Adaptive Selection/Maximal-Ratio Combining Based on Error Detection of Multidimensional Multicode DS-CDMA

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***Abstract*—A novel two-dimensional diversity combining is proposed for the uplink in a single cell which employs multidimensional multicode direct-sequence code-division multiple-access signaling and two receive antennas. First, the signaling is combined with precoding to obtain a constant envelope signal that is suitable for the uplink, and the resulting error detection is applied to the diversity combining. Based on the error detection, an adaptive selection-combining/maximal-ratio combining (SC/MRC) is performed, for which initial data detection is made by the SC to avoid the combining loss of the very noisy paths. When the initial SC is unreliable (indicated by the error detection), the MRC is attempted to fully exploit the space and multipath diversity. The adaptive SC/MRC is generalized to further increase the diversity gain over the MRC, offered by the error-detection capability. Through analysis and simulation results, it is shown that the adaptive SC/MRC and its generalized diversity receiver outperforms the other schemes in terms of the symbol-error rate, and also its bit-error rate can be lower than that of the variable spreading factor scheme using a single code.**

***Index Terms*—Diversity combining, error detection, multicode direct-sequence code-division multiple access (DS-CDMA), multidimensional (MD) signaling, precoding.**

I. Introduction

TO DATE, there has been much attention given to the study of multipath diversity combining, especially to bridge the gap between selection combining (SC) and maximal-ratio combining (MRC). It is well known that wideband signaling (e.g., spread-spectrum signal) gives several resolvable paths after the correlation/matched filtering. Hence, if they are properly combined at the receiver, the worst effect of multipath fading can be mitigated significantly. In particular, a generalized SC has been considered as an efficient means to combat multipath fading [1]–[4] where an arbitrary number of the strongest [highest signal-to-noise ratio (SNR)] resolvable paths are selected and combined. This is because MRC, combining all the resolvable paths, is more sensitive to channel estimation errors, while SC, choosing the path with the highest SNR, does not fully exploit the diversity gain offered by the channel. When a multilevel signal is adopted, the symbol-error rate (SER) is sensitive to fading, and the diversity combining that excludes the weakest SNR paths may avoid the *combining loss* of the very noisy (low SNR) paths. In fact, the combining loss will be observed throughout this paper where the multilevel signal is used to increase the data rate in the uplink.

On the other hand, there have been efforts to increase the data rate by proposing new signaling schemes, such as variable spreading factor (VSF) [5] and multicode direct-sequence code-division multiple access (DS-CDMA) [6], [7] in a cellular environment. The VSF scheme reduces the spreading factor per bit as the rate increases, while increasing the signal power to achieve the same bit-error rate (BER). But the uplink channel has limited dynamic range because of a cost-effective nonlinear amplifier. For this reason, as the rate further increases, the per-path SNR would be rather deteriorated, so that it is more likely to cause the combining loss. This phenomenon can be observed similarly in the multicode DS-CDMA, where the number of multiple orthogonal code (MOC) channels becomes large for high rates. Then, the multipath-induced interferences among the MOC channels are growing in proportion to the rate, and further, the envelope fluctuations, as a result of a linear sum of MOC channels, cause nonlinear distortion. These worst effects make the receiver observe the very noisy paths, resulting in the combining loss.

When a high rate is required in the uplink, it is necessary to minimize both the number of MOC channels and the envelope variations, while maintaining the spreading factor as a constant. For this, a multidimensional (MD) multicode DS-CDMA has been proposed [8], where both the code and time spaces are used to meet the above requirements and increase the rate at the same time. Here, a constant envelope signal can be obtained by a nonlinear block-coding technique, called precoding, for which redundancy is added. In this case, precoding incurs a loss in information rate, but the error-detection capability offered by precoding can be applied to improve the SER. Based on the error detection, this paper proposes a novel two-dimensional (space and multipath) diversity combining [9], namely, adaptive SC/MRC using two receive antennas, each followed by V resolvable paths. Initial data detection is based on the Vth-order SC, denoted by $\mathrm{SC}(V)$, which combines the V strongest paths among the $2V$ available ones. Then, error detection is performed on the initial data from the M-parallel MOC channels, and if there is no error detected, data detection is complete. But, if there is an error, data detection is repeated, using the MRC with $2V$ available paths.

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As a generalization of the SC/MRC, an iterative diversity combining is also developed, which is given by an iterative $\mathrm{SC}(v)/\mathrm{SC}(v+1), v=1,2,\ldots,2V-1$. Here, the successive $(v+1)$th-order SC is used only when the previous vth-order SC fails, provided undetected error events can be ignored. If the undetected error event is not negligible for a small v, it is necessary to properly select the initial diversity order v so that an overall SER can be minimized. Due to the complexity caused by this iterative combiner, this paper will focus on the adaptive SC/MRC, which is more compact and outperforms the MRC.

In general, the adaptive SC/MRC can be modified by replacing the Vth-order SC with a generalized SC, which selects an arbitrary number of the strongest paths, rather than V. Typically, in wideband CDMA (WCDMA) systems, a V-finger RAKE receiver is designed for each antenna, where the channel estimation block yields estimates on the path gains for diversity combining. Using the estimates from two antennas, the V strongest paths are selected and combined at one RAKE receiver, while the remaining V paths are combined at the other RAKE receiver if an error is detected. Thus, the proposed diversity system is compatible with WCDMA systems, only except for the additional sorting of $2V$ paths. Also, in some cases, the adaptive mode of operation can be switched to another type of diversity combining, depending on the channel conditions. For this reason, the performance of the proposed adaptive SC/MRC is analyzed, and compared with the other schemes, such as the Vth-order SC and MRC. Furthermore, the MD multicode DS-CDMA with adaptive SC/MRC is compared with the VSF scheme using MRC, especially when a high rate is used.

The rest of the paper is organized as follows. The system model describing a MD multicode DS-CDMA for a high-rate user and a symbol-by-symbol detection for J-parallel MOC channels is presented in Section II. Section III describes an adaptive SC/MRC diversity combining along with its generalized iterative combining $\mathrm{SC}(v)/\mathrm{SC}(v+1)$, both based on the two-level error detection offered by a constant envelope property. The SER performance is analyzed for both identical and nonidentical channel statistics in Section IV, along with the combining loss. Section V presents theoretical and simulation results to find the diversity gain of the adaptive SC/MRC and the iterative $\mathrm{SC}(v)/\mathrm{SC}(v+1)$ over the MRC, and to compare with the VSF scheme in view of the BER. Finally, concluding remarks are given in Section VI.

II. System Model

The uplink uses the MD multicode DS-CDMA signaling to support a high-rate data transmission, in which the baseband signal can be expressed by [8]

$$\begin{aligned} s(t) = \sum_{n=0}^{N_f-1}\sum_{g=0}^{G-1}\sum_{m=0}^{M-1} &\sqrt{P}[d_{I,m}(n)b_{I,\langle m\rangle}(g;n) \\ &\cdot c_I(t-gT_w-nT) + jd_{Q,m}(n)b_{Q,\langle m\rangle}(g;n) \\ &\cdot c_Q(t-gT_w-nT)]w_m(t-gT_w-nT). \end{aligned} \quad (1)$$

First, the conventional multicode DS-CDMA signal is obtained as a special form of the MD signaling $s(t)$ in (1), when $b_{O,\langle m\rangle}(g;n)=1(O=I,Q)$ is assumed for $g=0,1,\ldots,G-1$, all m and n. In this case, the parameter $G\geq 1$ will be determined by the spreading factor N per symbol assumed in the uplink, where $N=GM$. So the above signal can be viewed as a combination of the multicode DS-CDMA signal based on the M-parallel MOC channels, namely, $\{w_m(t), m=0,1,\ldots,M-1\}$ and the G-ary orthogonal signal $\{b_{O,\langle m\rangle}(g;n), g=0,1,\ldots,G-1\}$ for each subset of $J\ (\leq M)$ MOC channels.

Here, P is the signal power per channel, $\{d_{O,m}(n), m=0,1,\ldots,M-1\}$ is the M-parallel binary data at the nth signaling time, the index $\langle m\rangle \triangleq \lfloor m/J\rfloor$ is assumed to be $0,1,\ldots,Q-1$ for an integer $Q=M/J$ where $\lfloor x\rfloor$ is the greatest integer not exceeding x, T_w, and $T=GT_w$ denote the Walsh and data symbol times, respectively, and N_f is the frame length. The channelization code waveform $w_m(t), m=0,1,\ldots,M-1$, is given by

$$w_m(t)=\sum_{l=0}^{M-1} w_{m,l}\,p(t-lT_c) \quad (2)$$

where $p(t)$ is a rectangular chip pulse, occupied in $[0,T_c)$, with unit magnitude, and $(w_{m,0},w_{m,1},\ldots,w_{m,M-1})$ is the mth row vector of the Hadamard matrix, $H_M(M\times M)$ with elements ± 1. To have an in-phase(I)/quadrature(Q) code-multiplexed channel, the spreading code waveform is designed as

$$\begin{aligned} &c_O(t-gT_w-nT) \\ &\quad=\sum_{l=0}^{M-1} c_{O,nN+gM+l}\,p(t-(nN+gM+l)T_c) \end{aligned} \quad (3)$$

where $T_c=T/N=T_w/M$ with spreading factor $N=GM$ per symbol. The channel is assumed to observe the (high-rate) first user's signal $s(t)$ in (1) along with $K-1$ interfering (low-rate) signals, that is

$$s_k(t)=\sum_{l=0}^{NN_f}\sqrt{P_k}\left[a_{I,l}^{(k)}+ja_{Q,l}^{(k)}\right]p(t-lT_c), \quad k=2,\ldots,K \quad (4)$$

where P_k is the kth user's signal power and $\{a_{O,l}^{(k)}, l=0,1,\ldots\}$ are assumed to be random binary sequences. Note that the bit energy is set to $E_b=P_kN_kT_c$, and then N_k will determine the bit rate of the kth low-rate user.

