

# Multistage Selective ML Decoding for Multidimensional Multicode DS-CDMA with Precoding

Dong In Kim, *Senior Member, IEEE*

**Abstract**—A high-rate, reliable uplink transmission is designed using *multistage selective maximum-likelihood (ML) decoding for the multicode (MC) direct-sequence code-division multiple access (DS-CDMA) with precoding*. The precoding achieves a constant envelope MC signal by adding some redundancy bits, resulting in rate loss, but multidimensional signaling on top of MC DS-CDMA enables to further increase the data rate. The designed multistage selective ML decoding is shown to be promising for use in uplink high-rate and reliable data transmission.

**Index Terms**—Multicode (MC) DS-CDMA, multidimensional (MD) signaling, precoding, multistage selective ML decoding.

## I. INTRODUCTION

REALIZATION of high-rate data transmission has been a crucial issue in 3rd-generation (3G) cellular systems to accommodate asymmetric heavy-load downlink traffic. The high-speed downlink packet access (HSDPA) and high data rate (HDR) schemes for 3G-CDMA cellular systems are the examples, which can provide the rate up to several Mbps for downlink, by using *multi-level modulation* [1], [2].

However, multi-level modulation is not suitable for use in uplink, because the envelope fluctuation causes nonlinear distortion. For this reason, a constant envelope multicode (MC) signaling is preferred in uplink, but the rate loss due to precoding is a limiting factor. As an other option, the variable spreading factor (VSF) scheme [3] can be adopted to support the high rate but with reduced SF, which in turn requires higher signal power or additional processing gain to compensate for the reduced SF.

Multidimensional (MD) signaling on top of MC DS-CDMA was shown to increase the rate without reducing the SF and causing the nonlinear distortion. But, as the number of multicode increases, the lower precoding rate (or larger redundancy) limits the maximum achievable rate, even with MD signaling [4], [5]. Here, two-level precoding was used to send 18 bits/128 chips with 16-parallel MC channels, and the uplink was stabilized by adopting a two-stage interference cancellation scheme in [4] that requires three symbol estimates per channel with *group-wise* error detection in a successive manner.

To overcome this limitation, a new type of MD signaling with *multi-level* precoding is proposed to support 36 bits/128

chips with 16-parallel MC channels. Further, *multistage selective maximum-likelihood (SML) decoding*,<sup>1</sup> where only a desired symbol estimate per channel is needed to perform *group-wise* multistage cancellation and decoding in a successive manner, is designed to stabilize the uplink with manageable complexity. The designed multistage SML decoding is shown to further improve the symbol error probability (SEP), even at higher rate (e.g., 1 Mbps), compared to that in [4] and outperform the VSF scheme, in terms of the bit error probability (BEP).

## II. PROPOSED MDMC SIGNALING

The proposed *MDMC signaling* for a high-rate user is structured as

$$s(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{M-1} \sqrt{P} [d_{I,m}([n/N])b_{I,m}([n/M])c_{I,n} + jd_{Q,m}([n/N])b_{Q,m}([n/M])c_{Q,n}] w_{m,n} p(t - nT_c) \quad (1)$$

where  $w_{m,n} = w_{m,n+M}$  is the  $m$ th channelization code,  $(d_{I,m}, d_{Q,m})$  are the in-phase (I) and quadrature (Q) data at the  $m$ th channel and  $[n/N]$ th signaling time ( $[x]$  denotes the integer part of  $x$ ), and  $\mathbf{b}_{I,m} = [b_{I,m}(0), \dots, b_{I,m}(G-1)]$  represents the I-subchannel ( $m$ th channel)  $G$ -ary orthogonal sequence, being selected by  $\log_2 G$ -bit additional data, per signaling time with  $G = N/M$  and symbol time  $T = NT_c$  for the chip time  $T_c$ .  $\mathbf{b}_{Q,m}$  is the Q-subchannel  $G$ -ary sequence,  $(c_{I,n}, c_{Q,n})$  denote the I/Q-subchannel signature sequences to allow for multiple access, and  $p(t)$  is the unit-magnitude chip pulse of duration  $T_c$ . Note that  $P$  is the signal power per channel, and the total power of MC signal equals  $MP$  for the high-rate user.

