Selective Maximum-Likelihood Symbol-by-Symbol Detection for Multidimensional Multicode WCDMA With Precoding

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***Abstract*—To faciliate high-rate uplink transmission, a novel selective maximum-likelihood (SML) detection is realized on a set of parallel multicode (MC) channels as a result of MC wideband code-division multiple-access (WCDMA) signaling with precoding. Multidimensional (MD) signaling can be combined with MC to further increase the data rate with fixed spreading factor per symbol [5]. In connection with this MDMC signaling, the proposed SML detection achieves significantly improved symbol-error probability, compared with other detection methods, both without soft sequential decoders for a tractable analysis.**

***Index Terms*—Multicode (MC) wideband code-division multiple access (WCDMA), multidimensional (MD) signaling, precoding, selective maximum-likelihood (SML) detection.**

I. Introduction

REALIZING high-rate data transmission is a critical factor in evolving the third-generation wideband code-division multiple-access (WCDMA) cellular system which aims at providing wireless multimedia services [1]. Most of the recent research has focused more on high-rate downlink transmission to address the issue of asymmetric traffic loads. However, in order to efficiently provide interactive data services such as real-time video, it is equally important to achieve higher and multiple rates in both directions, for which multicode (MC) and variable spreading factor (VSF) WCDMA signaling options were considered [1].

Meanwhile, to adopt a cost-effective nonlinear amplifier at the mobile unit, it is required to have a constant envelope signal, especially for the MC option in uplink transmission. This has motivated the use of precoding in connection with MC signaling [2]–[4], which causes a certain loss in information rate due to the precoding redundancy. To compensate for this loss, a new multidimensional (MD) MC signaling has been proposed in [5], where an adaptive diversity-combining receiver was realized to exploit the error-detection capability offered by precoding. It was shown that the MDMC signaling offers better link performance than the VSF scheme, even at higher rates, by using the adaptive diversity receiver.

In this letter, the link performance, in terms of the symbol-error probability (SEP), is further improved with a novel *selective* maximum-likelihood (SML) detection, which is realized on a set of M-parallel MC and MDMC channels with precoding. The motivation behind the SML detection is to use the *more reliable* symbol estimates out of total M symbols, in terms of the likelihood function, without relying on the *less reliable* symbol estimates.[1] This is due to correct decoding of less reliable symbols being made possible by exploiting the precoding redundancy. In a real system, the soft sequential decoders need to be realized separately for each code channel, where the SML soft detection is first performed combined with the decoders (e.g., uplink in IS-95 CDMA [6]), after which, the precoding rule is applied to decode less reliable symbols.

II. System Description

A. MDMC Signaling

Fig. 1 shows the WCDMA system based on in-phase (I) and quadrature (Q) (I/Q) code multiplexing and MDMC signaling with $M = 4$-parallel codes per I/Q subchannels.

First, the *high-rate* MDMC signaling (refer to [5] for details) is modeled as

$$\begin{aligned} s(t) = &\sum_{n=0}^{\infty}\sum_{g=0}^{G-1}\sum_{m=0}^{M-1}\sqrt{P} \\ &\times [d_{I,m}(n)b_{I,m}(g;n)c_I(t-gT_w-nT) \\ &\quad + jd_{Q,m}(n)b_{Q,m}(g;n)c_Q(t-gT_w-nT)] \\ &\times w_m(t-gT_w-nT). \end{aligned} \tag{1}$$

Here, P is the signal power per code, T is the symbol time, and $\{d_{O,m}(n), m=0,1,\ldots,M-1\}(O=I,Q)$ is the M-parallel binary data at the nth signaling time that are encoded by the channelization codes (i.e., MC) $\{w_m(t), m=0,1,\ldots,M-1\}$, defined by

$$w_m(t) = \sum_{l=0}^{M-1} w_{m,l}\,p(t-lT_c) \tag{2}$$

for $w_{m,l} = \pm 1$ and unit-magnitude chip pulse $p(t)$ of duration T_c. The MC $w_m(t)$ of length $T_w = MT_c = T/G$ repeats G times, being weighted by the G-ary orthogonal sequence $\{b_{O,m}(g;n), g=0,1,\ldots,G-1\}$ that is selected by additional $\log_2 G$-bit data.