At the receiver, two receive antennas are adopted to provide space diversity where the V resolvable multipaths are modeled as

$$h_k^{(q)}(t)=\sum_{v=1}^{V}\beta_{k,v+qV}\,\delta(t-\tau_{k,v}) \quad (5)$$

for the qth $(q=0,1)$ antenna of kth user link, and $\{\tau_{k,v}\}$ are assumed to be uniformly distributed over $[0,T)$. It is assumed that $\{\beta_{k,v+qV}\}$ are mutually uncorrelated with fixed q, but either correlated or uncorrelated with fixed k and v, where $\{\beta_{k,v+qV}, k=1,\ldots,K; v=1,\ldots,V; q=0,1\}$ is a set of complex-valued Gaussian random variables. With this model, the received signal can be expressed by

$$r_q(t) = s(t) \otimes h_1^{(q)}(t) + \sum_{k=2}^{K} \left[s_k(t) \otimes h_k^{(q)}(t) \right] \quad (6)$$

for the qth antenna, $\otimes$ denoting the convolution.

An ideal channel estimation is assumed, for which the WCDMA system inserts a pilot signal that is code multiplexed to the M-parallel MOC channels [5]. In fact, a linear channel parameter estimation using the pilot signal causes a certain loss in the output SNR, which, in turn, degrades the SER performance [10]. However, the effect of the pilot signal on the SER is not taken into account here to focus on the effect of the interferences among users and MOC channels.

By virtue of the combined signaling, a symbol-by-symbol detection is made by choosing

$$\max_{\substack{(i=0,\ldots,2^J-1) \\ (e=0,\ldots,G-1)}} \left\{ \rho_{\langle m \rangle}(n; i, e) = \frac{1}{JG} \left(\alpha_i \cdot \mathbf{X}_{\langle m \rangle}(n) \cdot \mathbf{h}_e^T \right) \right\} \quad (7)$$

where the symbol α_i is an J-tuple row vector of elements ± 1, and $\mathbf{h}_e$ denotes the G-ary orthogonal sequence with size G and elements $\pm 1, T$ denoting the transpose. Here, the decision statistics $\mathbf{X}_{\langle m \rangle}(n)$ in matrix form will be formed by the adaptive SC/MRC diversity combining proposed.

III. Adaptive SC/MRC

For the uplink, it is desirable to achieve a constant envelope signal $s(t)$ in (1) using the two-level precoding in [8]. With $J = 4$, the symbol α_i in (7) is confined to the set of 4-tuple vectors having an odd weight of ones because of 1-bit precoding. For instance, when $M = 16$-parallel MOC channels are adopted for a high rate, the symbol-by-symbol detection in (7) is carried out for each subset of $J = 4$-parallel MOC channels. Then, the decision symbols are $[\alpha^0 | \alpha^1 | \alpha^2 | \alpha^3]$ for the $M = 16$-parallel binary data $\{d_{O,m}(n)\}$, while $[\mathbf{h}^0 | \mathbf{h}^1 | \mathbf{h}^2 | \mathbf{h}^3]$ for the G-ary orthogonal signals $\{b_{O,\langle m \rangle}(g; n)\}$, $\langle m \rangle = 0, 1, 2, 3$.

In general, when $M = 4^n$-parallel ($n \geq 1$ an integer) MOC channels are used for precoding, there exisits an $2(n-1)$-level error detection with $J = 4$, which makes it difficult to realize an error-check routine for large $n > 2$. Therefore, this paper intends to properly design the adaptive SC/MRC receiver based on the two-level error detection with $M = 16$, while the case $M = 4 (n = 1)$ is trivial, because the symbol-by-symbol detection in (7) fully recovers the signal energy, even with 1-bit precoding [8].

With MD multicode DS-CDMA signaling, the two-level error detection can be performed as [8]

$$\left[\frac{1}{2}(\alpha^0 H_4)\right] \odot \left[\frac{1}{2}(\alpha^1 H_4)\right] \odot \left[\frac{1}{2}(\alpha^2 H_4)\right] \odot \left[\frac{1}{2}(\alpha^3 H_4)\right] = -\mathbf{1}_4 \quad (8)$$

$$\mathbf{h}^0 \odot \mathbf{h}^1 \odot \mathbf{h}^2 \odot \mathbf{h}^3 = \mathbf{1}_G \quad (9)$$

where $H_4 = [(1,1,1,1)^T | (1,-1,1,-1)^T | (1,1,-1,-1)^T | (1,-1,-1,1)^T]$ and $\mathbf{1}_4 \triangleq (1,1,1,1), \odot$ denoting the vector product. Thus, if the above test fails, then errors can be detected, and otherwise, no error is declared, leading to an easy error-check routine. It is to be noted that with multicode DS-CDMA signaling $b_{O,\langle m \rangle}(g; n)$ remains a constant, resulting in one-level error detection, as given in (8).

Based on the error detection, the adaptive SC/MRC diversity combining can be designed as follows.

1) An initial symbol-by-symbol detection is made by selecting the highest SNR paths up to V among the $2V$ available ones, and combining them at one RAKE receiver.
2) The error-check routine in (8) and (9) is performed to find any possible errors at the initial data detection.
3) With no error detected, the initial data detection is declared correct, and otherwise, i.e., if there are some errors, the MRC with $2V$ paths is carried out at the two RAKE receivers following two antennas.

The motivation behind the adaptive SC/MRC is to fully exploit the diversity gain offered by the two-dimensional space and multipath channel. When a high-rate transmission is serviced in the uplink, the per-path SNR becomes relatively low because of an increase in the self-interference (SI) among the M-parallel MOC channels. In this case, the Vth-order SC, i.e., $\mathrm{SC}(V)$, may be suitable for use in the initial data detection because of the combining loss of the very noisy paths. Meanwhile, if the initial detection contains some errors, the V paths used in the Vth-order SC may not sufficiently be stronger than the remaining V paths in terms of the per-path SNR. This situation can often be observed in an interference-limited channel, where a high-rate transmission is required. This has motivated use of the MRC with $2V$ paths only when the initial data detection fails.

This idea can be generalized further to design an iterative combiner that is operated in an iterative manner conditioned on the successive error detection. If undetected error events can be ignored, each $\mathrm{SC}(v)$ with v paths can be verified by the error detection, so that the successive $\mathrm{SC}(v+1)$ based on $(v+1)$ paths (one more path included) is attempted to gradually improve the SER. This iterative combiner is formulated as $\mathrm{SC}(v)/\mathrm{SC}(v+1), v = 1, 2, \ldots, 2V-1$, for which the data detection may succeed if no error is detected with any $\mathrm{SC}(v), v \leq 2V-1$, and otherwise, require the MRC with $2V$ paths. It is observed that a proper selection of initial value v is prerequisite to the iterative combiner, since the error detection may not be reliable with small v. In fact, the iterative combiner is based on the MRC with significant paths *selected* in an iterative manner using the error-detection capability. Thus, the purpose of this combiner is to minimize the combining loss, especially when a multilevel signal is adopted. Meanwhile, the optimum combining (OC) proposed in [11] and [12] requires determination of the optimal weight vector to achieve the maximum output SNR, which is formidable for the chip-level wideband system and multipath fading. Furthermore, the iterative combiner based on the OC needs an iterative determination of the weight vector, depending on the error-detection events occurring in an iterative manner, mutually correlated.

For this reason, this paper focuses on the performance improvement by the adaptive SC/MRC and its generalized iterative combiner based on the MRC, rather than OC. If the OC can partially be combined with the iterative combiner, further improvement will be made, because the OC minimizes the effect of interferences.

Now, the decision statistics $\mathbf{X}_{\langle m\rangle}(n)$ in (7) can be formed by the adaptive SC/MRC, where the mth MOC channel yields statistics

$$X_{m,g}(n;v+qV)=\frac{1}{T_w\sqrt{P}}\int_{gT_w+nT+\tau_{1,v}}^{(g+1)T_w+nT+\tau_{1,v}} r_q(t) \cdot c_O(t-gT_w-nT-\tau_{1,v}) \cdot w_m(t-gT_w-nT-\tau_{1,v})dt \quad (10)$$

for $m=0,\ldots,M-1; g=0,\ldots,G-1; v=1,\ldots,V; q=0,1$. First, if the Vth-order SC is applied to the I-channel statistics, then

$$\mathbf{X}_{\langle m\rangle}(n)=\sum_{l=1}^{V}\Re\{\beta_{1,(l)}^{*}\mathbf{X}_{\langle m\rangle}[n;(l)]\} \quad (11)$$

where the matrix $\mathbf{X}_{\langle m\rangle}(n;v+qV)$ has the elements $\{X_{m,g}(n;v+qV)|m=\langle m\rangle J,\ldots,\langle m\rangle J+J-1; g=0,\ldots,G-1\}$ and $|\beta_{1,(1)}|>|\beta_{1,(2)}|>\cdots>|\beta_{1,(V)}|>|\beta_{1,(V+1)}|>\cdots>|\beta_{1,(2V)}|$ $[(l)\in\{1,2,\ldots,2V\}]$, $*$ denoting the complex conjugate. Similarly, the MRC with $2V$ paths produces the decision statistics as

$$\mathbf{X}_{\langle m\rangle}(n)=\sum_{v=1}^{2V}\Re\{\beta_{1,v}^{*}\mathbf{X}_{\langle m\rangle}(n;v)\}. \quad (12)$$

Note that with the Q channel statistics, the real part $\Re\{\cdot\}$ in (11) and (12) is replaced by the imaginary part $\Im\{\cdot\}$.