To achieve a constant envelope MDMC signal, a *multi-level* precoding is designed on the assumed  $M = 16$ -parallel channels for the I/Q subchannels, where the channelization code  $w_{m,n}$  is selected from each row vector of the Hadamard matrix  $H_M$ , as follows ( $O = I, Q$ ):

- 1-bit precoding is applied to each 4 data bits in group for  $m = 0, 1, \dots, 11$

$$d_{O,4j} \cdot d_{O,4j+1} \cdot d_{O,4j+2} \cdot d_{O,4j+3} = -1 \quad (2)$$

where the first 3 bits are input data and last bit is redundancy for  $j = 0, 1, 2$ , resulting in  $|(d_{O,4j}, d_{O,4j+1}, d_{O,4j+2}, d_{O,4j+3})H_4| = (2, 2, 2, 2)$  for  $H_4 = [(1, 1, 1, 1)^T | (1, -1, 1, -1)^T | (1, 1, -1, -1)^T | (1, -1, -1, 1)^T]$ ,  $T$  denoting the transpose.

<sup>1</sup>A single stage SML detection was designed in [6] with 4-parallel MC channels, combined with MD signaling.

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D. I. Kim was with the School of Engineering Science, Simon Fraser University, Burnaby, BC V5A 1S6, Canada. He is now with the School of Information and Communication Engineering, Sungkyunkwan University, Suwon 440-746, Korea (e-mail: dikim@ece.skku.ac.kr).

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TABLE I

MULTI-LEVEL PRECODING FOR  $M = 16$ -PARALLEL CHANNELS WHEN  $N = 64$  AND  $G = 4$ . THE SUM OF MC SIGNAL,  $SUM(\mathbf{h}_7) = \sum_{m=0}^{M-1} \alpha_{O,m}(g) h_{m,l} (l = 7)$  IS SHOWN FOR THE COLUMN VECTOR  $\mathbf{h}_7$  OF  $H_M$ .

ch.	$\alpha_{O,m}(g)$	$d_{O,m}$	$b_{O,m}(0)$	$b_{O,m}(1)$	$b_{O,m}(2)$	$b_{O,m}(3)$
i	$\alpha_{O,0}(g)$	+	1	-1	-1	1
i	$\alpha_{O,1}(g)$	-	1	1	-1	-1
i	$\alpha_{O,2}(g)$	+	1	-1	1	-1
p	$\alpha_{O,3}(g)$	+	1	1	1	1
i	$\alpha_{O,4}(g)$	-	1	1	-1	-1
i	$\alpha_{O,5}(g)$	-	1	-1	1	-1
i	$\alpha_{O,6}(g)$	+	1	1	1	1
p	$\alpha_{O,7}(g)$	-	1	-1	-1	1
i	$\alpha_{O,8}(g)$	+	1	-1	1	-1
i	$\alpha_{O,9}(g)$	+	1	1	1	1
i	$\alpha_{O,10}(g)$	-	1	-1	-1	1
p	$\alpha_{O,11}(g)$	+	1	1	-1	-1
p	$\alpha_{O,12}(g)$	+	1	1	-1	-1
p	$\alpha_{O,13}(g)$	-	1	-1	1	-1
p	$\alpha_{O,14}(g)$	+	1	1	1	1
p	$\alpha_{O,15}(g)$	+	1	-1	-1	1
$SUM(\mathbf{h}_7)$			+4	+4	-4	+4

- 2)  $\log_2 G$ -bit precoding is used for  $G$ -ary orthogonal sequences, ranging  $g = 0, 1, \dots, G - 1$

$$b_{O,4j}(g) \cdot b_{O,4j+1}(g) \cdot b_{O,4j+2}(g) \cdot b_{O,4j+3}(g) = 1 \quad (3)$$

where the first 3 sequences are selected by input data and last one is precoded sequence for  $j = 0, 1, 2$ , resulting in  $|\alpha_{O,4j}(g), \alpha_{O,4j+1}(g), \alpha_{O,4j+2}(g), \alpha_{O,4j+3}(g)| H_4 = (2, 2, 2, 2)$  for  $\alpha_{O,m}(g) = d_{O,m} \cdot b_{O,m}(g)$ ,  $m = 0, 1, \dots, 11$ .

