A Reduced Complexity Channel Estimation for OFDM Systems With Transmit Diversity in Mobile Wireless Channels

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***Abstract*—A reduced complexity channel estimation for OFDM systems with transmit diversity is proposed by exploiting the correlation of the adjacent subchannel responses. The sizes of the matrix inverse and the FFTs required in the channel estimation at every OFDM data symbol are reduced by half of the existing method for OFDM systems with nonconstant modulus subcarrier symbols or constant modulus subcarrier symbols with some guard tones. The complexity reduction of half FFTs size and some matrix multiplications is still achieved for constant modulus subcarrier symbols with no guard tones. The price for the complexity reduction is a slight BER degradation and for the channels with small relative delay spreads, the BER performance of the reduced complexity method becomes quite comparable to the existing method. An alternative approach for the number of significant taps required in the channel estimation is described which achieves a comparable performance to the case with the known suitable number of significant taps. A simple modification which reduces the lost leakage of the nonsample-spaced channel paths is also proposed. This modification achieves a substantial performance improvement over the existing method without any added complexity.**

***Index Terms*—OFDM, reduced complexity channel estimation, transmit diversity.**

I. Introduction

ONE OF the desirable features of OFDM is its robustness to the multipath induced intersymbol interference. On the other hand due to the frequency selective fading of the dispersive wireless channel, some subchannels may face deep fades and degrade the overall system performance. In order to compensate the frequency selectivity, techniques such as error correcting code and diversity have to be used [1]–[3]. Transmit diversity for wireless systems has been studied in research literature (e.g., [4], [5]). For OFDM systems, transmit diversity combined with Reed–Solomon code has been proposed for clustered OFDM in [2]. Recently, space–time coding [6] has been shown to give high code efficiency and good performance. Application of space–time coding in OFDM systems has been studied in [7] with perfect channel knowledge at the receiver.

The channel estimation for OFDM systems without transmit diversity has been studied by many researchers (e.g., [8]–[10]). However, for systems with transmit diversity, the received signal is a superposition of the different transmitted signals from all transmit antennas and consequently, the channel estimation becomes more complicated. Recently, the channel estimation for OFDM systems with transmit diversity using space–time coding has been proposed in [1]. In order to reduce the complexity associated with the matrix inverse operation, [1] also proposed a simplified approach. In this paper, we focus on reduced complexity channel estimation for OFDM systems with transmit diversity.

We propose a channel estimation method which has less complexity than the simplified method of [1]. By decoupling the effect of different transmit antennas, the sizes of the matrix inverse and FFTs required in the channel estimation for every OFDM data symbol are reduced by half. The significant tap catching approach of [1] requires the knowledge of the number of significant taps. In this paper, we describe an alternative approach which adaptively finds this number. Moreover, a simple modification which reduces the channel energy leakage lost in the channel estimation of both [1] and the reduced complexity method is also proposed. This modified approach achieves a substantial performance improvement.

The rest of the paper is organized as follows. In Section II, the system and the channel considered are described. In Section III, the reduced complexity channel estimation is derived and adaptively finding the number of significant taps is described. The performance in terms of the channel estimation mean square error (MSE) and the complexity comparison are presented in Section IV. The energy leakage of the channel path with nonsample-spaced delay is investigated in Section V. Based on this, a modified approach is proposed for further improvement. Simulation results are discussed in Section VI and conclusions are given in Section VII.

II. System and Channel Description

The considered OFDM system is the same as in [1, Fig. 1] with two-branch transmit diversity and two-branch receive diversity. At a transmission time n, a binary data block $\{b[n, k]: k = 0\,1, \ldots\}$ is space–time-coded (in fact, space–frequency-coded) into two blocks of frequency-domain subcarrier symbols, $\{t_i[n, k]: k = 0, 1, \ldots\}$ for $i = 1, 2$, which are simultaneously transmitted from the two antennas.

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The DFT output frequency-domain subcarrier symbols from receive antenna j can be expressed as

$$r_j[n, k] = \sum_{i=1}^{2} H_{ij}[n, k]\, t_i[n, k] + w_j[n, k] \quad (1)$$

where $w_j[n, k]$ is the additive complex Gaussian noise with zero mean and variance σ_n^2, on the jth receive antenna, that is uncorrelated for different ns, ks, or js, and $H_{ij}[n, k]$ is the channel frequency response for the kth tone at time n, corresponding to the ith transmit antenna and the jth receive antenna. For a system with L-transmit diversity, the average SNR at the receiver is defined as

$$\text{SNR} \triangleq \frac{E\left\{\sum_{i=1}^{L} |H_{ij}[n, k]|^2\right\}}{\sigma_n^2} \quad (2)$$

where $E\{|t_i|^2\} = 1$ is assumed. In our simulation, the channels are modeled such that $E\{|H_{ij}[n, k]|^2\} = 1$.

The channel impulse response of the mobile wireless channel can be given by

$$h(t, \tau) = \sum_k \gamma_k(t)\delta(\tau - \tau_k) \quad (3)$$

where τ_k is the delay of the kth path, $\gamma_k(t)$ is the corresponding complex gain, $\delta(\tau)$ is the Dirac Delta function, and the antenna indexes i, j are omitted for simplicity. The frequency response at time t is

$$\begin{aligned} H(t, f) &\triangleq \int_{-\infty}^{\infty} h(t, \tau)e^{-j2\pi f\tau}\, d\tau \\ &= \sum_k \gamma_k(t)e^{-j2\pi f\tau_k}. \end{aligned} \quad (4)$$

The path gains $\gamma_k(t)$s are modeled as independent wide-sense stationary (WSS), narrowband complex Gaussian processes with the time-domain correlation defined by the classical Doppler spectrum. The frequency-domain correlation of the channel is defined by the channel power delay profile.

With tolerable leakage, the channel frequency response can be expressed as [1]

$$H[n, k] \triangleq H(nT_f, k\Delta f) = \sum_{l=0}^{K_0-1} h[n, l]W_K^{kl} \quad (5)$$

where $h[n, l] \triangleq h(nT_f, lt_s)$, $W_K = \exp(-j2\pi/K)$, K_0 is the total number of sample-spaced channel taps, K is the number of tones (including guard tones), T_f and Δf are the total OFDM symbol interval and the tone spacing of the OFDM system, respectively, and $t_s = 1/(K\Delta f)$.

The considered channel power delay profile models are two-ray (equal average power on each ray), GSMs typical urban (TU) (6 taps), GSMs hilly terrain (HT) (6 taps), and JTCs indoor office channel model A (3 taps) [11].

