{corr}]$, the union of a pair of relative delays in (20) is shown to be uniformly distributed over the vulnerable period $I_R = [-T_{mds}, T_{corr}]$, such as

$$\theta_{j,j'}^{(1)} \cup \theta_{j'-1,j}^{(1)} - T_f \supset I_R$$

$$\theta_{j,j'}^{(2)} \cup \theta_{j'-1,j}^{(2)} - T_f + T_d^{(1)} \supset I_R$$

$$\theta_{j,j'}^{(3)} \cup \theta_{j'-1,j}^{(3)} - T_f - T_d^{(1)} \supset I_R$$

$$\theta_{j,j'}^{(1)} \cup \theta_{j'-1,j}^{(1)} - T_f - T_d^{(1)} \supset I_R$$

$$\theta_{j,j'}^{(2)} \cup \theta_{j'-1,j}^{(2)} - T_f - T_d^{(1)} \supset I_R$$

where the time-hopping codes $\{c_j^{(1)}, c_j^{(2)}, c_j^{(3)}\}$ have been taken the average so as to contribute uniformly to the delay distribution over $I_R$. The above uniform delay distribution also holds when $\{\theta_{j,j'}^{(1)}, \theta_{j'-1,j}^{(1)}\}$ are replaced by $\{\theta_{j,j'}^{(1)}, \theta_{j'-1,j}^{(1)}\}$.

Thus, if the expectation in (39) is taken with respect to the unions of pairs of relative delays, and by the fact that the resulting cross-terms disappear due to the zero-mean pseudorandom sequences $\{d_j^{(1)}\}$ and data modulation in (20), $\sigma_{I_{2,m}}^2$ can then be evaluated as given in (22).

**APPENDIX B: EVALUATION OF $\sigma_{I_{2,m}}^2$ IN (27) AND (28)**

Using (20), the self-MAI-times-MAI $I_{2,m,A}$ in (25) is first expressed as given in (40) at the next page. In (40), the cross-terms are weighted by alternating signs because of the balance property of the BTR signaling, which helps to mitigate the self-MAI-times-MAI.

Especially, the following pairwise cross-terms resulting from the IPI when $T_d^{(1)} < T_{mds}$

$$I_{2,m,A} = \frac{E_s}{8N_s} \sum_{k=2}^K \sum_{n=NI_s/2}^{(i+1)NI_s/2-1} h_{m,n}$$

$$\times \int_0^{T_{corr}} \left[ g^{(k)}(t-\theta_{j,j'}^{(1)}) - g^{(k)}(t-\theta_{j',j}^{(1)}) \right. \right.$$  

$$\left. - g^{(k)}(t+T_f-\theta_{j'-1,j}^{(1)})g^{(k)}(t+T_f-\theta_{j',j}^{(1)}) + T_d^{(1)}) + g^{(k)}(t+T_f-\theta_{j'-1,j}^{(1)})g^{(k)}(t-\theta_{j',j}^{(1)}) + T_d^{(1)}) \right.$$

$$\left. - g^{(k)}(t+T_f-\theta_{j',j}^{(1)}) - T_d^{(1)} \right.$$

$$\left. - g^{(k)}(t+T_f-\theta_{j',j}^{(1)}) - T_d^{(1)} \right.$$

$$\left. - g^{(k)}(t+T_f-\theta_{j',j}^{(1)}) - T_d^{(1)} \right.$$  

$$\left. \times \int_0^{T_{corr}} g^{(k)}(t+x)g^{(k)}(t+x+T_d^{(1)}-T_d^{(k)}) \, dt \right.$$  

$$\left. + \int_0^{T_{corr}} g^{(k)}(t+x)g^{(k)}(t+x+h_1T_c + h_2T_c + T_d^{(1)}) \, dt \right.$$  

$$\left. + \int_0^{T_{corr}} g^{(k)}(t+x)g^{(k)}(t+x+h_1T_c + h_2T_c + T_d^{(1)}) \, dt \right.$$  

$$\left. + \int_0^{T_{corr}} g^{(k)}(t+x)g^{(k)}(t+x+h_1T_c + h_2T_c + T_d^{(1)}) \, dt \right.$$  

$$\left. - T_d^{(k)} \right) \, dx \right\}.$$  \hspace{1cm} (41)

Note that, the cross-terms have been averaged out because the two adjacent indexes $n$ and $n+1$ correspond to $j = 2n+1$ and $j = 2n+3$, respectively. Thus, $\sigma_{I_{2,m,A}}^2$ can be found as given in (27).