- 3) Form 'bi-orthogonal'  $2G$ -ary sequences  $\{\alpha_{O,m}(g) | g = 0, 1, \dots, G - 1\}$  for  $m = 0, 1, \dots, 11$ , and the precoding is applied to the resulting envelopes  $\alpha_{O,m}^{(j)}(g) H_4$  (repeating step 1)<sup>2</sup>

$$\frac{1}{M} \left[ \left( \alpha_{O,m}^{(0)}(g) H_4 \right) \odot \left( \alpha_{O,m}^{(1)}(g) H_4 \right) \odot \left( \alpha_{O,m}^{(2)}(g) H_4 \right) \odot \left( \alpha_{O,m}^{(3)}(g) H_4 \right) \right] = -\mathbf{1} \quad (4)$$

where  $\alpha_{O,m}^{(j)}(g) = [\alpha_{O,4j}(g) | \alpha_{O,4j+1}(g) | \alpha_{O,4j+2}(g) | \alpha_{O,4j+3}(g)]$  ( $j = 0, 1, 2, 3$ ),  $\mathbf{1} = (1, 1, 1, 1)$ , and  $\odot$  denotes the vector product.

It is to be noted that, in step 3 above the precoded sequences  $\{\alpha_{O,m}(g) | g = 0, 1, \dots, G - 1\}$  for  $m = 12, 13, 14, 15$  (corresponding to  $j = 3$ ), are not necessarily bi-orthogonal sequences.

Table I illustrates the constructed information (i) and precoded (p) sequences for the  $M = 16$ -parallel channels when  $N = 64$  and  $G = 4$ , and also that their resulting MDMC signal has a constant envelope of  $\sqrt{M}$  via the *multi-level* precoding (2) - (4).

### III. MULTISTAGE SML DECODING

The MDMC signal in (1) for a high-rate user is interfered by concurrent low-rate user signals, which are equivalently

<sup>2</sup>If we view  $\pm H_4$  as elements like  $\pm 1$ , the Hadamard matrix  $H_M$  ( $M = 16$ ) reduces to the same form of  $H_4$ .

modeled as

$$s_k(t) = \sum_{n=0}^{\infty} \sqrt{P_k} [c_{I,n}^{(k)} + j c_{Q,n}^{(k)}] p(t - nT_c), \quad k = 2, \dots, K \quad (5)$$

with one bit (per subchannel) carried on a single code channel.

Assuming two receive antennas, a frequency-selective fading channel is viewed as

$$h_k^{(q)}(t) = \sum_{v=1}^V \beta_{k,v+qV} \delta(t - \tau_{k,v}) \quad (6)$$

at the  $q$ th ( $q = 0, 1$ ) antenna of  $k$ th user,  $\{\beta_{k,v}\}$  and  $\{\tau_{k,v}\}$  are  $V$ -path gains and delays, respectively. With this model, the received signal can be expressed as

$$r_q(t) = s(t) \otimes h_1^{(q)}(t) + \sum_{k=2}^K [s_k(t) \otimes h_k^{(q)}(t)] \quad (7)$$

at the  $q$ th antenna,  $\otimes$  denoting the convolution.