IV. Analysis of Diversity Combining

A compact-form formula is presented for the symbol-error probability (SEP) for both identically and nonidentically distributed channel statistics. The former assumes $\mathbf{E}\{|\beta_{k,v+qV}|^2\}=\sigma_\beta^2$ ($\mathbf{E}$ denotes the expectation), while the latter is modeled as

$$\mathbf{E}\{|\beta_{k,v+qV}|^2\}=\sigma_\beta^2(v+qV),\quad v=1,2,\ldots,V \quad (13)$$

for all k and $q=0,1$ with the constraint $\sum_{v=1}^{V}\sigma_\beta^2(v)=\sum_{v=1}^{V}\sigma_\beta^2(v+V)$. In addition, two antennas are assumed to be either correlated or uncorrelated between $\beta_{k,v}$ and $\beta_{k,v+V}(v=1,2,\ldots,V)$ with fixed k, while $\mathbf{E}\{\beta_{k,v}\beta_{k',l}^{*}\}=0$ for all $k\neq k'$ or $|v-l|\neq 0,V$.

First, the decision variable $\rho_{\langle m\rangle}(n;i,e)$ in (7) under the Vth-order SC can be formulated as

$$\rho_{\langle m\rangle}(n;i,e)=\sum_{l=1}^{V}|\beta_{1,(l)}|^2+\sum_{k=2}^{K}\mathrm{MAI}_k(i,e) +\sum_{l=1}^{V}\left[\mathrm{MAI}_{1,(l)}^{(I)}(i,e)+\mathrm{MAI}_{1,(l)}^{(Q)}(i,e)\right] \quad (14)$$

where $(\alpha_i,\mathbf{h}_e)$ was sent, and the second and third terms represent the other-user interference and the I/Q-channel self-interference, respectively. Since $\{\rho_{\langle m\rangle}(n;i,e)\}$ is a set of biorthogonal signals under the operator $\mathbf{E}$ [13], the probability of symbol error is

$$P(\epsilon)= 1-\Pr\left[\bigcap_{(i',e')\neq(i,e)}|\rho_{\langle m\rangle}(n;i',e')|<\rho_{\langle m\rangle}(n;i,e)\middle|\alpha_i,\mathbf{h}_e\right] \quad (15)$$

for $i=0,1,\ldots,2^{J-2}-1$, where $|\{\alpha_i\}|=2^{J-1}$ due to 1-bit precoding, and $e=0,1,\ldots,G-1$.

In this paper, the channel is assumed to be interference limited, in which the composite of interferences becomes dominant compared with noise, and is modeled as zero-mean Gaussian. For the Gaussian-modeled interference, $\{\rho_{\langle m\rangle}(n;i,e)\}$ are shown to be mutually independent due to the orthogonality among $\{\rho_{\langle m\rangle}(n;i,e)\}$ for $i=0,1,\ldots,2^{J-2}-1$ and $e=0,1,\ldots,G-1$ [8]. Therefore, it turns out [13] that

$$P(\epsilon)=1-\int_0^\infty[1-2Q(x)]^{G2^{J-2}-1}\mathbf{E}\{\phi(x-\sqrt{\gamma_{\mathrm{SC}}})\}dx \quad (16)$$

where $Q(x)=\int_x^\infty\phi(u)du$ for $\phi(u)=1/\sqrt{2\pi}\exp(-u^2/2)$, and the output *per-symbol* signal-to-interference ratio (SIR) is shown [8] to be

$$\gamma_{\mathrm{SC}}=\left(\frac{\sum_{l=1}^{V}|\beta_{1,(l)}|^2}{\sigma_\beta^2}\right) \cdot\left[\frac{\mathrm{JN}}{2[(K-1)V\varepsilon^2+M(V-1)]\mathbf{E}\{\bar{\eta}^2(\delta)\}}\right] \quad (17)$$

for uncorrelated/identically distributed channel statistics and $\varepsilon^2\triangleq P_k/P(k\geq 2)$. Here, the partial chip-pulse correlation is $\bar{\eta}(\delta)=1/T_c\int_0^\delta p(t)p(t-\delta+T_c)dt$, δ uniformly distributed over $[0,T_c)$.

Note that the MRC with $2V$ paths will produce the same form of SEP in (16) when γ_{SC} is replaced by γ_{MRC}. Using the relation $\gamma_{\mathrm{SC}}=\sum_{l=1}^{V}\gamma_{(l)}$ in (17), the output *per-symbol* SIR is $\gamma_{\mathrm{MRC}}=\sum_{v=1}^{2V}\gamma_v$ when $\sum_{l=1}^{V}|\beta_{1,(l)}|^2$ is replaced by $\sum_{l=1}^{2V}|\beta_{1,l}|^2$ in (17).

To analyze the adaptive SC/MRC proposed, it is necessary to invoke the set of partitioned events that are mutually exclusive, namely

$$P(\epsilon)=\sum_{\lambda\in S_V}P(\epsilon;\lambda) \quad (18)$$

where S_V is the set of all permutations of integers $\{1,2,\ldots,2V\}$, and $\lambda\in S_V$ denotes the particular selection $\lambda:(1,2,\ldots,2V)\rightarrow[(1),(2),\ldots,(2V)]$. Note that the cardinality of S_V is equal to $2V!$.

Proposition 1: With $M=16$ and $J=4(Q=4)$, the probability of symbol error for the adaptive SC/MRC,[1] conditioned on λ, is derived as

$$P(\epsilon;\lambda)=P(\epsilon;\lambda)|_{\mathrm{MRC}} -\mathbf{E}_{\boldsymbol{\gamma}_\lambda}\{[1-\Pr[E|\boldsymbol{\gamma}_\lambda]]^4\cdot P(\epsilon|C,\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}}\} \quad (19)$$

[1]The SEP is applied to both the conventional multicode DS-CDMA with precoding, and the MD multicode DS-CDMA proposed in [8].

where the subscript $|_{\rm MRC}$ denotes explicitly the symbol error due to the MRC with $2V$ paths, $\boldsymbol{\gamma}_\lambda = [\gamma_{(1)}, \gamma_{(2)}, \ldots, \gamma_{(2V)}]$ given λ, and the event E is on the initial detection error due to the Vth-order SC with $C = \bar{E}$, the complement of E. Here, *per-symbol* is defined on each subset of $J = 4$-parallel MOC channels, and a symbol error is assumed to occur independently on each subset because of the channel orthogonalization.

Proof of Proposition 1: See Appendix A.

Then, substituting (19) into (18), it follows that

$$P(\epsilon) < P(\epsilon)|_{\rm MRC} \tag{20}$$

which implies the superiority of the adaptive SC/MRC over the MRC, offered by the error-detection capability of precoding. Also, if the combining loss of the very noisy paths is expected to increase, then $P(\epsilon|C, \boldsymbol{\gamma}_\lambda)|_{\rm MRC}$ becomes large[2] so that the diversity gain over the MRC is increased.

This diversity gain can be increased further when the iterative combiner ${\rm SC}(v)/{\rm SC}(v+1), v = 1, 2, \ldots, 2V-1$ is employed along with error detection at each iteration. *Proposition 2* will be useful to generalize the SC/MRC and show the performance improvement.

Proposition 2: With $M = 16$ and $J = 4 (Q = 4)$, the iterative combiner ${\rm SC}(v)/{\rm SC}(v+1), v = 1, 2, \ldots, 2V-1$ outperforms the other type of diversity combining considered here, given undetected error events are ignored

$$P(\epsilon;\lambda)|_{{\rm SC}(v)/{\rm SC}(v+1)/{\rm SC}(v+2)} \le P(\epsilon;\lambda)|_{{\rm SC}(v+1)/{\rm SC}(v+2)} \tag{21}$$

$$P(\epsilon;\lambda)|_{{\rm SC}(v)/{\rm SC}(v+1)/{\rm SC}(v+2)} \le P(\epsilon;\lambda)|_{{\rm SC}(v)/{\rm SC}(v+2)} \tag{22}$$

where the subscript $|_{{\rm SC}(v)/{\rm SC}(v+1)/{\rm SC}(v+2)}$ represents the SEP of the iterative ${\rm SC}(v)/{\rm SC}(v+1)/{\rm SC}(v+2)$. Here, the successive ${\rm SC}(v+1)$ and ${\rm SC}(v+2)$ are attempted only if the previous ${\rm SC}(v)$ and ${\rm SC}(v+1)$ fail (indicated by the error detection), respectively.

Proof of Proposition 2: See Appendix B.

In general, analysis of the iterative combiner is not allowed because of the correlated events among successive SCs and resorts to the simulation, by which the diversity gain over the MRC is clearly justified. For this reason, the theoretical analysis here will focus on the adaptive SC/MRC.

To closely look into the combining loss mentioned in *Proposition 1*, and also for evaluation of the gain over the MRC, the following *Proposition 3* is prepared.

Proposition 3: The joint probability that the initial detection with ${\rm SC}(V)$ is correct but the following detection with the MRC is not, conditioned on $\boldsymbol{\gamma}_\lambda$, is evaluated as

$$\begin{aligned}\Pr[C|\boldsymbol{\gamma}_\lambda] \cdot P(\epsilon|C, \boldsymbol{\gamma}_\lambda)|_{\rm MRC} &= P(\epsilon; C|\boldsymbol{\gamma}_\lambda)|_{\rm MRC} \\ &= 1 - P(\epsilon|\boldsymbol{\gamma}_\lambda)|_{\rm SC} - \int_0^\infty \int_{-\kappa y}^\infty \Lambda^{4G-1}(x, y|\kappa) \\ &\quad \cdot \phi(x - \sqrt{\gamma_{\rm MRC} - \gamma_{\rm SC}})\phi(y - \sqrt{\gamma_{\rm SC}})dxdy \end{aligned} \tag{23}$$

[2]Given the initial detection is correct, the symbol error due to the MRC is incurred by the combining loss.

where the subscript $|_{\rm SC}$ indicates the initial detection error under the Vth-order SC, and $\Lambda(x, y|\kappa) = 2\int_0^y [1 - Q(x + \kappa y - \kappa z) - Q(x + \kappa y + \kappa z)]\phi(z)dz$ for $\kappa \triangleq \sqrt{\gamma_{\rm SC}}/\sqrt{\gamma_{\rm MRC} - \gamma_{\rm SC}}$.

Proof of Proposition 3: See Appendix C.