Thus, a symbol defined on a code channel carries $(1+\log_2 G)$ bits per symbol, where MC signaling carries one bit per code, while MD signaling carries $\log_2 G$ bits per code. Here, the spreading factor is defined by $N = M \cdot G$ chips per symbol.

[1]Note that some of the code channels having *less* interference would be selected for the SML detection.

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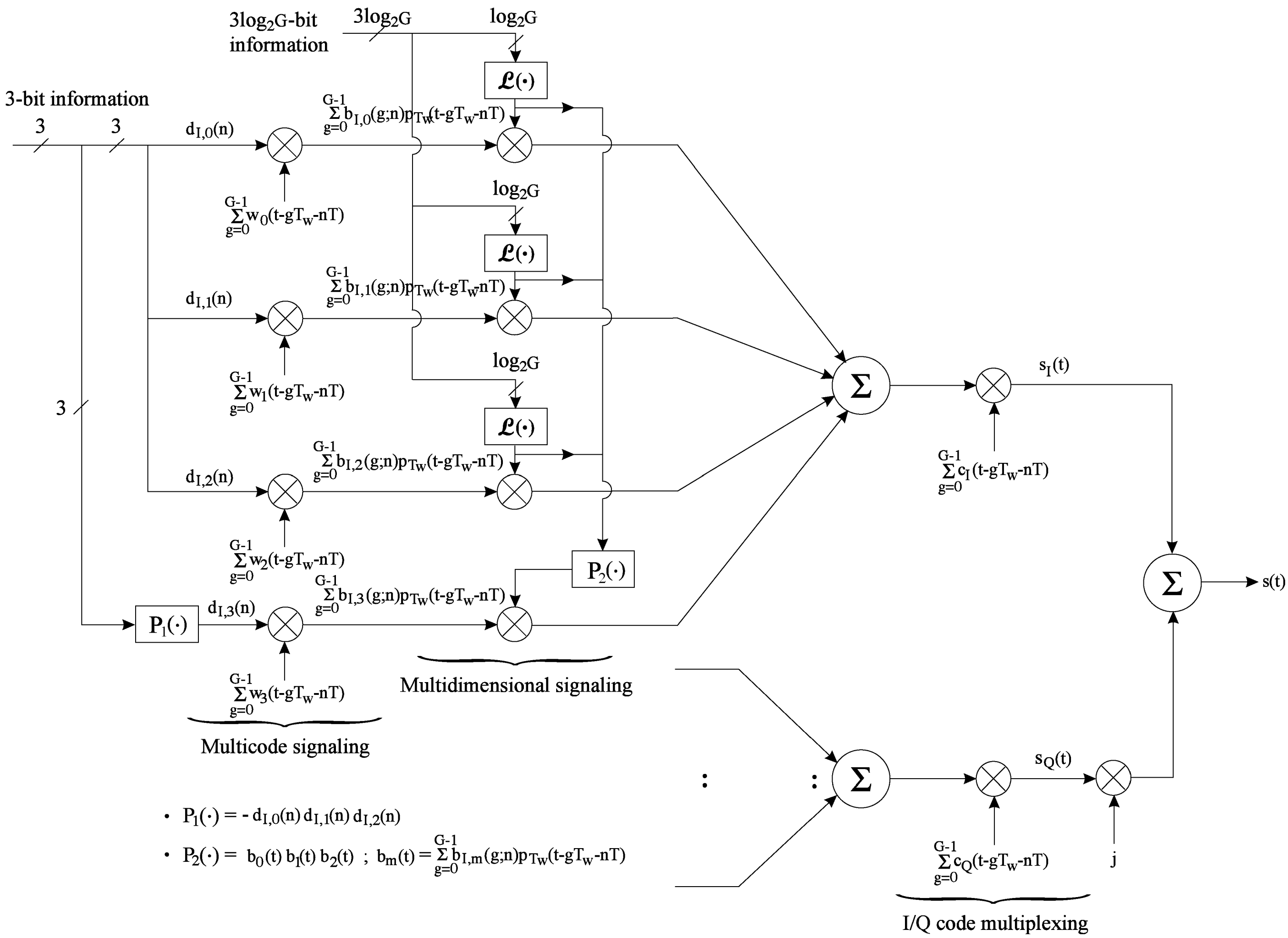