III. Reduced Complexity Channel Estimation

Since the channels considered have independent responses to different receive antennas, the channel estimation is independently performed for each receive antenna. The receive antenna index j will be omitted in the following. The frequency-domain subcarrier symbols from each receive antenna can be expressed as

$$r[n, k] = \sum_{i=1}^{2} H_i[n, k]\, t_i[n, k] + w[n, k]. \quad (6)$$

Due to the limited delay spread of the channel, the channel subcarrier responses are correlated. Generally, the smaller the ratio of the channel delay spread to the OFDM symbol interval, the more correlated the adjacent subcarrier responses are. In this paper, we attempt to find a reduced complexity channel estimation by exploiting the correlation of the subcarrier responses. In particular, we will assume in the derivation of the reduced complexity channel estimation that

$$H_i[n, 2m] = H_i[n, 2m+1]. \quad (7)$$

The above assumption would be a good approximation for the case with a very small ratio of channel delay spread to OFDM symbol interval. We will also investigate the applicability of this assumption for different channels with different delay spreads.

Let us define the following:

$$z_1[n, m] \triangleq \frac{r[n, 2m]}{t_2[n, 2m]} - \frac{r[n, 2m+1]}{t_2[n, 2m+1]} \quad (8)$$

$$z_2[n, m] \triangleq \frac{r[n, 2m]}{t_1[n, 2m]} - \frac{r[n, 2m+1]}{t_1[n, 2m+1]} \quad (9)$$

$$\varpi_1[n, m] \triangleq \frac{w[n, 2m]}{t_2[n, 2m]} - \frac{w[n, 2m+1]}{t_2[n, 2m+1]} \quad (10)$$

$$\varpi_2[n, m] \triangleq \frac{w[n, 2m]}{t_1[n, 2m]} - \frac{w[n, 2m+1]}{t_1[n, 2m+1]}. \quad (11)$$

With (7), we obtain

$$z_i[n, m] = H_i[n, 2m]v_i[n, m] + \varpi_i[n, m], \qquad i = 1, 2 \quad (12)$$

where

$$v_1[n, m] \triangleq \frac{t_1[n, 2m]}{t_2[n, 2m]} - \frac{t_1[n, 2m+1]}{t_2[n, 2m+1]} \quad (13)$$

$$v_2[n, m] \triangleq \frac{t_2[n, 2m]}{t_1[n, 2m]} - \frac{t_2[n, 2m+1]}{t_1[n, 2m+1]}. \quad (14)$$

From (12), it can be observed that the channel responses corresponding to different transmit antennas are decoupled. Hence, the channel estimation can be performed independently for each transmit–receive antenna pair by minimizing the following mean square error (MSE) cost function:

$$\begin{aligned} &C\left(\left\{\tilde{h}_i[n, l];\ i = 1, 2\right\}\right) \\ &\quad = \sum_{m=0}^{M-1}\left| z_i[n,m] - \sum_{l=0}^{K_0-1} \tilde{h}_i[n, l]\, W_M^{ml}\, v_i[n, m]\right|^2 \end{aligned} \quad (15)$$

where $M = K/2$, and $\{\tilde{h}_i[n, l]\}$ are trial values for the MMSE estimates of $\{h_i[n, l]\}$.

Solving the following

$$\frac{\partial C\left(\left\{\tilde{h}_i[n,l]\right\}\right)}{\partial \tilde{h}_i[n,l_0]} \triangleq \frac{1}{2}\left\{\frac{\partial C\left(\left\{\tilde{h}_i[n,l]\right\}\right)}{\partial \Re\left(\tilde{h}_i[n,l_0]\right)} - j\,\frac{\partial C\left(\left\{\tilde{h}_i[n,l]\right\}\right)}{\partial \Im\left(\tilde{h}_i[n,l_0]\right)}\right\} = 0 \quad (16)$$

where $\Re(*)$ and $\Im(*)$ denote the real and imaginary part of a complex number, respectively, and $l_0 = 0, 1, \ldots, K_0-1$, results in

$$\sum_{m=0}^{M-1}\left(z_i[n,m] - \sum_{l=0}^{K_0-1} \hat{h}_i[n,l]\,W_M^{ml}\,v_i[n,m]\right) \cdot v_i^*[n,m]\,W_M^{-ml_0} = 0 \quad (17)$$

where $\{\hat{h}_i[n,l]\}$ are the MMSE estimates.

Define

$$q_i[n,l] \triangleq \sum_{m=0}^{M-1} |v_i[n,m]|^2\,W_M^{-ml} \quad (18)$$

$$p_i[n,l] \triangleq \sum_{m=0}^{M-1} z_i[n,m]\,v_i^*[n,m]\,W_M^{-ml}. \quad (19)$$

Then, (17) can be expressed as

$$\sum_{l=0}^{K_0-1} \hat{h}_i[n,l]\,q_i[n,l_0-l] = p_i[n,l_0] \quad (20)$$

for $i = 1, 2$ and $l_0 = 0, 1, \ldots, K_0-1$. It can be expressed in matrix form as

$$\boldsymbol{Q}_i[n]\,\hat{\boldsymbol{h}}_i[n] = \boldsymbol{p}_i[n] \quad (21)$$

where

$$\boldsymbol{Q_i}[n] \triangleq \begin{pmatrix} q_i[n,0] & q_i[n,-1] & \cdots & q_i[n,-K_0+1] \\ q_i[n,1] & q_i[n,0] & \cdots & q_i[n,-K_0+2] \\ \vdots & \vdots & \ddots & \vdots \\ q_i[n,K_0-1] & q_i[n,K_0-2] & \cdots & q_i[n,0] \end{pmatrix} \quad (22)$$

$$\hat{\boldsymbol{h}}_i[n] \triangleq \left(\hat{h}_i[n,0], \hat{h}_i[n,1], \ldots, \hat{h}_i[n,K_0-1]\right)^T \quad (23)$$

$$\boldsymbol{p}_i[n] \triangleq (p_i[n,0], p_i[n,1], \ldots, p_i[n,K_0-1])^T. \quad (24)$$

Hence, the channel impulse response can be estimated by

$$\hat{\boldsymbol{h}}_i[n] = \boldsymbol{Q_i}^{-1}[n]\,\boldsymbol{p}_i[n]. \quad (25)$$

Following [1], the complexity involved in $\boldsymbol{Q}^{-1}$ can be further reduced and the channel estimation performance can be further improved by using only J significant channel taps whose indexes are denoted by $\{l_m\colon m = 1, 2, \ldots, J; (0 \le l_1 < l_2 < \cdots < l_J \le K_0-1)\}$. The value of J generally depends on the number of paths and the path delays, both of which are typically defined by the terrain environment. In [1], it is reported that the value of J typically ranges between 5 and 9 for 2-ray, TU, and HT channels. Due to the time-varying nature of the mobile wireless channel, the effective number of (sample-spaced) significant channel taps and/or their tap indexes can vary to some degree for different time instants. This prompts a question on the applicability of this significant tap catching approach in a fast time-varying channel. However, the results in [1] and also in this paper show that even with a Doppler frequency of 200 Hz for the OFDM system with 200 μs symbol interval and 10% training, the significant tap catching approach works well.