From *Propositions 1* and *3*, the combining loss can be measured via the formula

$$\begin{aligned}&P(\epsilon) - P(\epsilon)|_{\rm MRC} \\ &= -\sum_{\lambda \in S_V} \mathbf{E}_{\boldsymbol{\gamma}_\lambda}\{[1 - P(\epsilon|\boldsymbol{\gamma}_\lambda)|_{\rm SC}]^3 \cdot P(\epsilon; C|\boldsymbol{\gamma}_\lambda)|_{\rm MRC}\}\end{aligned} \tag{24}$$

where $\Pr[E|\boldsymbol{\gamma}_\lambda] = P(\epsilon|\boldsymbol{\gamma}_\lambda)|_{\rm SC}$ has been used. Then, it remains to evaluate the expectations in (16) and (24) for performance comparison.

First, the symbol-error performance is analyzed for uncorrelated/identically distributed channel statistics. Define $\bar{\gamma}_{\rm MRC} = \gamma_{\rm MRC}/\Omega$ in (17) with $\gamma_{\rm SC} \to \gamma_{\rm MRC}$, where

$$\Omega = \left[\frac{JN}{2[(K-1)V\varepsilon^2 + M(V-1)]\mathbf{E}\{\bar{\eta}^2(\delta)\}}\right]. \tag{25}$$

Then, the probability density function (pdf) $f_M(s)$ of $\bar{\gamma}_{\rm MRC}$ is given by [14]

$$f_M(s) = \frac{s^{2V-1}}{(2V-1)!}e^{-s}, \quad s \ge 0. \tag{26}$$

After a few manipulations, it can be evaluated as

$$\begin{aligned}&\mathbf{E}\{\phi(x - \sqrt{\gamma_{\rm MRC}})\} \\ &= \int_0^\infty \phi(x - \sqrt{\Omega s})f_M(s)ds \quad (27)\\ &= \frac{(4V-1)!}{2^{2V-1}(2V-1)!} \cdot \frac{e^{-x^2/2}}{(2+\Omega)^{2V}} \left[\frac{1}{\sqrt{2}\Gamma(2V+1/2)} \right.\\ &\quad \cdot {}_1F_1\left(2V, \frac{1}{2}, \frac{\Omega x^2}{4+2\Omega}\right) + \frac{x\sqrt{\Omega}}{(2V-1)!\sqrt{2+\Omega}} \\ &\quad \left. \cdot\, {}_1F_1\left(2V + \frac{1}{2}, \frac{3}{2}, \frac{\Omega x^2}{4+2\Omega}\right)\right] \quad (28)\end{aligned}$$

where $\Gamma(\cdot)$ is the gamma function, and ${}_1F_1(\cdot,\cdot,\cdot)$ the confluent hypergeometric function [15].

For the Vth-order SC, the *virtual branch* technique [3] is applied to evaluate $\mathbf{E}\{\phi(x - \sqrt{\gamma_{\rm SC}})\}$, where $\gamma_{\rm SC}$ in (17) is first transformed into

$$\gamma_{\rm SC} = \Omega \sum_{v=1}^V U_v + \Omega \sum_{v=1}^V \frac{V}{(V+v)} U_{V+v} \tag{29}$$

for independent, normalized exponential random variables $\{U_v\}$, i.e., $f_{U_v}(u) = e^{-u} (u \ge 0)$.

Proposition 4: The pdf $f_S(s)$ of $\bar{\gamma}_{\rm SC} = \gamma_{\rm SC}/\Omega$ is derived as $(s \ge 0)$

$$\begin{aligned}f_S(s) = C_V^{2V} \sum_{v=1}^V \frac{e^{-s}}{\varphi_v} &\left\{(-1)^V (V-1)!(V/v)^V e^{-sv/V} \right.\\ &\left. + \sum_{n=0}^{V-1} (-1)^n C_n^{V-1} n!(V/v)^{n+1} s^{V-1-n}\right\}\end{aligned} \tag{30}$$

where $C_n^k = k!/(k-n)!/n!$ and $\varphi_v = \prod_{\substack{l=1\\l\neq v}}^{V}(l-v)$.

Proof of Proposition 4: See Appendix D.

Using the above pdf $f_S(s)$, it can be evaluated as

$$\begin{aligned}\mathbf{E}\{\phi(x-\sqrt{\gamma_{\rm SC}})\} &= \int_0^\infty \phi(x-\sqrt{\Omega s})f_S(s)ds\\ &= C_V^{2V}e^{-x^2/2}\sum_{v=1}^{V}\frac{1}{\varphi_v}\left\{(-1)^V\frac{(V-1)!(V/v)^V}{(2+\Omega+2v/V)}\right.\\ &\quad\cdot\left[\frac{1}{\sqrt{2}\Gamma(3/2)}\,{}_1F_1\left(1,\frac{1}{2},\frac{\Omega x^2}{4+2\Omega+4v/V}\right)\right.\\ &\quad\left.+\frac{x\sqrt{\Omega}}{\sqrt{2+\Omega+2v/V}}\,{}_1F_1\left(\frac{3}{2},\frac{3}{2},\frac{\Omega x^2}{4+2\Omega+4v/V}\right)\right]\\ &\quad+\sum_{n=0}^{V-1}(-1)^nC_n^{V-1}n!(V/v)^{n+1}\frac{(2V-2n-1)!}{2^{V-n-1}(2+\Omega)^{V-n}}\\ &\quad\cdot\left[\frac{1}{\sqrt{2}\Gamma(V-n+1/2)}\,{}_1F_1\left(V-n,\frac{1}{2},\frac{\Omega x^2}{4+2\Omega}\right)\right.\\ &\quad\left.\left.+\frac{x\sqrt{\Omega/(2+\Omega)}}{(V-n-1)!}\,{}_1F_1\left(V-n+\frac{1}{2},\frac{3}{2},\frac{\Omega x^2}{4+2\Omega}\right)\right]\right\}.\end{aligned} \tag{31}$$

A direct evaluation of the combining loss in (24) is not allowed, and hence, resorts to the numerical approximation as

$$\begin{aligned}&P(\epsilon)-P(\epsilon)|_{\rm MRC}\\ &\quad\approx -[1-P(\epsilon)|_{\rm SC}]^3\\ &\quad\quad\cdot\left[1-P(\epsilon)|_{\rm SC}-\sum_{\lambda\in S_V}\mathbf{E}_{\boldsymbol{\gamma}_\lambda}\right.\\ &\quad\quad\cdot\left\{\int_0^\infty\int_{-\kappa y}^\infty \Lambda^{4G-1}(x,y|\kappa)\right.\\ &\quad\quad\cdot\phi(x-\sqrt{\gamma_{\rm MRC}-\gamma_{\rm SC}})\\ &\quad\quad\left.\left.\cdot\phi(y-\sqrt{\gamma_{\rm SC}})dxdy\right\}\right]\end{aligned} \tag{32}$$

using the Jensen's inequality [16] where $\mathbf{E}\{f[g(X)]\}\approx f[\mathbf{E}\{g(X)\}]$ for the functions $f(\cdot)$ and $g(\cdot)$. Then, the expectation $\mathbf{E}_{\boldsymbol{\gamma}_\lambda}\{\cdot\}$ is further approximated as

$$\begin{aligned}&1-\sum_{\lambda\in S_V}\mathbf{E}_{\boldsymbol{\gamma}_\lambda}\left\{\int_0^\infty\int_{-\kappa y}^\infty\Lambda^{4G-1}(x,y|\kappa)\right.\\ &\quad\quad\left.\cdot\phi(x-\sqrt{\gamma_{\rm MRC}-\gamma_{\rm SC}})\phi(y-\sqrt{\gamma_{\rm SC}})dxdy\right\}\\ &\approx\Delta\left[1-\int_0^\infty\int_{-\bar\kappa y}^\infty\Lambda^{4G-1}(x,y|\bar\kappa)\right.\\ &\quad\quad\left.\cdot\phi(x-\mathbf{E}\{\sqrt{\gamma_{\rm MRC}-\gamma_{\rm SC}}\})\phi(y-\mathbf{E}\{\sqrt{\gamma_{\rm SC}}\})dxdy\right]\end{aligned} \tag{33}$$

using $\mathbf{E}\{g(X)\}\approx\Delta g(\mathbf{E}\{X\})$ for $X=\sqrt{\gamma}$, where the factor Δ is defined by

$$\Delta \triangleq \frac{\mathbf{E}\{g'(\sqrt{\gamma})\}}{g'(\mathbf{E}\{\sqrt{\gamma}\})}$$

and $\bar\kappa \triangleq \mathbf{E}\{\sqrt{\gamma_{\rm SC}}\}/\mathbf{E}\{\sqrt{\gamma_{\rm MRC}-\gamma_{\rm SC}}\}$. Note that $g'(\cdot)$ can simply be replaced by $P(\epsilon)$ in (16), since $1-\sum_{\lambda\in S_V}\mathbf{E}_{\boldsymbol{\gamma}_\lambda}\{\cdot\}$ is on the order of $P(\epsilon)$.

Using the pdf $f_S(s)$ in (30), it can be evaluated as

$$\begin{aligned}\mathbf{E}\{\sqrt{\gamma_{\rm SC}}\} &= C_V^{2V}\sqrt{\Omega}\sum_{v=1}^{V}\frac{1}{\varphi_v}\left[(-1)^V(V-1)!(V/v)^V\right.\\ &\quad\cdot\frac{\sqrt{\pi}}{2(1+v/V)^{3/2}}+\sum_{n=0}^{V-1}(-1)^nC_n^{V-1}n!\\ &\quad\left.\cdot(V/v)^{n+1}(2V-2n-1)!!\frac{\sqrt{\pi}}{2^{V-n}}\right]\end{aligned} \tag{34}$$

where $(2n-1)!! = 1\cdot3\cdot5\cdots(2n-1)$. Similarly, it can be shown that the pdf $f_{M-S}(s)$ of $\bar\gamma_{\rm MRC}-\bar\gamma_{\rm SC}=\sum_{v=1}^{V}v/(V+v)U_{V+v}$, where $\bar\gamma_{\rm MRC}=\sum_{v=1}^{2V}U_v$ and $\bar\gamma_{\rm SC}$ is given in (29), is

$$f_{M-S}(s)=\frac{C_V^{2V}}{V^{V-1}}\sum_{v=1}^{V}\frac{e^{-s(1+V/v)}}{\zeta_v},\quad s\geq 0 \tag{35}$$

with $\zeta_v=\prod_{\substack{l=1\\l\neq v}}^{V}(1/l-1/v)$, and then

$$\mathbf{E}\{\sqrt{\gamma_{\rm MRC}-\gamma_{\rm SC}}\}=\frac{C_V^{2V}\sqrt{\Omega}}{V^{V-1}}\sum_{v=1}^{V}\frac{\sqrt{\pi}}{2\zeta_v(1+V/v)^{3/2}}. \tag{36}$$

Therefore, the combining loss in (32) is estimated using (33), (34), and (36), which will be validated by the simulation.