Now, the decision statistics  $X_m(g; v + qV)$  for *bi-orthogonal* detection on the  $m$ th code channel (I-subchannel) can be formed as

$$X_m(g; v + qV) = \frac{1}{MT_c \sqrt{P}} \int_{gMT_c + \tau_{1,v}}^{(g+1)MT_c + \tau_{1,v}} r_q(t) \times \sum_{n=gM}^{(g+1)M-1} c_{I,n} w_{m,n} p(t - nT_c - \tau_{1,v}) dt \quad (8)$$

for  $m = 0, \dots, M - 1$ ;  $g = 0, \dots, G - 1$ ;  $v = 1, \dots, V$ ;  $q = 0, 1$ . Then, the maximal-ratio combining (MRC) with  $2V$  paths available produces<sup>3</sup>

$$\mathbf{X}_m = \sum_{v=1}^{2V} \text{Re} \{ \beta_{1,v}^* \mathbf{X}_m(v) \} \quad (9)$$

where  $\mathbf{X}_m(v) = [X_m(0; v) | \dots | X_m(G - 1; v)]$  and  $*$  denotes the complex conjugate. Hence, the bi-orthogonal ML symbol detection is performed by choosing the maximum

$$\max_{(i=0,1; e=0,\dots,G-1)} \left\{ \rho_m(i, e) = \frac{1}{G} (a_i \cdot \mathbf{X}_m \cdot \mathbf{h}_e^T) \right\} \quad (10)$$

where  $a_i = \pm 1$  is the binary data ( $d_{I,m}$ ) and  $\mathbf{h}_e$  is the  $G$ -ary orthogonal sequence ( $\mathbf{b}_{I,m}$ ) to be chosen from a row vector of the Hadamard matrix  $H_G$ .

For the proposed MDMC signaling with precoding, *multi-stage* SML decoding is designed on the assumed  $M = 16$ -parallel channels as follows:

- 1) The first 12 channels are divided into 3 groups, each of which contains 4 channels associated with two-level precoding in (2) and (3).
- 2) Choose the most *reliable* group in terms of the maximum-likelihood (ML) as

$$\max_{(j=0,1,2)} \left\{ \sum_{n=0}^2 \rho_m(i_m, e_m) \right\}$$

<sup>3</sup>The MRC produces a *coherent* matched-filter output (i.e., matched to the  $m$ th code (I-subchannel) and multipath channel, *complex-valued*, gain coefficients  $\{\beta_{1,v}\}$ ), which forms the log-likelihood function except a constant in (10).

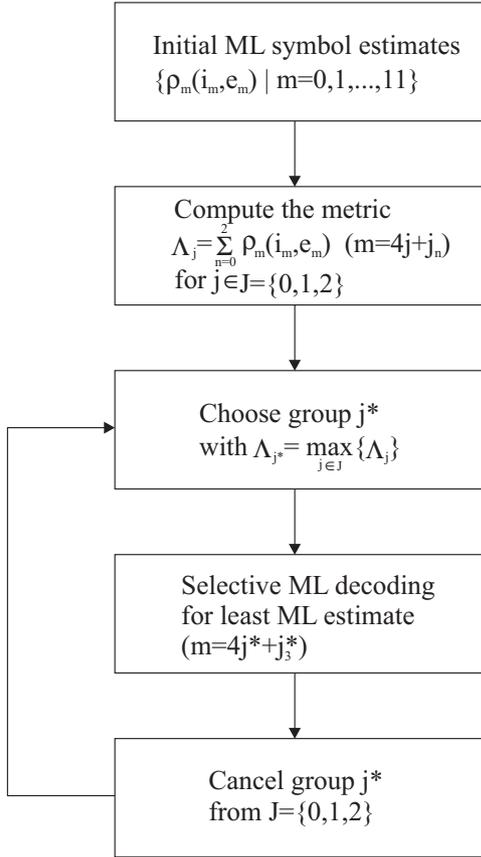


Fig. 1. Flow diagram for *multistage* selective ML decoding on  $M = 16$ -parallel channels.

where  $(i_m, e_m)$  denote the indexes of the bi-orthogonal symbol estimate in (10) on the  $m(=4j+j_n)$ th channel for  $0 \leq j_n \leq 3$ , and in each group the symbol estimate with least ML, i.e.,  $(4j+j_3)$ th channel, is excluded.