Fig. 1. WCDMA system based on MDMC signaling with $M = 4$-parallel codes per I/Q subchannels, where $\mathcal{L}(\cdot)$ is the one-to-one mapping and $p_{T_w}(t)$ is the unit-magnitude pulse of duration T_w.

To allow for anti-multipath capability and to distinguish among users, the MDMC signal is *concatenated* by the I/Q *independent* spreading codes[2] which are

$$c_O(t-gT_w-nT)=\sum_{l=0}^{M-1} c_{O,nN+gM+l}\,p\,(t-(nN+gM+l)T_c) \tag{3}$$

for $c_{O,l}=\pm 1$ and symbol time $T=NT_c=GT_w$.

To achieve a constant envelope MDMC signaling, *two-level precoding* is applied to $s(t)$ in (1), which is defined for $M=4^n$, $n=1,2,\ldots$ [5]. As illustrated in Fig. 1 with $M=4$

$$d_{O,0}(n)\cdot d_{O,1}(n)\cdot d_{O,2}(n)\cdot d_{O,3}(n) = -1 \tag{4}$$

$$b_{O,0}(g;n)\cdot b_{O,1}(g;n)\cdot b_{O,2}(g;n)\cdot b_{O,3}(g;n)=1. \tag{5}$$

The two-level precoding rule results from [3] and [5], where the *row-closure* property holds for the orthogonal Hadamard matrix $H_{G\times G}$ with elements ± 1, to be used for the G-ary orthogonal signaling via $\{b_{O,m}(g;n)\}$. It should be noted that $\{d_{O,m}(n)b_{O,m}(g;n), g=0,1,\ldots,G-1\}$ form the $(2G)$-ary *biorthogonal* signal for the mth code channel.

[2]Note that complex spreading used in the WCDMA system [1] is not considered here to simplify analysis.

B. SML Detection

As described in Fig. 1, the MDMC signaling with precoding may be detected for the entire codeword, using the ML rule. However, the entire codeword requires $2^{rM(1+\log_2 G)}$-ary symbol-by-symbol detection when the precoding of rate $r<1$ is used.

For example, when $M=4$, $(r=3/4)$, and $G=32$ are assumed (the entire codeword carries 18 b), the ML rule should be defined for 2^{18}-ary symbol-by-symbol detection.

To avoid the huge complexity with *optimum* ML detection, a symbol-by-symbol detection is realized on each code channel, which is *suboptimum* but requires manageable complexity (i.e., 64-ary biorthogonal demodulation).

Following the system model in [5], the channel model and an expression for the baseband-equivalent received signal are found in [5, Sec. II, eqs. (5) and (6)]. Then, a symbol-by-symbol (biorthogonal) detection at the mth code channel is based on the decision variable as follows:

$$\text{choose} \max_{\substack{(i=0,1)\\(e=0,\ldots,G-1)}} \left\{\rho_m(n;i,e)=\frac{1}{G}\left(\alpha_i\cdot \mathbf{X}_m(n)\cdot \mathbf{h}_e^T\right)\right\} \tag{6}$$

where $\alpha_i = \pm 1$, $\mathbf{h}_e$ is the row vector of $H_{G\times G}$ as the G-ary orthogonal symbol, $\mathbf{h}^T$ denotes the transpose of $\mathbf{h}$, and the decision vector $\mathbf{X}_m(n) \triangleq [X_{m,0}(n), \ldots, X_{m,G-1}(n)]$ can be formulated as in [5, Sec. III, eqs. (10) and (12)] with $J = 1$ (i.e., $\langle m \rangle = m$) considered here.