Next, the symbol-error performance is derived for uncorrelated/nonidentically distributed channel statistics. Note that $P(\epsilon)$ in (16) is evaluated through the expectations in (28) and (31) for the MRC and the Vth-order SC, respectively. Here, the pdf $f_M(\gamma)$ of $\gamma_{\rm MRC}=\sum_{v=1}^{2V}\gamma_v$ can be shown to be [17]

$$f_M(\gamma)=\sum_{v=1}^{2V}\frac{\pi_v}{\Omega_v}e^{-\gamma/\Omega_v},\quad \gamma\geq 0 \tag{37}$$

where

$$\mathbf{E}\{\gamma_v\}\triangleq\Omega_v=\frac{\sigma_\beta^2(v)}{1/V\sum_{n=1}^{V}\sigma_\beta^2(n)}\Omega \tag{38}$$

because of the constraint $\sum_{v=1}^{V}\sigma_\beta^2(v)=\sum_{v=1}^{V}\sigma_\beta^2(v+V)$, and $\pi_v=\prod_{\substack{l=1\\l\neq v}}^{2V}\Omega_v/(\Omega_v-\Omega_l)$ with $\Omega_v\neq\Omega_l$. Then, it turns out that

$$\begin{aligned}\mathbf{E}\{\phi(x-\sqrt{\gamma_{\rm MRC}})\} &= 2e^{-x^2/2}\sum_{v=1}^{2V}\pi_v\left\{\frac{1}{\sqrt{2\pi}(2+\Omega_v)}\right.\\ &\quad+\frac{x\sqrt{\Omega_v}}{(2+\Omega_v)^{3/2}}\exp\left[\frac{x^2\Omega_v}{2(2+\Omega_v)}\right]\\ &\quad\left.\cdot\left[1-Q\left(\frac{x\sqrt{\Omega_v}}{\sqrt{2+\Omega_v}}\right)\right]\right\}.\end{aligned} \tag{39}$$

Similarly, the *virtual branch* technique [4] is applied to the Vth-order SC, where $\gamma_{\rm SC}$ is first transformed into

$$\gamma_{\rm SC} = \sum_{n=1}^{2V} b_n U_n \tag{40}$$

and then the joint pdf of $\{U_n\}$ is expressed by

$$f(\{U_n\}) = \sum_{\lambda \in S_V} \Pr[\lambda] \prod_{n=1}^{2V} f_{U_n|\lambda}(u|\lambda) \tag{41}$$

for $f_{U_n|\lambda}(u|\lambda) = 1/\tilde{\Omega}_n e^{-u/\tilde{\Omega}_n} (u \geq 0)$. Here, it has been assumed that

$$b_n = \begin{cases} \left[\frac{1}{n}\sum_{v=1}^{n} \Omega_v^{-1}\right]^{-1}, & n \leq V \\ \frac{V}{n}\left[\frac{1}{n}\sum_{v=1}^{n} \Omega_v^{-1}\right]^{-1}, & n \geq V+1 \end{cases} \tag{42}$$

$$\tilde{\Omega}_n = \left[\frac{1}{n}\sum_{v=1}^{n} \Omega_v^{-1}\right]\left[\frac{1}{n}\sum_{l=1}^{n} \Omega_{(l)}^{-1}\right]^{-1} \tag{43}$$

$$\Pr[\lambda] = \prod_{m=1}^{2V} \Omega_{(m)}^{-1}\left[\sum_{l=1}^{m} \Omega_{(l)}^{-1}\right]^{-1}. \tag{44}$$

Based on the above results, the pdf $f_S(\gamma)$ of $\gamma_{\rm SC}$ in (40) can be derived as

$$f_S(\gamma) = \sum_{\lambda \in S_V} \Pr[\lambda]\left[\sum_{n=1}^{2V} \frac{\tilde{\pi}_n}{b_n \tilde{\Omega}_n} e^{-\gamma/(b_n \tilde{\Omega}_n)}\right], \quad \gamma \geq 0 \tag{45}$$

where $\tilde{\pi}_n = \prod_{m=1, m\neq n}^{2V} b_n\tilde{\Omega}_n/(b_n\tilde{\Omega}_n - b_m\tilde{\Omega}_m)$. Then, it follows that

$$\mathbf{E}\{\phi(x-\sqrt{\gamma_{\rm SC}})\} = \sum_{\lambda \in S_V} \Pr[\lambda]\mathbf{E}\{\phi(x-\sqrt{\gamma_{\rm MRC}})\}|_{\substack{\pi_v \to \tilde{\pi}_n \\ \Omega_v \to b_n\tilde{\Omega}_n}} \tag{46}$$

where $\mathbf{E}\{\phi(x-\sqrt{\gamma_{\rm MRC}})\}$ is as given in (39).

To find the combining loss in (32), it is further evaluated as

$$\mathbf{E}\{\sqrt{\gamma_{\rm SC}}\} = \sum_{\lambda \in S_V} \Pr[\lambda] \sum_{n=1}^{2V} \frac{\tilde{\pi}_n\sqrt{\pi}}{2}\sqrt{b_n\tilde{\Omega}_n}. \tag{47}$$

Also, the pdf $f_{M-S}(\gamma)$ of $\gamma_{\rm MRC} - \gamma_{\rm SC} = \sum_{n=1}^{V} b_{n+V}U_{n+V}$, where $\gamma_{\rm MRC} = \sum_{n=1}^{2V}[(1/n)\sum_{v=1}^{n}\Omega_v^{-1}]^{-1}U_n$ and $\gamma_{\rm SC}$ is given in (40), can be derived as

$$f_{M-S}(\gamma) = \sum_{\lambda \in S_V} \Pr[\lambda]\left[\sum_{n=1}^{V} \frac{\tilde{\pi}_{n+V}}{b_{n+V}\tilde{\Omega}_{n+V}} e^{-\gamma/(b_{n+V}\tilde{\Omega}_{n+V})}\right] \tag{48}$$

for $\gamma \geq 0$, where

$$b_{n+V} = \frac{n}{n+V}\left[\frac{1}{n+V}\sum_{v=1}^{n+V} \Omega_v^{-1}\right]^{-1}, \quad n \leq V \tag{49}$$

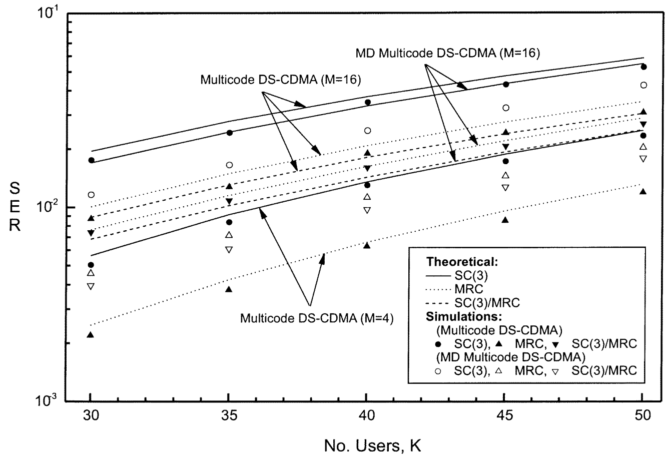

Fig. 1. SER versus K for SC(3), MRC, and SC(3)/MRC with uncorrelated/identical channel statistics.

$\tilde{\pi}_{n+V} = \prod_{m=1, m\neq n}^{V} b_{n+V}\tilde{\Omega}_{n+V}/(b_{n+V}\tilde{\Omega}_{n+V} - b_{m+V}\tilde{\Omega}_{m+V})$, and $\tilde{\Omega}_{n+V}$ is as given in (43) with n replaced by $n+V$. This implies that

$$\mathbf{E}\{\sqrt{\gamma_{\rm MRC} - \gamma_{\rm SC}}\} = \sum_{\lambda \in S_V} \Pr[\lambda]\sum_{n=1}^{V} \frac{\tilde{\pi}_{n+V}\sqrt{\pi}}{2} \cdot \sqrt{b_{n+V}\tilde{\Omega}_{n+V}}. \tag{50}$$

Meanwhile, the symbol-error performance for correlated antennas is complicated due to the expectation $\mathbf{E}\{\phi(x-\sqrt{\gamma_{\rm SC}})\}$, and resorts to the simulation to see the combining loss.

V. Results

A symbol-by-symbol detection with $J = 4$ is assumed when $M = 16$-parallel MOC channels are adopted. The two-level precoding [8] is employed to have a constant envelope multicode signal with rate $9/16$, while a total information rate is 18 bits per $N = 128$ chips by the MD signaling, i.e., $G = N/M = 8$-ary orthogonal signaling conveying an additional three bits per each subset of $J = 4$-parallel MOC channels. Here, the last $J = 4$-parallel MOC channels are used as the precoding channels, so that the 9/16-rate multicode carries nine bits, and the 8-ary signaling sends a total of nine bits for the first 12-parallel channels, resulting in a total of 18 bits. Hence, the power ratio $\varepsilon^2 \triangleq P_k/P(k \geq 2)$ is set to the value $\varepsilon^2 = 8/9$ with $M = 16$, because $P_k = MP/R$ at the rate $R = 18$ bits per $N = 128$ chips, given one bit per $N_k = 128$ chips for a low-rate user in (4).