Next, the cross-MAI-times-MAI $I_{2,m,B}$ in (26) is expressed
\[ I_{2,m,A} = \frac{E_s}{8 N_s} \sum_{k=2}^{K} \sum_{n=1}^{(i+1) N_s/2} h_{m,n} \int_0^{T_{corr}} \left\{ u^{(k)}_{j'j'}(t - \theta^{(k)}_{j'j'}) + u^{(k)}_{j'-1}(t + T_f - \theta^{(k)}_{j'j'}) + v^{(k)}_{j'-1}(t - \theta^{(k)}_{j'j'}) + v^{(k)}_{j'+2}(t + T_f - \theta^{(k)}_{j'j'}) + \alpha_{j'j'}^{(k)} u^{(k)}_{j'j'}(t + T_f - \theta^{(k)}_{j'j'}) + \alpha_{j'j'}^{(k)} u^{(k)}_{j'-1}(t + T_f - \theta^{(k)}_{j'j'}) + \alpha_{j'j'}^{(k)} v^{(k)}_{j'-1}(t - \theta^{(k)}_{j'j'}) + \alpha_{j'j'}^{(k)} v^{(k)}_{j'+2}(t + T_f - \theta^{(k)}_{j'j'}) + \alpha_{j'j'}^{(k)} g^{(k)}(t - \theta^{(k)}_{j'j'}) + \gamma^{(k)}(t + T_f - \theta^{(k)}_{j'j'}) + \gamma^{(k)}(t + T_f - \theta^{(k)}_{j'j'}) + \gamma^{(k)}(t + T_f - \theta^{(k)}_{j'j'}) + \gamma^{(k)}(t + T_f - \theta^{(k)}_{j'j'}) \right\} dt \]  

APPENDIX C: EVALUATION OF \( \sigma_{I_3,m}^2 \) IN (31)

From (30), \( \sigma_{I_3,m}^2 \) can be formulated as

\[ \sigma_{I_3,m}^2 = \frac{E_s}{2 N_s} \sum_{n=1}^{(i+1) N_s/2} \int_0^{T_{corr}} \int_0^{T_{corr}} \left\{ \left\{ g^{(1)}(t) \right\} i_{j'+1}(s + \hat{\tau}(1)) i_{j'}(s + \hat{\tau}(1)) \right\} dt ds. \]  

Since \( i_{j'+1}(\cdot) i_{j'}(\cdot) \) contain four independent unions of pairs of relative delays, respectively, it turns out that

\[ \sigma_{I_3,m}^2 = \frac{E_s^2}{32 N_s} \sum_{k=2}^{K} \frac{8}{T_f} \int_{-T_{corr}}^{T_{corr}} \int_{-T_{corr}}^{T_{corr}} \left\{ \left\{ \int_0^{T_{corr}} g^{(1)}(t) \right\} \times g^{(k)}(t + x) dt \right\} dx dy. \]  

Rearranging and simplifying the above, \( \sigma_{I_3,m}^2 \) can be found as given in (28).

REFERENCES


Dong In Kim (S’89-M’91-SM’02) received the B.S. and M.S. degrees in Electronics Engineering from Seoul National University, Seoul, Korea, in 1980 and 1984, respectively, and the M.S. and Ph.D. degrees in Electrical Engineering from University of Southern California (USC), Los Angeles, in 1987 and 1990, respectively.

From 1984 to 1985, he was a Researcher with Korea Telecom Research Center, Seoul. From 1986 to 1988, he was a Korean Government Graduate Fellow in the Department of Electrical Engineering, USC. From 1991 to 2002, he was with the University of Seoul, Seoul, leading the Wireless Communications Research Group. From 2002 to 2007, he was a tenured Full Professor in the School of Engineering Science, Simon Fraser University, Burnaby, BC, Canada. From 1999 to 2000, he was a Visiting Professor at the University of Victoria, Victoria, BC. Since 2007, he has been with Sungkyunkwan University (SKKU), Suwon, Korea, where he is a Professor and SKKU Fellow in the School of Information and Communication Engineering. Since 1988, he is engaged in the research activities in the areas of cellular radio networks and spread-spectrum systems. His current research interests include 4G systems, ultra-wideband (UWB) multi-gigabyte short-range transmission, cooperative communications and cognitive radios, and cross-layer design.

Dr. Kim was an Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS: WIRELESS COMMUNICATIONS SERIES and also a Division Editor for the JOURNAL OF COMMUNICATIONS AND NETWORKS. He is currently an Editor for Spread Spectrum Transmission and Access for the IEEE TRANSACTIONS ON COMMUNICATIONS and an Area Editor for Transmission Technology III for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He also serves as Co-Editor-in-Chief for the JOURNAL OF COMMUNICATIONS AND NETWORKS.