By virtue of the redundancy added by precoding, the receiver can choose *more reliable* symbol estimates, in terms of the likelihood function, and then apply the precoding rule to decode a *less reliable* symbol as follows ($M = 4$).

1) Find the biorthogonal symbol estimate $(\alpha_{i_m}, \mathbf{h}_{e_m})$ using the ML rule in (6) for the mth code channel, $m = 0, \ldots, M-1$.
2) Discard the symbol estimate $(\alpha_{i_{m'}}, \mathbf{h}_{e_{m'}})$ for the m'th code channel corresponding to
$$\hat{\rho}_{m'}(n; i_{m'}, e_{m'}) = \min_{(m=0,\ldots,M-1)} \{\hat{\rho}_m(n; i_m, e_m)\}$$
where the decision variable $\hat{\rho}_m(n; i_m, e_m)$, defined by $\max\{\rho_m(n; i, e)\}$, is found in (6).
3) Decode the less reliable symbol $(\alpha_{i_{m'}}, \mathbf{h}_{e_{m'}})$ using the precoding rule in (4) and (5) as
$$\alpha_{i_{m'}} = -\prod_{m=0, m\neq m'}^{M-1} \alpha_{i_m} \text{ and } \mathbf{h}_{e_{m'}} = \bigodot_{m=0, m\neq m'}^{M-1} \mathbf{h}_{e_m}$$
where $\bigodot$ denotes the vector product.

The above-proposed SML detection can easily be generalized to the MC/MDMC signaling with precoding of arbitrary rate $(M-f)/M$ for $f > 1$. In this case, f out of M symbol estimates are discarded in step 2 in an ascending order of $\{\hat{\rho}_m(n; i_m, e_m), m = 0, \ldots, M-1\}$, and then the f (less reliable) symbols are decoded by the specified precoding rule in step 3.

III. Analysis of Link Performance

To evaluate the performance of ML detection in step 1 above, the decision variable $\rho_m(n; i, e)$ in (6) is formulated [from [5, Secs. II and III, eqs. (5), (6), (10), and (12)]] as

$$\rho_m(n; i, e) = \sum_{v=1}^{2V} |\beta_{1,v}|^2 + \sum_{k=2}^{K} \mathrm{MAI}_k(i, e) + \sum_{v=1}^{2V} \left[\mathrm{MAI}_{1,v}^{(I)}(i, e) + \mathrm{MAI}_{1,v}^{(Q)}(i, e)\right] \quad (7)$$

where $(\alpha_i, \mathbf{h}_e)$ was assumed sent, $\{\beta_{1,v}\}$ denotes the *uncorrelated* path gains for two receive antennas, each following a V-path Rayleigh fading channel, and MAI_k represents the multiple-access interference (MAI) due to kth user along with the I/Q self-interference $\mathrm{MAI}_{1,v}^{(O)}$.

For the Gaussian-modeled MAI,[3] the SEP in each code channel with biorthogonal signaling ($G \geq 2$) and hard detection for a tractable analysis [5], [7] is

$$P_1(\epsilon) = 1 - \Pr\left[\bigcap_{(i',e')} |\rho_m(n; i', e')| < \rho_m(n; i, e) | \alpha_i, \mathbf{h}_e\right] = 1 - \int_0^\infty [1 - 2Q(x)]^{G-1} \mathbf{E}\{\phi(x - \sqrt{\gamma})\} dx \quad (8)$$

where $e' \in \{0, 1, \ldots, G-1\}$, $e' \neq e$, $\mathbf{E}$ is the expectation operator, and $Q(x) = \int_x^\infty \phi(u) du$ for $\phi(u) = 1/\sqrt{2\pi} \exp(-u^2/2)$. Note that γ represents the signal-to-interference ratio (SIR) as a function of the path gains $\{\beta_{1,v}, v = 1, \ldots, 2V\}$ in (7).