An alternative approach in the significant tap catching rather than using fixed value of J is described in the following. Suppose the maximum value of J for which the complexity can be afforded be J_m. Let $\{\hat{h}_{sort}[l]\}$ be the sorted version of $\{\hat{h}[l]\}$ in descending order, and J_i be the number of taps satisfying the following condition:

$$\left|\hat{h}[l]\right|^2 > \eta \cdot \frac{1}{\alpha}\sum_{l=0}^{\alpha-1}\left|\hat{h}_{sort}[l]\right|^2. \quad (26)$$

Then, the value of J is chosen as the minimum of J_i and J_m. In the above equation, η is a threshold value. Although $\alpha = 1$ is applicable, it was observed in our investigation that a little larger value (say, 3 or 4) gives more robustness to the noise. The threshold value η depends on the SNR value and is chosen according to

$$\eta_2 = \eta_1\,\frac{\mathrm{SNR}_1}{\mathrm{SNR}_2} \quad (27)$$

where η_i is for SNR_i. We observed that the value of η about the range of 0.01 to 0.04 works well for a SNR value of 10 dB. This threshold value can be set according to the designed received SNR or by means of the sync detection metric such as received power measurement. Due to the varying value of J at every training symbol, this approach will be denoted by "adaptive" in contrast to the fixed value of J. For channel environments where a suitable fixed value of J is unknown, this adaptive approach would be an alternative. It is noted that in the adaptive approach, the value of J need not be the same for the different transmit antennas.

IV. Performance Analysis

A. MSE Performance

In the previous section, a reduced complexity channel estimation for OFDM systems with transmit diversity is proposed by assuming that (7) holds. In this section, the MSE performance of the proposed method in a multipath fading channel not satisfying (7) is analyzed. For simplicity, the time index n is omitted in the following analysis. Define

$$\Delta H_i[m] \triangleq H_i[2m] - H_i[2m+1] = \sum_{l=0}^{K_0-1} h_i[l](1 - W_K^l)\,W_K^{2ml} \quad (28)$$

$$\Delta H_{e1}[m] \triangleq \Delta H_2[m] + \Delta H_1[m]\,\frac{t_1[2m+1]}{t_2[2m+1]}. \quad (29)$$

Then,

$$z_1[m] = \Delta H_{e1}[m] + H_1[2m]v_1[m] + \varpi_1[m] \quad (30)$$

$$p_1[l] = X_1[l] + \sum_{k=0}^{K_0-1} q_1[l-k]\,h_1[k] + \mathcal{W}_1[l] \quad (31)$$

where

$$X_1[l] \triangleq \sum_{m=0}^{M-1} \Delta H_{e1}[m]\, v_1^*[m]\, W_M^{-ml} \qquad (32)$$

$$\mathcal{W}_1[l] \triangleq \sum_{m=0}^{M-1} \varpi_1[m]\, v_1^*[m]\, W_M^{-ml}. \qquad (33)$$

In matrix form,

$$\boldsymbol{p}_1 = \boldsymbol{X}_1 + \boldsymbol{\mathcal{W}}_1 + \boldsymbol{Q_1}\, \boldsymbol{h}_1 \qquad (34)$$

where

$$\begin{aligned} \boldsymbol{p}_1 &\triangleq (p_1[0], p_1[1], \ldots, p_1[K_0-1])^T \\ \boldsymbol{X}_1 &\triangleq (X_1[0], X_1[1], \ldots, X_1[K_0-1])^T \\ \boldsymbol{\mathcal{W}}_1 &\triangleq (\mathcal{W}_1[0], \mathcal{W}_1[1], \ldots, \mathcal{W}_1[K_0-1])^T \\ \boldsymbol{h}_1 &\triangleq (h_1[0], h_1[1], \ldots, h_1[K_0-1])^T. \end{aligned}$$

The channel impulse response estimate corresponding to transmit antenna 1 is given by

$$\hat{\boldsymbol{h}}_1 = \boldsymbol{Q}_1^{-1}\boldsymbol{p}_1 = \boldsymbol{Q_1}^{-1}(\boldsymbol{X}_1 + \boldsymbol{\mathcal{W}}_1) + \boldsymbol{h}_1. \qquad (35)$$

The MSE of the channel impulse response estimate can be given by

$$\begin{aligned} \text{MSE} &\triangleq E\left\{\left\|\hat{\boldsymbol{h}}_1 - \boldsymbol{h}_1\right\|^2\right\} \\ &= E\left\{\left[\boldsymbol{Q}_1^{-1}(\boldsymbol{X}_1 + \boldsymbol{\mathcal{W}}_1)\right]^H \boldsymbol{Q}_1^{-1}(\boldsymbol{X}_1 + \boldsymbol{\mathcal{W}}_1)\right\} \\ &= \text{Trace}\left\{\boldsymbol{Q}_1^{-1}E\{\boldsymbol{X}_1\boldsymbol{X}_1^H\}\boldsymbol{Q}^{-1^H}\right\} \\ &\quad + \text{Trace}\left\{\boldsymbol{Q}_1^{-1}E\{\boldsymbol{\mathcal{W}}_1\boldsymbol{\mathcal{W}}_1^H\}\boldsymbol{Q}^{-1^H}\right\}. \end{aligned} \qquad (36)$$

The elements of $E\{\boldsymbol{X}_1\,\boldsymbol{X}_1^H\}$ are given by

$$\begin{aligned} &E\{X_1[l_0]\, X_1^*[l_1]\} \\ &\quad = \sum_{l=1}^{K_0-1}\left(1 - W_K^l\right)\sum_{k=1}^{K_0-1}\left(1 - W_K^{-k}\right) \\ &\qquad \cdot \left(E\{h_1[l]h_1^*[k]\}\zeta_1[l_0-l]\zeta_1^*[l_1-k]\right. \\ &\qquad\quad \left. + E\{h_2[l]h_2^*[k]\}V_1[l_0-l]V_1^*[l_1-k]\right) \end{aligned} \qquad (37)$$

with

$$V_1[k] \triangleq \sum_{m=0}^{M-1} v_1^*[m]\, W_M^{-mk} \qquad (38)$$

$$\zeta_1[k] \triangleq \sum_{m=0}^{M-1} v_1^*[m]\, \frac{t_1[2m+1]}{t_2[2m+1]}\, W_M^{-mk}. \qquad (39)$$