- 3) Perform selective ML decoding for the *least* ML symbol estimate in the most reliable group (say  $j = 0$ ) by the two-level precoding rule [6]

$$\begin{aligned} a_{i_{j_3}} &= -a_{i_{j_0}} \cdot a_{i_{j_1}} \cdot a_{i_{j_2}} \\ \mathbf{h}_{e_{j_3}} &= \mathbf{h}_{e_{j_0}} \odot \mathbf{h}_{e_{j_1}} \odot \mathbf{h}_{e_{j_2}} \end{aligned}$$

- 4) Cancel the most reliable group from  $M = 16$ -parallel channels by using the bi-orthogonal symbol estimates decoded above.
- 5) Repeat steps 2, 3 and 4 for the remaining groups.

Fig. 1 shows a flow diagram for implementing the *multistage* (group-wise) selective ML decoding on  $M = 16$ -parallel channels, which can easily be generalized for large  $M$ .

#### IV. ANALYSIS OF SML DECODING

Since the multistage SML decoding involves cancellation of the most reliable groups in a successive manner, it seems to be not tractable to find a closed-form error performance. Hence, the error performance of selective ML decoding is evaluated by nicely deriving the MRC combined output signal-to-interference ratio (SIR), unlike the approach in [5], [6].

From (10), the decision variable  $\rho_m(i, e)$  can be formulated as

$$\rho_m(i, e) = \sum_{v=1}^{2V} |\beta_{1,v}|^2 + \sum_{k=2}^K MAI_k(i, e) + \sum_{v=1}^{2V} MAI_{1,v}(i, e) \quad (11)$$

where  $(a_i, \mathbf{h}_e)$  was sent, the second and third terms represent the other-user interference and the self-interference (SI), respectively. Then, we find the following lemma useful in evaluating the error performance of selective ML decoding below.

**Lemma 1:** The MRC combined output SIR in (11) is derived as

$$\gamma = \frac{N}{2\mathbf{E}\{\bar{\eta}^2(\delta)\}} \left[ \left( \frac{\sum_{v=1}^{2V} |\beta_{1,v}|^2}{(K-1)V\varepsilon^2} \right)^{-1} + \left( \frac{\Omega V}{M(V-1)} \right)^{-1} \right]^{-1} \quad (12)$$

where  $\mathbf{E}$  denotes the expectation,  $\varepsilon^2 \triangleq P_k/P$  ( $k \geq 2$ ),  $\bar{\eta}(\delta) = 1/T_c \int_0^\delta p(t)p(t-\delta+T_c)dt$ ,  $\delta$  uniformly distributed over  $[0, T_c]$ , and  $\Omega$  is nicely approximated to<sup>4</sup>

$$\Omega \cong \frac{(A+B)^2}{A^2+B^2} \rightarrow 2$$

where  $A = \sum_{v=1}^V |\beta_{1,v}|^2 \rightarrow V/n$  and  $B = \sum_{v=1}^V |\beta_{1,v+V}|^2 \rightarrow V/n$  for  $n \geq 1$ . Note that  $n = 1$  corresponds to no fading with equal average path power normalized to 1 (i.e.,  $\mathbf{E}\{|\beta_{k,v}|^2\} = 1$ ).

**Proof of Lemma 1:** See Appendix.

For Gaussian-modeled interference, the probability of symbol error on a *single channel* is first evaluated as [5], [7]

$$\begin{aligned} P_1(\epsilon) &= 1 - \Pr \left[ \bigcap_{(i', e')} |\rho_m(i', e')| < \rho_m(i, e) \mid a_i, \mathbf{h}_e \right] \\ &= 1 - \int_0^\infty [1 - 2Q(x)]^{G-1} \mathbf{E}\{\phi(x - \sqrt{\gamma})\} dx \quad (13) \end{aligned}$$

where  $i' = 0$ ,  $e' \neq e$ , and  $Q(x) = \int_x^\infty \phi(u) du$  for  $\phi(u) = 1/\sqrt{2\pi} \exp(-u^2/2)$ .