It is to be noted that the results here are *uncoded* SER and BER for MD multicode DS-CDMA, so that *coded* BER must be much lower. Also, the high-rate uplink transmission is for packet-based data transmission, and a powerful coding scheme should be used to provide a specific target BER, depending on the applications (i.e., types of traffic).

Figs. 1 and 2 show the SER, $P(\epsilon)$ versus $K = 30$–50 (number of users) for identical and nonidentical channel statistics, where $V = 3$-path and two-antenna diversity combining are used. Here, equal path gain denotes σ_β^2, while unequal path gain is given by $\sigma_\beta^2(1) = 0.51, \sigma_\beta^2(2) = 0.31, \sigma_\beta^2(3) = 0.18, \sigma_\beta^2(4) =$

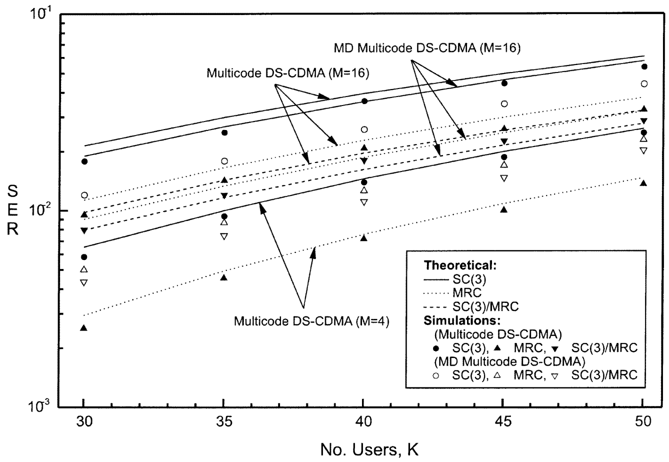

Fig. 2. SER versus K for SC(3), MRC, and SC(3)/MRC with uncorrelated/nonidentical channel statistics.

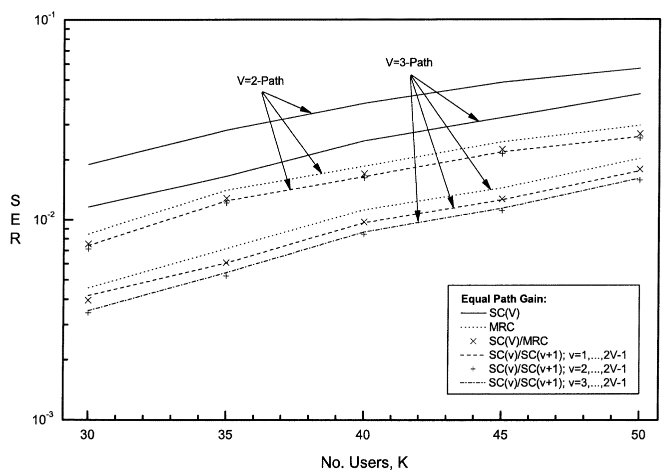

Fig. 3. SER versus K for $\mathrm{SC}(V)$, MRC, $\mathrm{SC}(V)/\mathrm{MRC}$, and iterative $\mathrm{SC}(v)/\mathrm{SC}(v+1)(v = 1,\ldots,2V-1)$ with uncorrelated/identical channel statistics.

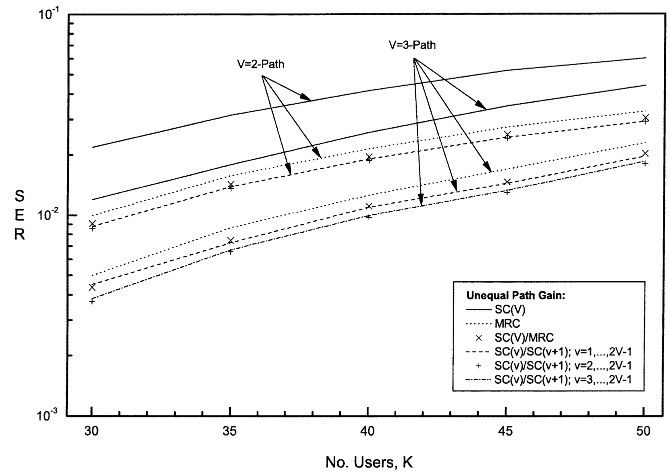

Fig. 4. SER versus K for $\mathrm{SC}(V)$, MRC, $\mathrm{SC}(V)/\mathrm{MRC}$, and iterative $\mathrm{SC}(v)/\mathrm{SC}(v+1)(v = 1,\ldots,2V-1)$ with uncorrelated/nonidentical channel statistics.

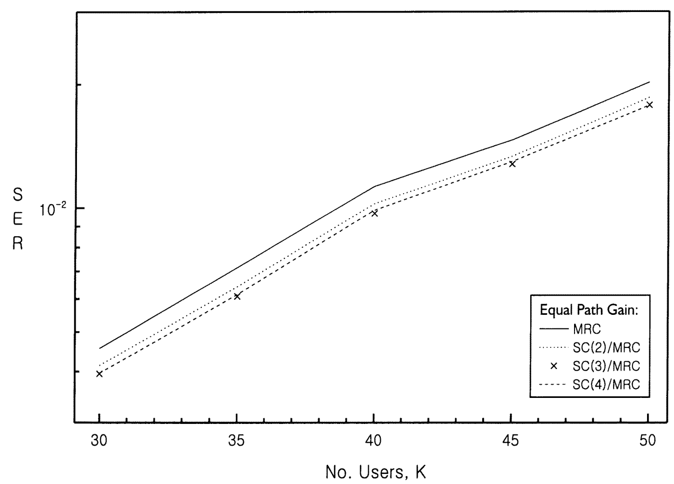

Fig. 5. SER versus K for MRC and $\mathrm{SC}(v)/\mathrm{MRC}(v = 2,3,4)$ with uncorrelated/identical channel statistics.

$0.49, \sigma_\beta^2(5) = 0.32$, and $\sigma_\beta^2(6) = 0.19$. It is seen that there exists some gap between the SER performances, but the relative performance behavior fits well enough to predict the combining loss. It is interesting to see the performance improvement over the MRC, because it results from the error-detection capability to avoid the very noisy paths in diversity combining. As expected, SC(3) is much inferior to the MRC, since there is a certain loss in diversity gain.

Here, the difference between theoretical and simulation results is due to the following facts. The desired signal is observed in the presence of the SI and the multiple-access interference (MAI). The SI is caused by the *high-rate* multicode signal with different multipath delays, while the MAI is caused by $K-1$ *low-rate* interfering signals. It should be noted that the multicode signal has the constant envelope of $\sqrt{MP}$ by precoding, and the $K-1$ interfering signals have much lower envelope of $\sqrt{P_k} \ll \sqrt{MP}$, subject to a fixed power per bit. When $M = 16$ in Figs. 1 and 2, the power ratio $\varepsilon^2(= P_k/P) = 8/9(k \geq 2)$ for the MD multicode signaling, while $\varepsilon^2 = 16/9$ for the multicode signaling with precoding of rate $9/16$. So the envelope imbalance between SI and MAI is much increased when the MD signaling is used, compared with the multicode signaling. Therefore, the MAI approaches the Gaussian distribution as K increases, but the sum of nonidentical SI and MAI would not.

To further validate this observation, the results for $M = 4$ are also presented in Figs. 1 and 2, for which the difference becomes negligible, since the envelope imbalance is much reduced. Here, the constant envelope of $\sqrt{MP}$ is reduced to half, and $\varepsilon^2 = 4/3$ for the multicode signaling with precoding of rate $3/4$.

To closely look into the diversity gain over the MRC that is offered by the error-detection capability, the iterative $\mathrm{SC}(v)/\mathrm{SC}(v+1), v = 1,\ldots,2V-1$ is compared with the MRC and the adaptive $\mathrm{SC}(V)/\mathrm{MRC}$ when $V = 2, 3$ paths are assumed in Figs. 3 and 4. First, the diversity gain is maximized when the $\mathrm{SC}(v)/\mathrm{SC}(v+1)$ with initial value $v = 2$ is adopted, since SC(1) as the initial detection may be unreliable and cause some undetected errors. Second, the diversity gain is increased as the number of paths increases, since the multipath diversity is fully exploited along with error detection. Third, the adaptive $\mathrm{SC}(V)/\mathrm{MRC}$ performs well with a modest increase in SER but a simple realization, compared with the iterative combiner.

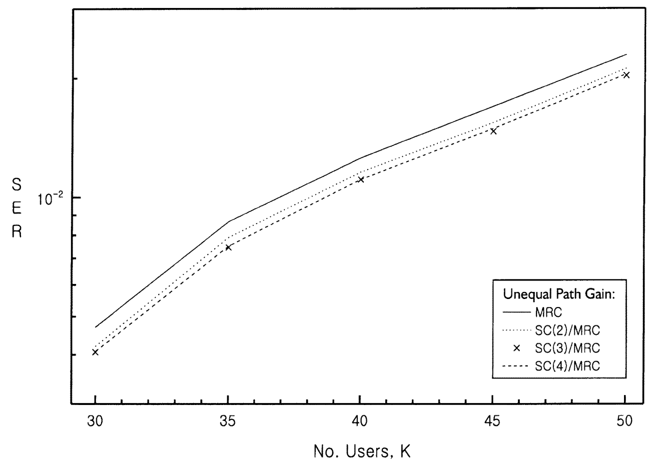

Fig. 6. SER versus K for MRC and $\mathrm{SC}(v)/\mathrm{MRC}(v = 2, 3, 4)$ with uncorrelated/nonidentical channel statistics.