The elements of $E\{\boldsymbol{\mathcal{W}}_1\,\boldsymbol{\mathcal{W}}_1^H\}$ are given by

$$E\{\mathcal{W}_1[l_0]\,\mathcal{W}_1^*[l_1]\} = \sigma_n^2\,\psi_1[l_0 - l_1] \qquad (40)$$

where

$$\psi_1[l] \triangleq \sum_{m=0}^{M-1}\left(\frac{1}{|t_2[2m]|^2} + \frac{1}{|t_2[2m+1]|^2}\right)|v_1[m]|^2\, W_M^{-ml}. \qquad (41)$$

By defining

$$\Psi_1 \triangleq \begin{pmatrix} \psi_1[0] & \psi_1[-1] & \cdots & \psi_1[-K_0+1] \\ \psi_1[1] & \psi_1[0] & \cdots & \psi_1[n, -K_0+2] \\ \vdots & \vdots & \ddots & \vdots \\ \psi_1[K_0-1] & \psi_1[K_0-2] & \cdots & \psi_1[0] \end{pmatrix} \qquad (42)$$

we can express that

$$E\left\{\boldsymbol{\mathcal{W}}_1\,\boldsymbol{\mathcal{W}}_1^H\right\} = \sigma_n^2\,\Psi_1. \qquad (43)$$

For constant modulus subcarrier symbols ($\{|t_i[m]|\}$ are constant), if $\{v_i[m]\}$ are constant, $E\{\boldsymbol{\mathcal{W}}_1 \quad \boldsymbol{\mathcal{W}}_1^H\}$ and $\boldsymbol{Q}_1$ become scaled identity matrices and the diagonal elements of $E\{\boldsymbol{X}_1 \quad \boldsymbol{X}_1^H\}$ become $\{|1 - W_K^l|^2 K^2 (P_1[l] + P_2[l])$: $l = 0, 1, \ldots, K_0 - 1\}$ where $P_i[l] \triangleq E\{|h_i[l]|^2\}$ is the effective sample-spaced power delay profile of the channel corresponding to the transmit antenna i. One criterion that satisfies the above condition is given by $t_1[m] = (-1)^m\, t_2[m]$ which is the same as the optimal training design for [1]. Under this condition, the MSE (36) of the reduced complexity method becomes

$$\text{MSE} = \frac{1}{2}\sum_{l=1}^{K_0-1}\left|1 - W_K^l\right|^2 P[l] + \frac{K_0\,\sigma_n^2}{K|t_i[m]|^2} \qquad (44)$$

where the index i from $P[l]$ is omitted for simplicity since generally all channels have the same power delay profile.

Although the above MSE analysis is derived for $\boldsymbol{h}_1$, it also holds for other channels and hence is the average MSE. The second term in (44) is the same as the MSE for [1] at the optimal condition. Hence, the reduced complexity method has a larger channel estimation MSE than [1] due to the assumption of (7). The extra term in the MSE expression also indicates the dependence of the reduced complexity channel estimation MSE performance on the channel power delay profile. Particularly, the larger delay taps have more impact on the MSE. This fact is intuitively justified since the larger delay taps would cause more frequency selectivity of the channel.

It should be mentioned that the above MSE expressions for [1] and the reduced complexity method are derived assuming that the channel gains remain constant over one OFDM symbol, and all subchannels are used (i.e., without guard tones). If the channel gain varies during one OFDM symbol, there may be some degradation due to the inter-subchannel interference (ICI). If guard tones are used, $\boldsymbol{Q}_{ii}$ of [1] would not be exactly a diagonal matrix, and there may be some degradation, too.

From (44), the channel estimation MSE mainly depends on $\sigma_n^2/|t_i[m]|^2$ which in turn indicates that the channel estimation MSE performance evaluation by computer simulation depends on how to model $\{E\{|H_{ij}[n, k]|^2\}\}$. For a SNR value and fixed $E\{|t_i[m]|^2\}$, if $\{E\{|H_{ij}[n, k]|^2\}\}$ are modeled very small, σ_n^2 will be very small and consequently, the obtained channel estimation MSE will be very small. Hence, a suitable simulation model for channel estimation performance evaluation should be adopted. There are two possible approaches: the first one is to model $E\{\sum_{i=1}^{L}|H_{ij}[n, k]|^2\} = 1$, and the second is to model $E\{|H_{ij}[n, k]|^2\} = 1$. Due to the similar reasoning of

TABLE I
CHANNEL ESTIMATION COMPLEXITY

# Rx. Ant.	Condition	Method	Complex Multiplications
1	Constant modulus with $K_u = K$	[1]	$3K + 4J^2 + 3J^3 + 3\ FFT_K + [\cdot]^{-1}_{J \times J}$
		RC	$4.5K + 2J^2 + 3\ FFT_{K/2} + [\cdot]^{-1}_{J \times J}$
	Constant modulus with $K_u \neq K$	[1]	$3K_u + 4J^2 + 3\ FFT_K + [\cdot]^{-1}_{2J \times 2J}$
		RC	$4.5K + 2J^2 + 3\ FFT_{K/2} + [\cdot]^{-1}_{J \times J}$
	Non-constant modulus	[1]	$5K_u + 4J^2 + 5\ FFT_K + [\cdot]^{-1}_{2J \times 2J}$
		RC	$6K_u + 2J^2 + 4\ FFT_{K/2} + 2\ [\cdot]^{-1}_{J \times J}$
2	Constant modulus with $K_u = K$	[1]	$5K + 8J^2 + 6J^3 + 5\ FFT_K + 2\ [\cdot]^{-1}_{J \times J}$
		RC	$7.5K + 4J^2 + 5\ FFT_{K/2} + 2\ [\cdot]^{-1}_{J \times J}$
	Constant modulus with $K_u \neq K$	[1]	$5K_u + 8J^2 + 5\ FFT_K + 2\ [\cdot]^{-1}_{2J \times 2J}$
		RC	$7.5K_u + 4J^2 + 5\ FFT_{K/2} + 2\ [\cdot]^{-1}_{J \times J}$
	Non-constant modulus	[1]	$7K_u + 8J^2 + 7\ FFT_K + 2\ [\cdot]^{-1}_{2J \times 2J}$
		RC	$9K_u + 4J^2 + 6\ FFT_{K/2} + 4\ [\cdot]^{-1}_{J \times J}$

the MSE's dependence on channel power gain and to be comparable with different number of diversity branches, the second approach of $E\{|H_{ij}[n, k]|^2\} = 1$ is adopted in this paper.