Next, the selective ML decoding is performed *group-wise* (on 4 channels), from which the *least* ML symbol estimate is decoded. Hence, if a symbol error happens to occur on the least ML channel, the two-level precoding rule allows to correctly decode it. In this case, conditioned on  $\gamma$ , the probability of symbol error can be evaluated as

$$\begin{aligned} P(\epsilon|\gamma) &\cong \frac{\varphi}{3} \left\{ 1 - [1 - P_1(\epsilon|\gamma)]^4 - 4\xi P_1(\epsilon|\gamma) \right. \\ &\quad \left. \times [1 - P_1(\epsilon|\gamma)]^3 \right\} \quad (14) \end{aligned}$$

where  $\xi$  represents the probability of having a single error on the least ML channel, given the single error and no error on the other 3 channels. Note that the second term in brackets indicates no error on 4 channels, in which case the terms in brackets account for the word error probability (WEP) per group. There are 3 information symbols per word, where the

<sup>4</sup>Note that  $\Omega = (A+B)/V \rightarrow 2/n$  in [5], [6], causing the output SIR to become smaller, as  $n$  increases (i.e., severe fading). It should be noted that, here the SI is treated as one being conditioned on the *desired* first user's channel gain coefficients  $\{\beta_{1,v}\}$ , unlike [5], [6] where the SI was evaluated in *average sense* (like the other-user interference).

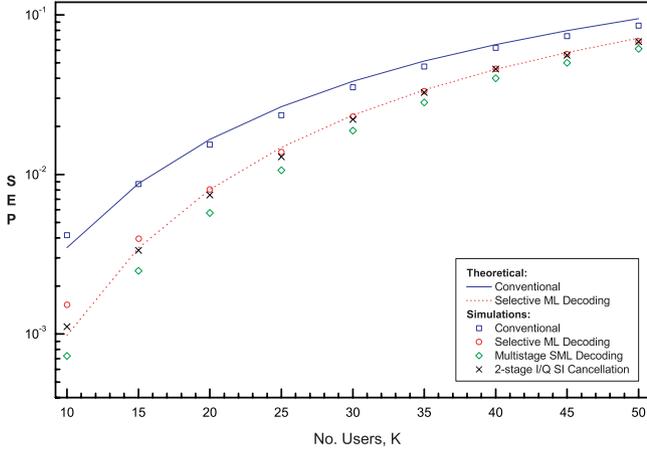


Fig. 2. SEP  $P(\epsilon)$  versus  $K$  for MDMC signaling when multipath ( $V = 3$ ) Rayleigh fading with equal gain ( $\mathbf{E}\{|\beta_{k,v}|^2\} = 1$ ) is assumed with 2 receive antennas.

average symbol errors are about  $\varphi = 1.5$  per word due to the possible double errors by precoding, so that the factor  $\varphi/3$  converts the WEP to the SEP. To obtain the SEP  $P(\epsilon) = \mathbf{E}\{P(\epsilon|\gamma)\}$ , the expectation is numerically taken for the SIR  $\gamma$  in (12), where  $\xi$  is found in (13) of [6].