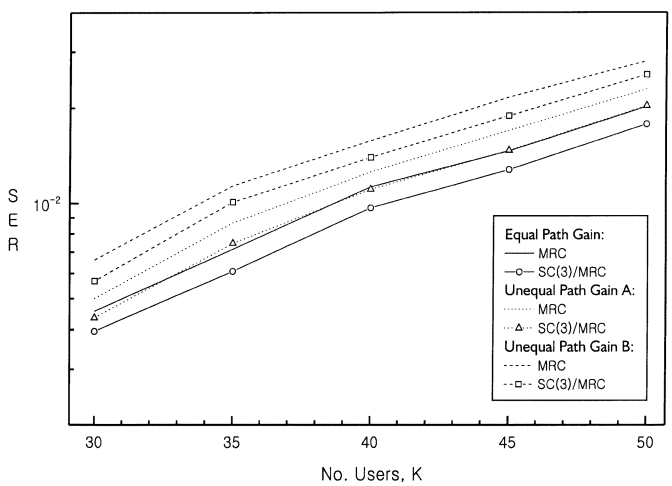

Fig. 7. SER versus K for MRC and SC(3)/MRC with uncorrelated/varying channel statistics.

Next, the initial data detection is compared, assuming the generalized SC with varying order of diversity, namely, 2, 3, and 4 strongest resolvable paths are selected and combined. It is observed in Figs. 5 and 6, for identical and nonidentical channel statistics, that SC(3)/MRC performs better than SC(2)/MRC, and is comparable to or even better than SC(4)/MRC when K ranges from 30 to 50. It implies that SC(3) is a reasonable selection, and also preferred for use in WCDMA systems when combined with the MRC by the error detection. Also, Fig. 7 shows that SC(3)/MRC provides a certain diversity gain over the MRC for the worst channel conditions, where unequal path gain B is given by $\sigma_\beta^2(1) = 0.61, \sigma_\beta^2(2) = 0.31, \sigma_\beta^2(3) = 0.08$, $\sigma_\beta^2(4) = 0.59, \sigma_\beta^2(5) = 0.32$, and $\sigma_\beta^2(6) = 0.09$. Note that unequal path gain A is the same as given in Figs. 2 and 6.

Meanwhile, the SER performance versus K = 30–50 is investigated for the two correlated antennas with power correlation coefficient ρ

$$\rho = \frac{\mathbf{E}\{\gamma_v \gamma_{v+V}\} - \mathbf{E}\{\gamma_v\}\mathbf{E}\{\gamma_{v+V}\}}{\sqrt{\left(\mathbf{E}\left\{\gamma_v^2\right\} - \mathbf{E}^2\{\gamma_v\}\right)\left(\mathbf{E}\left\{\gamma_{v+V}^2\right\} - \mathbf{E}^2\{\gamma_{v+V}\}\right)}}$$

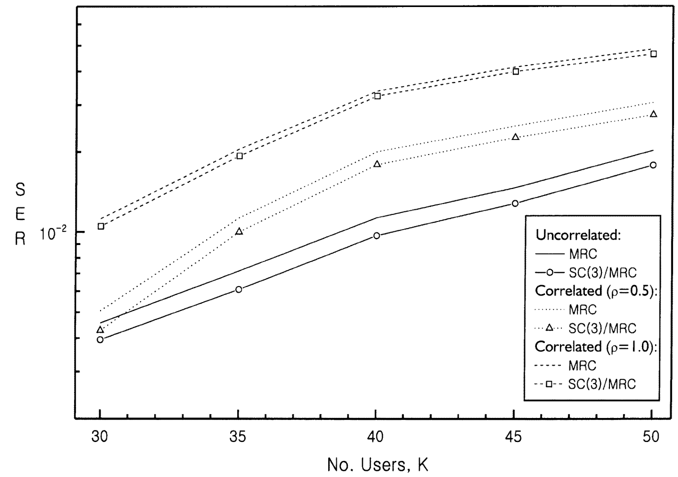

Fig. 8. SER versus K for MRC and SC(3)/MRC with correlated/identical channel statistics.

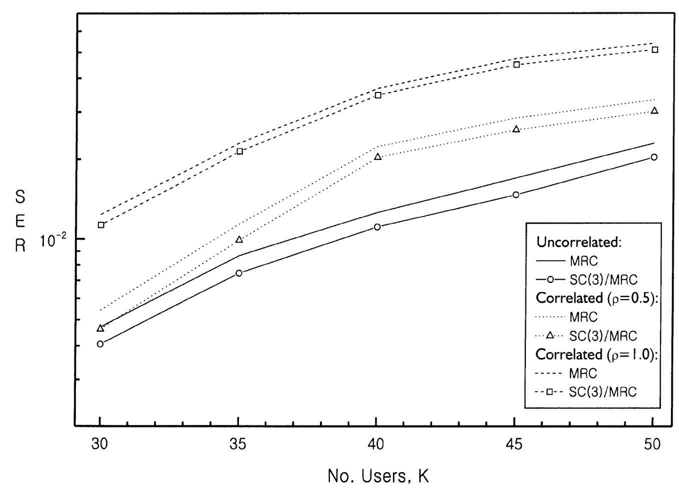

Fig. 9. SER versus K for MRC and SC(3)/MRC with correlated/nonidentical channel statistics.

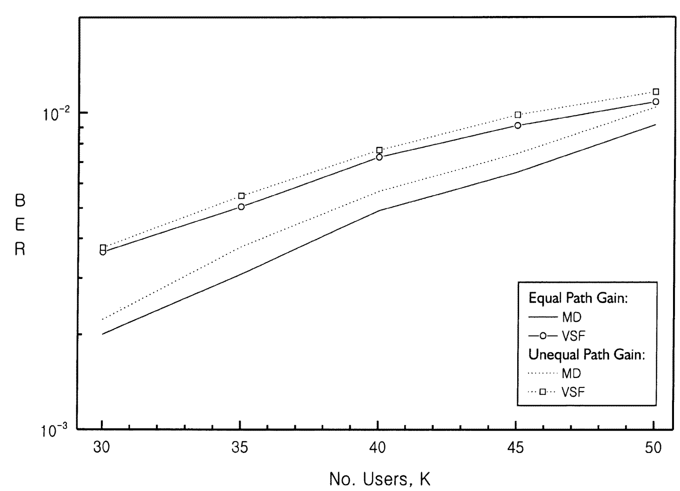

Fig. 10. BER versus K for MD (SC(3)/MRC) and VSF (MRC) with uncorrelated/identical channel statistics.

for $v = 1, 2, 3$, where $V = 3$ paths are assumed for each antenna. As the amount of correlation ρ increases, the diversity gain offered by SC(3)/MRC over the MRC is reduced, as shown in Figs. 8 and 9, but the gain can be maintained when typical values of $\rho < 0.5$ are considered.

Finally, the MD multicode DS-CDMA system with SC(3)/MRC is compared with the VSF scheme using MRC in terms of the BER, assuming $\rho = 0$ (uncorrelated). Here, the data rate of the VSF scheme is assumed to carry 16 bits per $N = 128$ chips, which means the (integer) spreading factor $N_b = 8$ per bit. For instance, if the low-rate signal with one bit per $N_k = 128$ chips is set to 8 kb/s, the rate 128 kb/s can be supported by the VSF, while 144 kb/s are supported by the MD multicode, conveying an additional 16 kb/s. Even at the higher rate, the MD multicode system outperforms the VSF scheme as shown in Fig. 10, with a modest increase in receiver complexity. This is because the former needs $J = 4$-parallel RAKE receivers when compared with the latter, requiring a single RAKE receiver per antenna.

VI. Conclusion

A two-dimensional diversity combining has been proposed for the MD multicode DS-CDMA, based on the error detection of precoding. By virtue of the error detection, the MRC was used selectively in order to fully exploit the diversity gain offered by the space and multipath channel. Initial data detection has been performed by using the Vth-order SC, i.e., $\mathrm{SC}(V)$, that allows avoiding the combining loss of the very noisy paths. Thus, the proposed $\mathrm{SC}(V)/\mathrm{MRC}$, being generalized to the iterative combiner $\mathrm{SC}(v)/\mathrm{SC}(v+1)$, is operated adaptively depending on the channel conditions, resulting in increased diversity gain over the MRC.

Theoretical and simulation results showed that SC(3)/MRC performs better than the MRC in an interference-limited channel, considering both identical and nonidentical channel statistics. Even though the correlation between two antennas reduces the diversity gain, such loss was almost negligible for a typical range of correlation. Especially, the MD multicode DS-CDMA with SC(3)/MRC was shown to outperform the VSF with MRC in terms of the BER, even at a higher rate. It implies that the MD signaling is preferred for use in the high-rate uplink transmission, combined with the adaptive $\mathrm{SC}(V)/\mathrm{MRC}$.

Appendix A
Proof of *Proposition 1*

For the proposed adaptive SC/MRC, a symbol error occurs on the $\langle m\rangle$th subset of $J = 4$-parallel MOC channels as follows. The initial detection by the Vth-order SC and the following detection by the MRC with $2V$ paths successively fails; the initial detection is correct, but the following detection may fail when a symbol error is detected on the other subset ($\neq \langle m\rangle$) of $J = 4$-parallel MOC channels. With $M = 16$-parallel MOC channels, in which case, there exist $Q = 4$ subsets, each carrying *per-symbol*, the SEP conditioned on $\boldsymbol{\gamma}_\lambda$ is

$$\begin{aligned} P(\epsilon|\boldsymbol{\gamma}_\lambda) = \Pr[E|\boldsymbol{\gamma}_\lambda]P(\epsilon|E,\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}} \\ + \{1-[1-\Pr[E|\boldsymbol{\gamma}_\lambda]]^3\} \\ \cdot \Pr[C|\boldsymbol{\gamma}_\lambda]P(\epsilon|C,\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}}. \end{aligned} \tag{51}$$

Using the relation

$$\begin{aligned} P(\epsilon|\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}} = \Pr[E|\boldsymbol{\gamma}_\lambda]P(\epsilon|E,\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}} \\ + \Pr[C|\boldsymbol{\gamma}_\lambda]P(\epsilon|C,\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}} \end{aligned} \tag{52}$$

and taking the expectation $\mathbf{E}\{P(\epsilon|\boldsymbol{\gamma}_\lambda)\} = P(\epsilon;\lambda)$, (51) simply reduces to (19).