B. Channel Estimation Complexity

For channel estimation based on the training symbol, the required matrix inverse can be pre-computed. Hence, the discussion on the complexity will be focused on the channel estimation based on the decision-directed reference symbol. The number of complex multiplications will be used as a performance measure. Operations such as K-point FFT and $K_0 \times K_0$ matrix inversion have complexity order of $N \log_2(N)$ and K_0^3, respectively. However, the exact complexity can vary depending on their implementation. Hence, we will use FFT_K and $[\cdot]^{-1}_{K_0 \times K_0}$, respectively in the complexity expression rather than the number of complex multiplications for them. For a two transmit antenna space–time coded OFDM system with K total subcarriers and K_u used subcarriers, Table I lists the complexity measures of [1] and the reduced complexity method (denoted by RC), both using J significant taps. The significant tap catching is independently performed at each receive antenna and its complexity is not included in the complexity expression. It is noted that if the significant tap catching is performed by averaging over different receive antennas, the number of matrix inverse for the two receive antenna case will be halved for all methods.

For constant modulus subcarrier symbols with no guard tones, the $2J \times 2J$ matrix inversion required in [1] can be reduced to a $J \times J$ matrix inversion and some matrix multiplications (see [1, eq. 24a]). Due to the spectrum requirement, OFDM systems generally use some guard tones. In this case, the above simplification [1, eq. 24a] does not hold. From Table I, it can be observed that RC method achieves complexity reduction for all conditions. More complexity reduction is observed for the cases of nonconst modulus subcarrier symbols and constant modulus subcarrier symbols with guard tones. It is also noted that the size of FFT for RC is $K/2$ while that for [1] is K for all conditions. Moreover, the matrix inverse size for RC is half of that for [1] for all conditions except constant modulus subcarrier symbols with no guard tones.

V. FURTHER IMPROVEMENT ON CHANNEL ESTIMATION

In previous sections, the channel response is estimated based on the sample-spaced impulse response $[h[0], h[1], \ldots, h[K_0 - 1]]$. Generally, the channel path delays are not sample-spaced which causes the channel energy leakage to other sample-spaced taps besides the adjacent sample-spaced taps. In the following, the effect of nonsample-spaced path delay on the channel estimation is investigated.

By using (4), the sample-spaced channel tap gains can be given by

$$\begin{aligned} h[l] &= \frac{1}{K} \sum_{m=0}^{K-1} \sum_k \gamma_k \, e^{-j2\pi m \lambda_k / K} \, e^{j2\pi m l / K} \\ &= \sum_k \gamma_k \, g_k[l] \end{aligned} \tag{45}$$

where $\lambda_k \triangleq \tau_k K \Delta f$, $\{g_k[l]\}$ are the contribution of the unity gain channel path with delay τ_k over the sampled-spaced taps and given by

$$\begin{aligned} g_k[l] &\triangleq \frac{1}{K} \sum_{m=0}^{K-1} e^{j2\pi m (l - \lambda_k)/K} \\ &= \begin{cases} \delta[l - \lambda_k], & \text{if } \lambda_k = \text{integer} \\ \dfrac{\sin(\pi(l - \lambda_k))}{K \sin(\pi(l - \lambda_k)/K)} \, e^{j(K-1)\pi(l-\lambda_k)/K} & \\ \qquad \text{otherwise.} & \end{cases} \end{aligned} \tag{46}$$

The plots of $\{|g_k[l]|: l = 0, 1, \ldots, K-1\}$ for $0 < \lambda_k < 1$ and $4 < \lambda_k < 5$ are shown in Fig. 1(a) and (b), respectively. Firstly, the effect of different λ_k values on a sample-spaced tap will be discussed. Consider the range $(l' < \lambda_k < l' + 1)$ with l' being an integer. As can be expected, the energy distribution to tap l' is larger if λ_k is closer to l'. The same holds for tap $l' + 1$. To the other taps away from these two adjacent taps l' and $l' + 1$, the largest energy distribution (energy leakage) occurs at $\lambda_k = l' + 0.5$. In other words, the energy leakage to not-nearby taps is maximum when $\lambda_k = l' + 0.5$.

Secondly, the energy distribution of a unity gain path with delay τ_k over the sample-spaced taps $\{l = 0, 1, \ldots, K - 1\}$ is discussed. As can be expected, the larger energy is distributed to the nearer tap. However, since $g_k[l] = g_k[l - K]$, the path with λ_k near 0 will also cause leakage to the taps $(K-1, K-2, \ldots)$ where tap $K-1$ will get the largest leakage among them. Consequently, the channel estimation based on $[h[0], h[1], \ldots, h[K_0 - 1]]$ will lose some leakage energy especially from taps $(K-1, K-2, \ldots)$. From Fig. 1(a) and (b), it can be observed that if λ_k can be intentionally increased by some amount, the leakage to those taps $(K-1, K-2, \ldots)$ can be substantially reduced and the channel estimation can be improved. A simple way of establishing this is to use a pre-advancement of the timing point. A timing pre-advancement of β samples will effectively increase λ_k to $(\tau_k K \Delta f + \beta)$. The value of β should not only be less than the ISI-free guard interval but also be small enough that most channel energy is contained in the $[h[0], h[1], \ldots, h[K_0 - 1]]$. If necessary, the value of K_0 may be increased in account for the timing pre-advancement. By