## V. RESULTS AND REMARKS

The proposed MDMC signaling with precoding is tested under the system conditions that MC channels are  $M = 16$ , spreading factor (SF) is  $N = 128$ , and  $G = N/M = 8$ -ary orthogonal sequences are used in the 9 information channels (see Table I) to send 3 additional bits, resulting in 4 bits per channel. Thus, a high-rate user, as given in (1) can achieve the data rate of  $4 \times 9 = 36$  bits/ $N = 128$  chips (per subchannel), and if the chip rate is 3.84 Mcps (e.g., WCDMA in [3]), the data rate is about 1 Mbps for uplink transmission.<sup>5</sup> To assess the multi-user performance,  $(K - 1)$  low-rate users in (5) act as interference with the power ratio  $\varepsilon^2 \triangleq P_k/P = M/36 = 4/9$ . The channel is assumed to be  $V = 3$ -path Rayleigh fading with equal gain, and 2 receive antennas are used for MRC diversity combining where spatial correlation is assumed to be zero and otherwise, *selection diversity gain* is gradually reduced but the gain can be maintained for the power correlation coefficient  $< 0.5$  in [5].

In Fig. 2, the Gaussian approximation to the SEP, as evaluated in (13) and (14) is compared with the simulation results for conventional (symbol) detection and selective ML decoding, respectively. When  $K$  is sufficiently large, the approximation closely follows the simulation results for both cases so that the output SIR nicely derived in (12) (unlike the one used in [5], [6]) is meaningful for preliminary analysis. As anticipated, the approximation tends to be somewhat optimistic in the region of relatively small  $K$ , which complies with prior observations in CDMA literatures. Also, the performance improvement with proposed *multistage* SML decoding

<sup>5</sup>For illustration, a half-rate channel coding is assumed to map a serial input data into two parallel I/Q subchannels. Further, if  $N = 32$  is used instead, the *achievable* rate is doubled (i.e., 2 Mbps) with a certain loss in SIR.

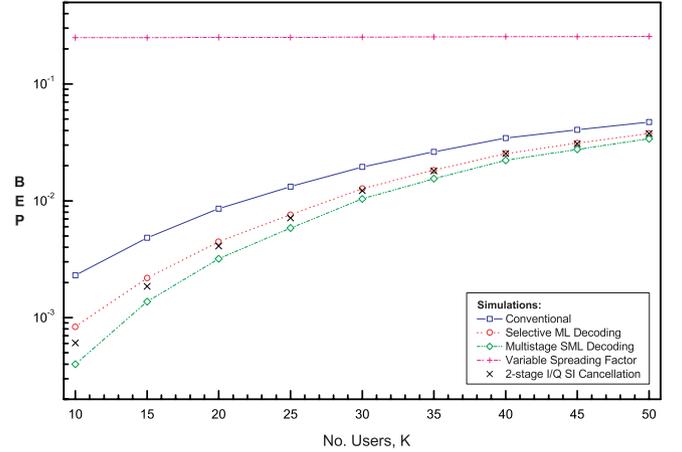


Fig. 3. BEP comparison for MDMC signaling and VSF scheme (32 bits/ $N = 128$  chips) when multipath ( $V = 3$ ) Rayleigh fading with equal gain ( $\mathbf{E}\{|\beta_{k,v}|^2\} = 1$ ) is assumed with 2 receive antennas.

is apparent compared to the conventional (symbol) detection, and further it outperforms the I/Q SI cancellation in [4] with *group-wise* error detection iterated twice (i.e., 2-stage), as well as in terms of the BEP in Fig. 3. Note that complexity increases further with the SI cancellation, because it requires three symbol estimates per channel due to multipath-delayed signals. Unlike this, the multistage SML decoding requires one symbol estimate per channel for the current symbol estimates *desired* in each group.

Fig. 3 shows the BEP for the MDMC signaling and variable spreading factor (VSF) scheme, in which the VSF scheme uses the SF of 4 per bit, resulting in 32 bits/ $N = 128$  chips. Hence, the processing gain is not sufficient to yield an acceptable BEP with VSF scheme, whereas the MDMC signaling with *multistage* SML decoding offers a high-rate, reliable uplink transmission (i.e., BEP  $< 10^{-3}$  at 1 Mbps when  $K = 10$  users are present.) This implies that the proposed MDMC signaling, combined with *multistage* SML decoding (or SML decoding with manageable complexity), paves the way for high-speed wireless multimedia services.