Since a direct evaluation of the SEP for the SML detection is complicated due to precoding, the probability of a *codeword* of $M = 4$ symbols ($f = 1$) being in error is first derived as

$$\begin{aligned} P_C(\epsilon|\gamma) = 1 &- \Pr\left[\bigcap_{m=0}^{M-1} \hat{\rho}_m(n; i_m, e_m) | \{\alpha_{i_m}\}, \{\mathbf{h}_{e_m}\}\right] \\ &- M \Pr\left[\bigcap_{m\neq m'} \hat{\rho}_m(n; i_m, e_m) \times \bigcap \hat{\rho}_{m'}(n; i, e) | \{\alpha_{i_m}\}, \{\mathbf{h}_{e_m}\}\right] \\ &\times \Pr\left[\bigcap_{m\neq m'} \hat{\rho}_m(n; i_m, e_m) > \hat{\rho}_{m'}(n; i, e) | \{\hat{\rho}_m(n; i_m, e_m), m \neq m'\}, \hat{\rho}_{m'}(n; i, e), (i, e) \neq (i_{m'}, e_{m'})\right] \end{aligned} \quad (9)$$

where $\hat{\rho}_m(n; i_m, e_m) \triangleq \rho_m(n; i_m, e_m) > \max\{|\rho_m(n; i', e')|, (i', e') \neq (i_m, e_m)\}$ indicates the correct detection of the mth symbol ($m = 0, \ldots, M-1$), given $(\alpha_{i_m}, \mathbf{h}_{e_m})$ was sent.

In (9), the second term denotes the probability of all M symbols being detected correctly, while the third term represents the joint probability that a single symbol error, i.e., $(\alpha_i, \mathbf{h}_e) \neq (\alpha_{i_{m'}}, \mathbf{h}_{e_{m'}})$, occurs on a particular m'th code channel in M ways and its ML $\hat{\rho}_{m'}(n; i, e)$ is minimum, so as to be discarded in step 2 above. Thus, the sum of second and third terms accounts for the probability that there is no symbol error after the SML detection, namely, the probability of correct codeword reception. With $M - f$ information symbols ($f = 1$), the SEP for the SML detection, denoted by $P_M(\epsilon|\gamma)$, is first-order approximated to

$$P_M(\epsilon|\gamma) \approx \frac{\varphi}{M - f} P_C(\epsilon|\gamma) \quad (10)$$

for the average number of symbols in error, φ. This approximation is based on the observation that a single symbol error (i.e., $\varphi = 1$) is the most probable error for a high-rate system, since

[3]The I/Q independent code multiplexing with multipath renders the MAI statistics approximately Gaussian.

the system must be operated near the target bit-error probability (BEP). But the second-order effect due to precoding may cause double symbol errors, leading to $\varphi \cong 1.5$.

To generalize the derivation in (9) for $M > 4$, the additional terms need to be evaluated such that two or more symbol errors are discarded in step 2 above.

Since the M code channels are mutually orthogonal, and the MAI in (7) is zero mean, $\{\rho_m(n; i, e)\}$ become uncorrelated between any two code channels $(m \neq m')$, which, in turn, become independent if the MAI can be modeled as Gaussian. Therefore, it follows that

$$\Pr\left[\bigcap_{m=0}^{M-1} \hat{\rho}_m(n; i_m, e_m) | \{\alpha_{i_m}\}, \{\mathbf{h}_{e_m}\}\right] = [1 - P_1(\epsilon|\gamma)]^M \tag{11}$$