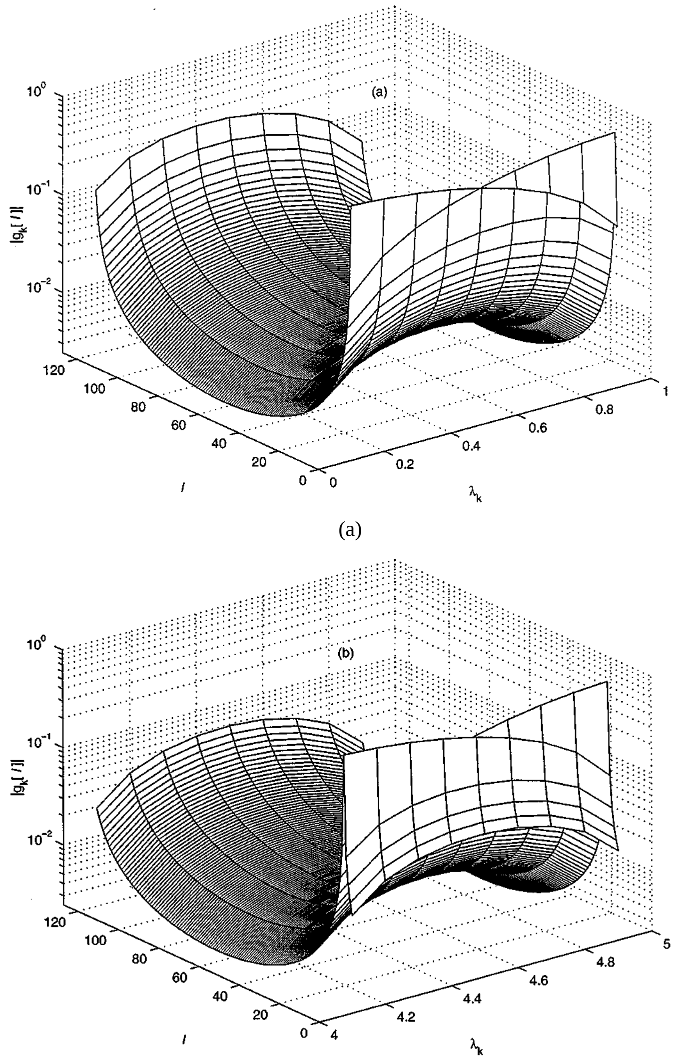

Fig. 1. The leakage of a unity gain channel path with nonsample-spaced path delay. l is the index of sample-spaced channel taps, λ_k is the normalized delay of the kth path.

using (46), a worst-case leakage may be found over $0 < \lambda_k < 1$. Fig. 2 shows this worst-case leakage for an OFDM system with $K = 128$ subcarriers. Based on this, β may be chosen for a designed SNR value. For example, the use of $\beta = 4$ will include all leakage down to approximately -23 dB of the total energy of the path with $0 < \lambda_k < 1$ in the channel estimation.

VI. Simulation Results and Discussions

The OFDM system parameters used in the simulation are the same as [1]: 128 subchannels, 4 guard-subchannels on each end, 1/160 -μs subchannel spacing, and 40 -μs guard interval. The channel models considered are two-ray and TU channel models with delay spread of 1.06 μs, two-ray and HT channel models with delay spread of 5.04 μs, and JTC channel model with delay spread of 34.8 ns. The classical Doppler spectrum with the maximum frequency of 40 Hz and 200 Hz are considered. The space–time codes used are of 16 states with 4-PSK and 16-QAM [6]. The number of significant taps used in all channel estimation methods is 7.

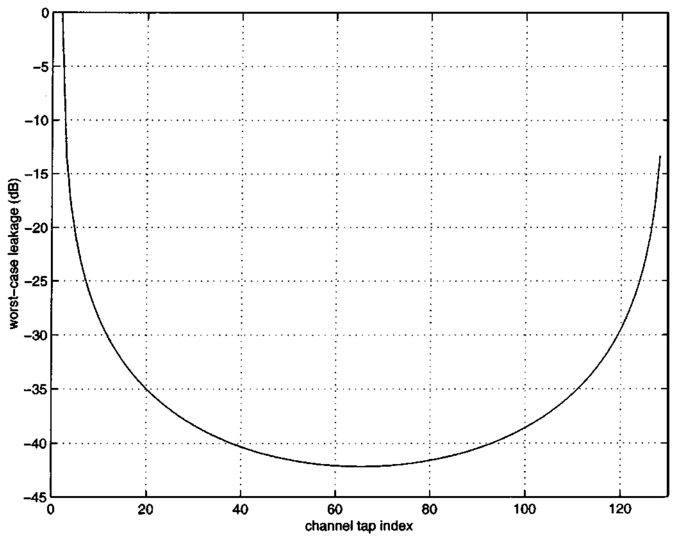

Fig. 2. The worst-case leakage of a unity gain channel path with nonsample-spaced path delay.

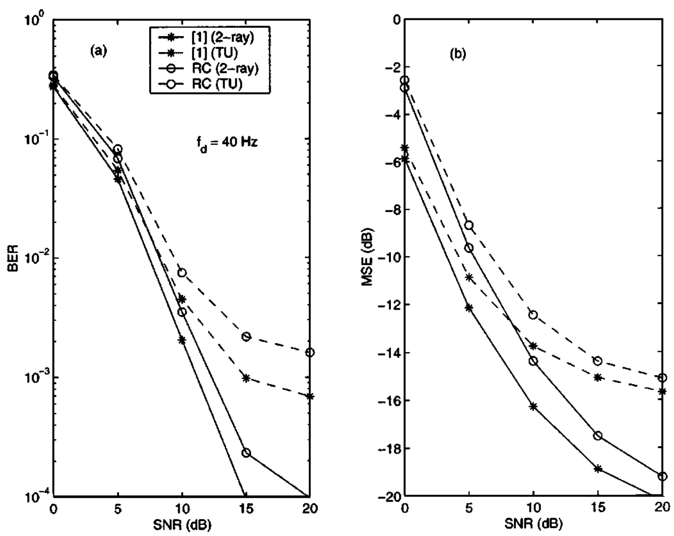

Fig. 3. Performance of the channel estimation methods in OFDM with 4-PSK, 16 states space–time code. The rms delay spread is 1.06 μs, the Doppler frequency is 40 Hz.

In the following, the performance degradation associated with the complexity reduction is investigated. For the channel estimation based on the training symbol, [1] is also used in the reduced complexity method. The different methods are applied only for the channel estimation based on the decision directed reference symbols. Figs. 3–7 show the BER and MSE performance for 4-PSK, 16 states space–time code in different channel models with different delay spreads and Doppler frequencies. For the same delay spread, two-ray model shows a better performance due to the less channel energy leakage. The channels with larger delay spread may not necessarily show a larger MSE. For example, the MSE of [1] for two-ray model with delay spread of 1.06 μs is larger than that with delay spread of 5.04 μs. The reason can be ascribed to the much larger channel energy leakage of the former to the taps $(K - 1, K - 2, \ldots)$ (see previous Section). By comparing the MSE in the JTC, TU and HT models, it is observed that

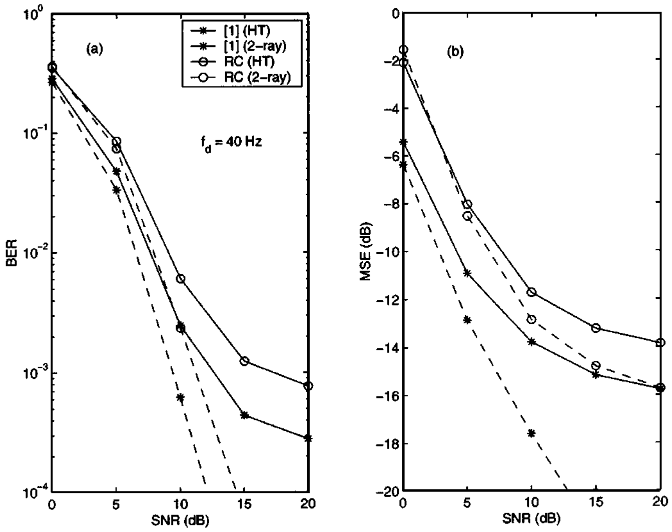

Fig. 4. Performance of the channel estimation methods in OFDM with 4-PSK, 16 states space–time code. The rms delay spread is 5.04 μs, the Doppler frequency is 40 Hz.