Appendix B
Proof of *Proposition 2*

From *Proposition 1*, the SEP of the iterative combiner is

$$\begin{aligned} &P(\epsilon;\lambda)|_{\mathrm{SC}(v)/\mathrm{SC}(v+1)/\mathrm{SC}(v+2)} \\ &\quad = P(\epsilon;\lambda)|_{\mathrm{SC}(v+1)/\mathrm{SC}(v+2)} - \mathbf{E}_{\boldsymbol{\gamma}_\lambda}\{[1-\Pr[E_v|\boldsymbol{\gamma}_\lambda]]^4 \\ &\qquad \cdot P(\epsilon|C_v,\boldsymbol{\gamma}_\lambda)|_{\mathrm{SC}(v+1)/\mathrm{SC}(v+2)}\} \end{aligned} \tag{53}$$

where the event E_v denotes explicitly the initial detection error due to $\mathrm{SC}(v)$ with $C_v = \bar{E}_v$. It implies that the iterative $\mathrm{SC}(v)/\mathrm{SC}(v+1)/\mathrm{SC}(v+2)$ is superior to an $\mathrm{SC}(v+1)/\mathrm{SC}(v+2)$, which can easily be generalized to any type of iterative $\mathrm{SC}(v)/\mathrm{SC}(v+1), v = 1,2,\ldots,2V-1$. Similarly, *Proposition 1* is applied to the $\mathrm{SC}(v+1)/\mathrm{SC}(v+2)$ in (53), which yields

$$\begin{aligned} &P(\epsilon;\lambda)|_{\mathrm{SC}(v+1)/\mathrm{SC}(v+2)} \\ &\quad = P(\epsilon;\lambda)|_{\mathrm{SC}(v+2)} - \mathbf{E}_{\boldsymbol{\gamma}_\lambda}\{[1-\Pr[E_{v+1}|\boldsymbol{\gamma}_\lambda]]^4 \\ &\qquad \cdot P(\epsilon|C_{v+1},\boldsymbol{\gamma}_\lambda)|_{\mathrm{SC}(v+2)}\} \end{aligned} \tag{54}$$

$$\begin{aligned} &P(\epsilon|C_v,\boldsymbol{\gamma}_\lambda)|_{\mathrm{SC}(v+1)/\mathrm{SC}(v+2)} \\ &\quad = P(\epsilon|C_v,\boldsymbol{\gamma}_\lambda)|_{\mathrm{SC}(v+2)} - [1-\Pr[E_{v+1}|\boldsymbol{\gamma}_\lambda]]^4 \\ &\qquad \cdot P(\epsilon|C_v,C_{v+1},\boldsymbol{\gamma}_\lambda)|_{\mathrm{SC}(v+2)}. \end{aligned} \tag{55}$$

Now, combining (54) and (55) with (53) gives

$$\begin{aligned} &P(\epsilon;\lambda)|_{\mathrm{SC}(v)/\mathrm{SC}(v+1)/\mathrm{SC}(v+2)} \\ &\quad \leq P(\epsilon;\lambda)|_{\mathrm{SC}(v+2)} - \mathbf{E}_{\boldsymbol{\gamma}_\lambda}\{[1-\Pr[E_v|\boldsymbol{\gamma}_\lambda]]^4 \\ &\qquad \cdot P(\epsilon|C_v,\boldsymbol{\gamma}_\lambda)|_{\mathrm{SC}(v+2)}\} \end{aligned} \tag{56}$$

for $P(\epsilon|C_{v+1},\boldsymbol{\gamma}_\lambda)|_{\mathrm{SC}(v+2)} \geq P(\epsilon|C_v,C_{v+1},\boldsymbol{\gamma}_\lambda)|_{\mathrm{SC}(v+2)}$. Since the right-hand side of (56) is $P(\epsilon;\lambda)|_{\mathrm{SC}(v)/\mathrm{SC}(v+2)}$, it turns out that the iterative $\mathrm{SC}(v)/\mathrm{SC}(v+1)/\mathrm{SC}(v+2)$ outperforms an $\mathrm{SC}(v)/\mathrm{SC}(v+2)$, which can also be generalized to prove *Proposition 2*.

Appendix C
Proof of *Proposition 3*

Since

$$\Pr[C|\boldsymbol{\gamma}_\lambda]P(\epsilon|C,\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}} = \Pr[C|\boldsymbol{\gamma}_\lambda][1-P(c|C,\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}}] \tag{57}$$

for $P(c) \triangleq 1-P(\epsilon)$, it might be easy to evaluate

$$\begin{aligned} &\Pr[C|\boldsymbol{\gamma}_\lambda]P(c|C,\boldsymbol{\gamma}_\lambda)|_{\mathrm{MRC}} \\ &\quad = \Pr\left[\bigcap_{(i',e')\neq(i,e)} \left|\rho_{\langle m\rangle}^{\mathrm{MRC}}(n;i',e')\right| \right. \\ &\qquad < \rho_{\langle m\rangle}^{\mathrm{MRC}}(n;i,e) \bigcap_{(i',e')\neq(i,e)} \left|\rho_{\langle m\rangle}^{\mathrm{SC}}(n;i',e')\right| \\ &\qquad \left. < \rho_{\langle m\rangle}^{\mathrm{SC}}(n;i,e) \middle| \alpha_i, \mathbf{h}_e, \boldsymbol{\gamma}_\lambda \right]. \end{aligned} \tag{58}$$

Define $X+Y \triangleq \rho^{\mathrm{MRC}}_{\langle m\rangle}(n;i',e')$ and $Y \triangleq \rho^{\mathrm{SC}}_{\langle m\rangle}(n;i',e')$, then (58) is formulated as

$$\begin{aligned}\mathbf{E}\Big\{ \Pr{}^{4G-1} \Big[&|X+Y|<\psi+\xi, |Y|<\xi \,\Big|\, \rho^{\mathrm{MRC}}_{\langle m\rangle}(n;i,e) \\ &= \psi+\xi>0, \rho^{\mathrm{SC}}_{\langle m\rangle}(n;i,e)=\xi>0, \boldsymbol{\gamma}_\lambda\Big]\Big\} \\ &= \int_0^\infty\int_{-\xi}^\infty \Pr{}^{4G-1}[|X+Y|<\psi+\xi,|Y|<\xi|\boldsymbol{\gamma}_\lambda] \\ &\cdot f_\Psi(\psi) f_\Xi(\xi)\, d\psi\, d\xi \qquad (59)\end{aligned}$$

because of the channel orthogonalization and $|\{(i',e') \neq (i,e)\}| = G2^{J-2}-1 = 4G-1$ $(J=4)$, in which

$$\begin{aligned}\Pr[|X+Y|&<\psi+\xi,|Y|<\xi|\boldsymbol{\gamma}_\lambda] \\ &= \int_{-\xi}^{\xi}\int_{-y-(\psi+\xi)}^{-y+(\psi+\xi)} f_X(x) f_Y(y)\, dx\, dy. \qquad (60)\end{aligned}$$

Here, $\{X,Y,\Psi,\Xi\}$ are modeled as Gaussian with statistics such as $\mathbf{E}\{X\} = \mathbf{E}\{Y\} = 0, \mathbf{E}\{\Psi\} = \sum_{v=1}^{2V}|\beta_{1,v}|^2 - \sum_{l=1}^{V}|\beta_{1,(l)}|^2, \mathbf{E}\{\Xi\} = \sum_{l=1}^{V}|\beta_{1,(l)}|^2$, and $\mathrm{var}\{X\} = \mathrm{var}\{\Psi\} = \mathbf{E}^2\{\Psi\}/(\gamma_{\mathrm{MRC}}-\gamma_{\mathrm{SC}}), \mathrm{var}\{Y\} = \mathrm{var}\{\Xi\} = \mathbf{E}^2\{\Xi\}/\gamma_{\mathrm{SC}}$ (see [8, App. A] for details). Hence, combining the above results simply yields the formula in (23).

APPENDIX D
PROOF OF *PROPOSITION 4*

Since $\bar{\gamma}_{\mathrm{SC}} = \sum_{v=1}^{V} U_v + \sum_{v=1}^{V} V/(V+v) U_{V+v}$ in (29) and $\{U_v\}_{v=1}^{2V}$ are independent and identically distributed, the characteristic function method is used to derive the pdf of $\bar{\gamma}_{\mathrm{SC}}$, in which

$$\mathbf{E}\left\{\exp\left[jw\left(\frac{V}{V+v}\right)U_{V+v}\right]\right\} = \frac{1}{1-jwV/(V+v)}. \qquad (61)$$

Define $Y = \sum_{v=1}^{V} V/(V+v)U_{V+v}$, and then

$$C_Y(jw) \triangleq \mathbf{E}\{\exp(jwY)\} = \prod_{v=1}^{V}\frac{1}{1-jwV/(V+v)}. \qquad (62)$$

Thus, the pdf of Y is derived as

$$\begin{aligned} f_Y(y) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} C_Y(jw) e^{-jwy}\, dw \\ &= \frac{2V!}{V!V}\sum_{v=1}^{V}\frac{\exp[-(1+v/V)y]}{\varphi_v}, \quad y\geq 0 \qquad (63)\end{aligned}$$

where the Cauchy's residue theorem [18] has been used.

Next, let $X = \sum_{v=1}^{V} U_v$, which has the pdf $f_X(x) = x^{V-1}/(V-1)!e^{-x} (x \geq 0)$ [14], and the pdf $f_S(s)$ of $\bar{\gamma}_{\mathrm{SC}} = X+Y$ becomes

$$f_S(s) = f_X(x)\otimes f_Y(y) = \int_0^s f_X(x) f_Y(s-x)\, dx. \qquad (64)$$

Finally, $f_S(s)$ is derived as given in (30).

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