$$\Pr\left[\bigcap_{m\neq m'} \hat{\rho}_m(n; i_m, e_m) \bigcap \hat{\rho}_{m'}(n; i, e) | \{\alpha_{i_m}\}, \{\mathbf{h}_{e_m}\}\right] = P_1(\epsilon|\gamma)\,[1 - P_1(\epsilon|\gamma)]^{M-1} \tag{12}$$

where $P_1(\epsilon|\gamma)$ is found in (8) for a given γ without $\mathbf{E}$. In the Appendix, it is shown that

$$\Pr\left[\bigcap_{m\neq m'} \hat{\rho}_m(n; i_m, e_m) > \hat{\rho}_{m'}(n; i, e) | \right. \\ \left. \{\hat{\rho}_m(n; i_m, e_m), m \neq m'\}, \hat{\rho}_{m'}(n; i, e), (i, e) \neq (i_{m'}, e_{m'})\right] \\ = \int_0^{\infty} \left[1 - \frac{\eta(x)}{1 - P_1(\epsilon|\gamma)}\right]^{M-1} \left[\frac{1 - Q(x - \sqrt{\gamma})}{P_1(\epsilon|\gamma)}\right] \\ \cdot 2(G-1)\phi(x)\,[1 - 2Q(x)]^{G-2}\,dx \tag{13}$$

where the event of making a symbol error to $(-\alpha_{i_{m'}}, \mathbf{h}_{e_{m'}})$ (farthest signal point) is ignored, and $\eta(x) \triangleq \int_0^x [1 - 2Q(u)]^{G-1} \phi(u - \sqrt{\gamma})du$, converging to $\lim_{x\to\infty} \eta(x) = 1 - P_1(\epsilon|\gamma)$.

Finally, to evaluate the *unconditional* SEP $P_M(\epsilon)$, an expectation should be numerically taken with respect to the SIR γ in (9), which is found in [5] $(J = 1)$ as

$$\gamma = \left(\frac{\sum_{v=1}^{2V} |\beta_{1,v}|^2}{\sigma_\beta^2}\right) \cdot \left[\frac{N}{2\left[(K-1)V\varepsilon^2 + M(V-1)\right]\mathbf{E}\{\bar{\eta}^2(\delta)\}}\right] \tag{14}$$

where $\mathbf{E}\{|\beta_{1,v}|^2\} = \sigma_\beta^2$ for identical path gains. Here, the user power ratio is $\varepsilon^2 \triangleq P_k/P$ $(k \geq 2)$ and the partial chip-pulse correlation is given by $\bar{\eta}(\delta) = 1/T_c \int_0^\delta p(t)p(t - \delta + T_c)dt$, where $\mathbf{E}\{\bar{\eta}^2(\delta)\} = 1/3$ for the rectangular pulse. Note that the probability density function $f_M(s)$ of $s \triangleq \sum_{v=1}^{2V} |\beta_{1,v}|^2/\sigma_\beta^2$ for identical/uncorrelated Rayleigh-faded path gains is

$$f_M(s) = \frac{s^{2V-1}}{(2V-1)!}e^{-s}, \quad s \geq 0. \tag{15}$$

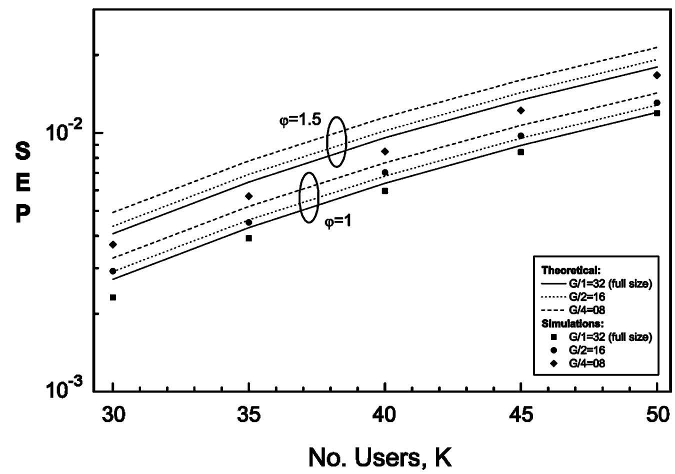

Fig. 2. SEP $P_M(\epsilon)$ versus K for MDMC signaling when multipath $(V = 3)$ Rayleigh fading with equal gain $(\sigma_\beta^2 = 1/3)$ is assumed with two receive antennas.