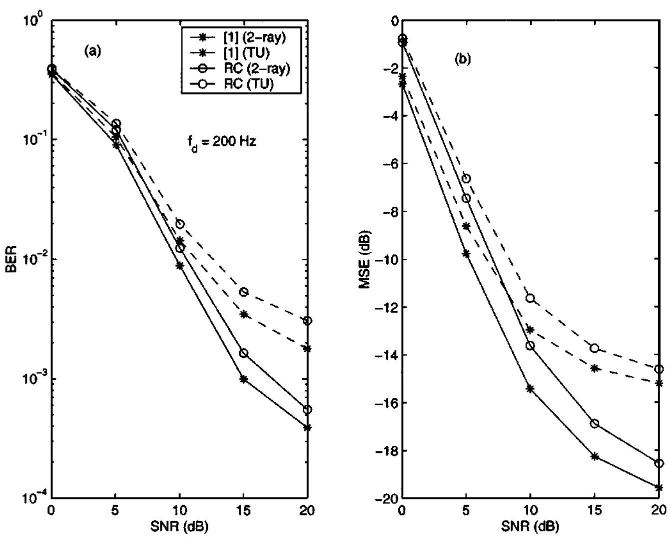

Fig. 5. Performance of the channel estimation methods in OFDM with 4-PSK, 16 states space–time code. The rms delay spread is 1.06 μs, the Doppler frequency is 200 Hz.

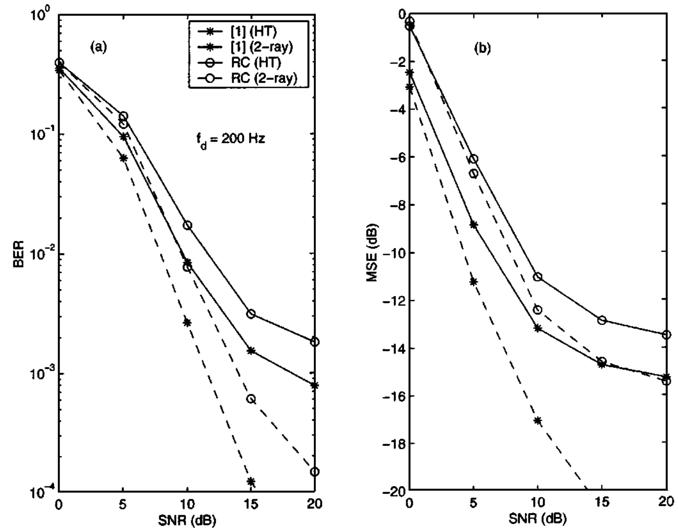

Fig. 6. Performance of the channel estimation methods in OFDM with 4-PSK, 16 states space–time code. The rms delay spread is 5.04 μs, the Doppler frequency is 200 Hz.

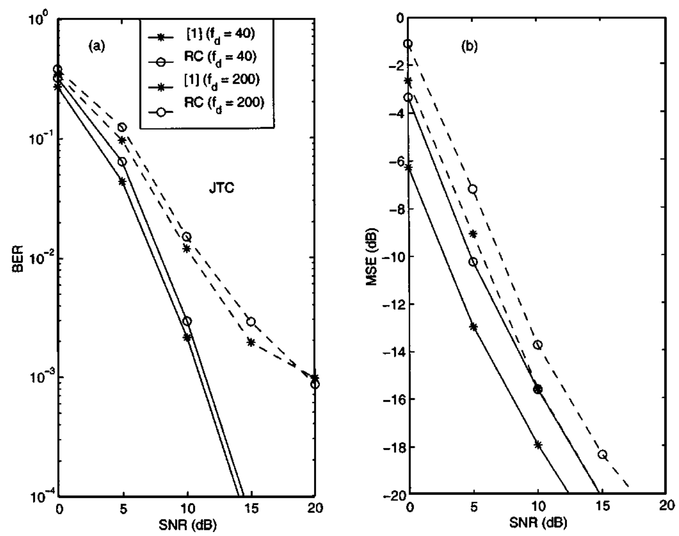

Fig. 7. Performance of the channel estimation methods in OFDM with 4-PSK, 16 states space–time code. The rms delay spread is 34.8 ns.

at the same MSE level, RC method has more degradation in the channel with larger delay spread. This is due to the fact that the larger delay spread channel is more deviated from the assumption (7). For the same delay spread, RC method has more degradation in two-ray, especially if compared with HT model. The reason can be deduced from (44). Although the maximum delay spread of HT is larger than that of two-ray, the last path's power is larger in two-ray than HT. This causes a larger value of the first term in (44), hence a larger MSE.

When comparing for different Doppler frequencies, (more or less) a larger MSE degradation for the higher Doppler frequency is observed at lower SNR values in all channel models. This may be ascribed to a larger reference symbol error caused by the larger time selectivity of the channel with a larger Doppler spread, in addition to the low SNR condition and ICI. However, on the contrary, a larger BER degradation is observed at higher SNR values. This indicates that the effect of the channel estimation errors on BER performance becomes more dominant at higher SNR values.

Another observation is that when comparing performance between JTC and other models, although JTC model has a larger MSE gap between [1] and RC than the other models at the same SNR value, it has a smaller BER gap between [1] and RC than the other models. This indicates that if the MSE is much smaller than the SNR value, (say, a difference of 6 dB or more), provided that the ICI is negligible, then further reduction in MSE would not improve the BER performance very much.

From the above investigation, we can observe that the cost of RC method's complexity reduction is just a slight BER degradation for all SNR values in a channel with very small delay spread such as JTC model. For channels with larger delay spreads such as TU and HT, a small BER degradation is observed for low to moderate SNR values. Hence, if the complexity is affordable, [1] is a better solution. If the complexity is of concern, RC method can be an alternative.