## APPENDIX PROOF OF LEMMA 1 :

First, the  $k$ th ( $k \geq 2$ ) user interference  $MAI_k(i, e)$  in (11) can be formulated as

$$MAI_k(i, e) = \frac{1}{N} \sum_{v=1}^{2V} \sum_{v'=1+qV}^{(1+q)V} \operatorname{Re} \left\{ \beta_{1,v}^* \beta_{k,v'} \varepsilon \times \sum_{n=0}^{N-1} a_i \Gamma_k(n) c_{I,n} w_{m,n} h_{e,[n/M]} \right\} \quad (15)$$

where  $q = [(v - 1)/V]$  and the complex-valued partial correlation function  $\Gamma_k(n)$  is

$$\Gamma_k(n) = \tilde{c}_{n-1-l_{k,v'}}^{(k)} \bar{\eta}(\delta_{k,v'}) + \tilde{c}_{n-l_{k,v'}}^{(k)} \bar{\eta}(T_c - \delta_{k,v'}) \quad (16)$$

for  $\tilde{c}_n^{(k)} = c_{I,n}^{(k)} + j c_{Q,n}^{(k)}$ ,  $l_{k,v'} = [(\tau_{k,v'} - \tau_{1,v})/T_c]$ , and  $\delta_{k,v'} = \tau_{k,v'} - \tau_{1,v} - l_{k,v'} T_c$ .

Similarly, the self-interference  $MAI_{1,v}(i, e)$  in (11) is put in the form

$$MAI_{1,v}(i, e) = \frac{1}{N} \sum_{\substack{v'=1+qV \\ v' \neq v}}^{(1+q)V} \operatorname{Re} \left\{ \beta_{1,v}^* \beta_{1,v'} \right. \\ \left. \times \sum_{n=0}^{N-1} a_i \Gamma_1(n) c_{I,n} w_{m,n} h_{e,[n/M]} \right\} \quad (17)$$

where the partial correlation function  $\Gamma_1(n) = \tilde{\mu}_{n-1} \bar{\eta}(\delta_{1,v'}) + \tilde{\mu}_n \bar{\eta}(T_c - \delta_{1,v'})$  for the complex-valued MC signal  $\tilde{\mu}_n = \mu_{I,n} + j\mu_{Q,n}$ , in which

$$\mu_{I,n} = \sum_{m=0}^{M-1} d_{I,m}([(n-l_{1,v'})/N]) b_{I,m}([(n-l_{1,v'})/M]) \\ \times c_{I,n-l_{1,v'}} w_{m,n-l_{1,v'}}. \quad (18)$$

After a few steps, the corresponding second-order moments can be evaluated as

$$\mathbf{E}\{MAI_k^2(i, e)\} = \frac{2}{N} \sum_{v=1}^{2V} |\beta_{1,v}|^2 V \varepsilon^2 \mathbf{E}\{\bar{\eta}^2(\delta)\} \quad (19)$$

$$\mathbf{E}\{MAI_{1,v}^2(i, e)\} = \frac{2}{N} \sum_{\substack{v'=1+qV \\ v' \neq v}}^{(1+q)V} |\beta_{1,v}|^2 |\beta_{1,v'}|^2 M \mathbf{E}\{\bar{\eta}^2(\delta)\} \quad (20)$$

where if  $V \gg 1$ , the latter is closely approximated to

$$\mathbf{E}\{MAI_{1,v}^2(i, e)\} \cong \frac{2}{N} \sum_{v'=1+qV}^{(1+q)V} |\beta_{1,v}|^2 |\beta_{1,v'}|^2 \frac{(V-1)}{V} \\ \times M \mathbf{E}\{\bar{\eta}^2(\delta)\}. \quad (21)$$

Since the cross-terms in evaluating the variance of (11) average out, the MRC combined output SIR,  $\gamma \triangleq \left( \sum_{v=1}^{2V} |\beta_{1,v}|^2 \right)^2 / \operatorname{var}[\rho_m(i, e)]$  is derived as (12).

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