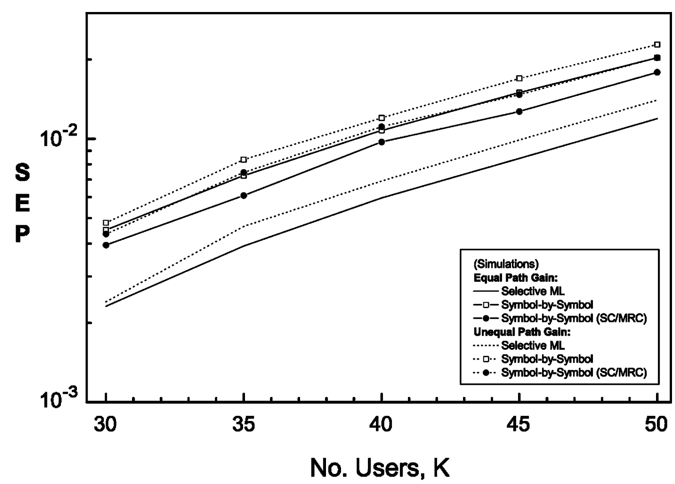

Fig. 3. SEP $P_M(\epsilon)$ versus K for MDMC signaling when multipath $(V = 3)$ Rayleigh fading with equal gain $(\sigma_\beta^2 = 1/3)$ and unequal gains $(\sigma_\beta^2(1) = 0.51, \sigma_\beta^2(2) = 0.31, \sigma_\beta^2(3) = 0.18, \sigma_\beta^2(4) = 0.49, \sigma_\beta^2(5) = 0.32$, and $\sigma_\beta^2(6) = 0.19)$ are assumed with two receive antennas.

IV. Results and Remarks

The key parameters assumed here are as follows: the spreading factor $N = 128$, the MDMC signaling with $M = 4$ parallel codes and precoding of rate 3/4, where $G = N/M = 32$-ary orthogonal signaling carries 5 additional bits per code channel, resulting in total 18 bits over the entire codeword for a high-rate user. For γ in (14), $(K - 1)$ interfering low-rate users carry a single bit over $N = 128$ chips, yielding the power ratio $\varepsilon^2 = M/18 = 2/9$.

Fig. 2 shows the SEP of the first-order approximation in (10) with $\varphi = 1, 1.5$, where a subset of (G/m)-ary orthogonal sequences are used for the MD signaling to carry $[1 + \log_2(G/m)]$ bits per symbol for $m = 1, 2, 4$. It is seen that the approximation follows the simulation results, but there is a gap, mainly because the second-order effect due to precoding affects the SEP, and also the Gaussian approximation to the MAI statistics is somewhat corrupted by the constant envelope property existing in the MDMC signaling.

Fig. 3 clearly demonstrates that the proposed SML detection further improves the SEP, compared with the adaptive selection combining/maximal-ratio combining (SC/MRC) for the