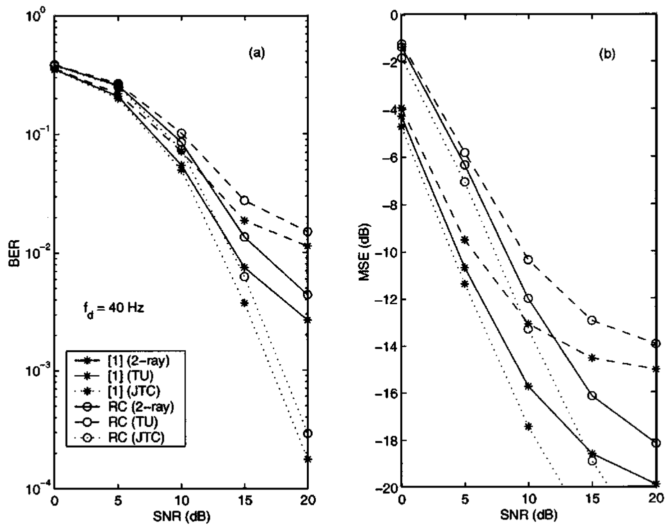

Fig. 8. Performance of the channel estimation methods in OFDM with 16-QAM, 16 states space–time code. The rms delay spread is 1.06 μs for two-ray and TU, 34.8 ns for JTC, the Doppler frequency is 40 Hz.

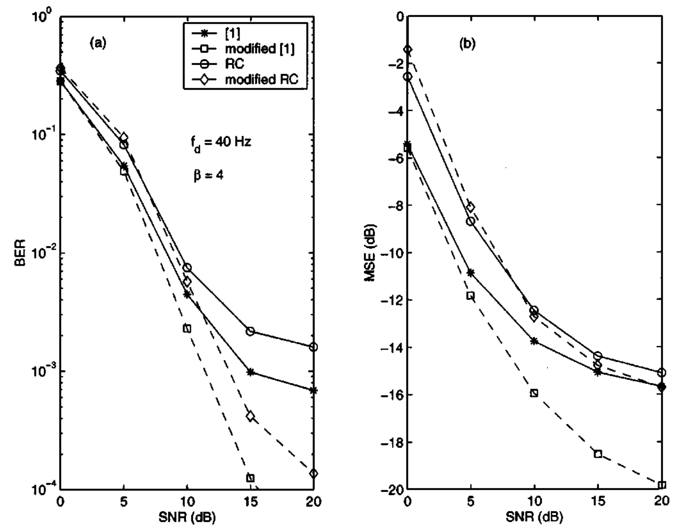

Fig. 10. Performance of the modified channel estimation methods in TU channel with the Doppler frequency of 40 Hz for OFDM with 4-PSK, 16 states space–time code.

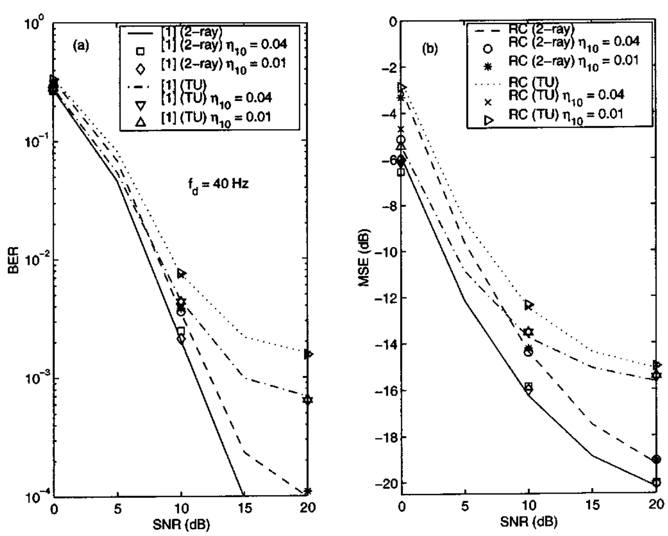

Fig. 9. Performance of the channel estimation methods with adaptive significant tap selection ($J_m = 7,\ \alpha = 4$) in two-ray and TU channels with delay spread of 1.06 μs, and the Doppler frequency of 40 Hz for OFDM with 4-PSK, 16 states space–time code.

In Fig. 8, the performances for 16-QAM, 16 states space–time coded OFDM system in two-ray, TU, and JTC channel models are presented. A higher MSE gap between [1] and RC is observed for 16-QAM than for 4-PSK case. The reason is that due to the nonconstant modulus subcarrier symbols, the MMSE criterion in RC method loses its optimality in the maximum likelihood (ML) sense while [1] still holds the ML optimality. However, in terms of BER performance, only a small BER gap is observed. This indicates that RC method can be an alternative approach also for nonconstant modulus subcarrier symbols when the complexity is an issue.

In Fig. 9, the performance of the adaptive selection of the number of significant taps is presented for two-ray and TU channel models with delay spread of 1.06 μs, and the Doppler frequency of 40 Hz. The threshold values used at a SNR of 10 dB, denoted by η_{10}, are 0.04 and 0.01. The other parameters are $J_m = 7$ and $\alpha = 4$. This adaptive approach has almost the same performance as the case with the knowledge of suitable fixed number of significant taps. For low SNR value, the adaptive approach has a slight performance improvement since the adaptive selection would quite often choose a smaller number of taps and hence would suppress the effect of the dominant noise to a larger extent. The results also show that threshold value around the range of 0.01 to 0.04 at a SNR of 10 dB works well. It is also noted that if J in the approach with fixed J value and J_m in the approach with adaptive J value are the same, the adaptive approach saves some complexity.

As discussed in the previous section, both [1] and RC methods can be modified by introducing some timing pre-advancement of β samples. The results of this modified approach with $\beta = 4$ as an example are presented in Fig. 10 for TU channel model with a Doppler frequency of 40 Hz. The results clearly show that for moderate to high SNR values, the modified approach achieves quite substantial improvement without any added complexity. This is a result of less unused leakage energy of the modified approach in the channel estimation.

VII. Conclusion

In this paper, we have investigated a reduced complexity channel estimation for an OFDM system with space–time coding in time-varying, dispersive multipath fading channels. In particular, the motivation was to reduce the complexity in the matrix inversion needed for every OFDM data symbols. The method is developed based on a channel with relatively small delay spread. By decoupling the channel responses from different transmit antennas, much complexity reduction is achieved. In particular, the sizes of the matrix inverse and the FFTs required in the channel estimation for every OFDM data symbol is reduced by half. The simulation results show that the price for the complexity reduction is just a slight BER degradation for channels with relatively small delay spreads. For cases where the complexity is of a major concern, the reduced complexity method can be considered as a good alternative. Adaptively finding the number of significant taps based on a threshold decision is also shown to have a comparable performance to the case where the suitable number of significant taps is known. We have also investigated the effect of the channel path with nonsample-spaced delay on the channel estimation based on the sample-spaced channel taps. A simple modification by means of the timing pre-advancement is proposed in order to reduce the energy leakage of the paths with nonsample-spaced delays lost in the channel estimation. This modified approach brings about a substantial improvement without any added complexity.

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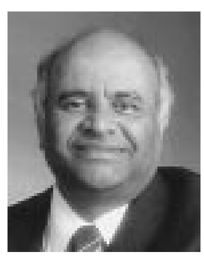

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