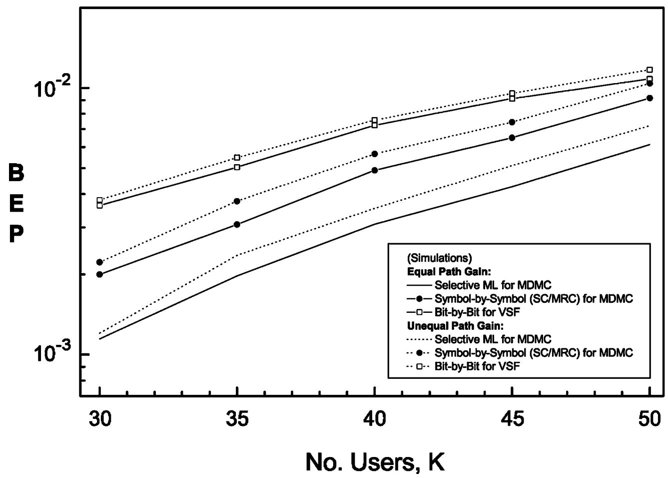

Fig. 4. BEP comparison for MDMC signaling and VSF scheme (16 bits/$N = 128$ chips) when multipath ($V = 3$) Rayleigh fading with equal gain ($\sigma_\beta^2 = 1/3$) and unequal gains ($\sigma_\beta^2(1) = 0.51$, $\sigma_\beta^2(2) = 0.31$, $\sigma_\beta^2(3) = 0.18$, $\sigma_\beta^2(4) = 0.49$, $\sigma_\beta^2(5) = 0.32$, and $\sigma_\beta^2(6) = 0.19$) are assumed with two receive antennas.

MDMC signaling in [5], considering both equal and unequal path gains at the same data rate (i.e., 18 bits/$N = 128$ chips). This implies that the proposed SML detection can realize high-rate uplink transmission by maximizing the effective diversity gain via precoding.

In Fig. 4, the MDMC signaling in connection with SML detection provides a significant improvement in BEP, compared with the conventional VSF scheme with bit-by-bit detection, while the complexity incurred by the MDMC signaling/detection is on the order of the IS-95 uplink system (i.e., 64-ary Walsh orthogonal signaling), especially when $N = 128$.

Appendix
Derivation of the Probability in (13)

Let us define $X \triangleq \hat{\rho}_{m'}(n; i, e) = \max\{|\rho_{m'}(n; i', e')|, (i', e') \neq (i_{m'}, e_{m'})\}$, conditioned on $X > Z \triangleq \rho_{m'}(n; i_{m'}, e_{m'})$, whose probability distribution function (PDF) can be derived as

$$F_X(x|X > Z) = \frac{\Pr[X \leq x, X > Z]}{\Pr[X > Z]} = \frac{\int_0^x f_X(u) F_Z(u) du}{P_1(\epsilon|\gamma)}. \quad (16)$$

Here, $F_X(u) = [1 - 2Q(u)]^{G-1}$ with $f_X(u) = dF_X(u)/du$ [8], and $F_Z(u) = \int_{-\infty}^{u} \phi(v - \sqrt{\gamma}) dv$. The PDF of $Y_m \triangleq \hat{\rho}_m(n; i_m, e_m)$, given $Y_m > W \triangleq \max\{|\rho_m(n; i', e')|, (i', e') \neq (i_m, e_m)\}$, is

$$F_{Y_m}(y|Y_m > W) = \frac{\int_0^y f_{Y_m}(u) F_W(u) du}{1 - P_1(\epsilon|\gamma)} \quad (17)$$

where $f_{Y_m}(u) = \phi(u - \sqrt{\gamma})$ and $F_W(u) = [1 - 2Q(u)]^{G-1}$.

Then, the probability in (13) is equivalent to

$$\begin{aligned} &\Pr\left[\bigcap_{m \neq m'} Y_m > X | X > Z, Y_m > W\right] \\ &= \int_0^\infty \Pr\left[\bigcap_{m \neq m'} Y_m > x | Y_m > W\right] dF_X(x|X > Z) \\ &= \int_0^\infty [1 - F_{Y_m}(x|Y_m > W)]^{M-1} dF_X(x|X > Z) \quad (18) \end{aligned}$$

where $\{X, Y_m, m \neq m'\}$ are mutually independent for the Gaussian-modeled MAI in (7), due to the channel orthogonalization.

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