Combined Binary Pulse Position Modulation/Biorthogonal Modulation for Direct-Sequence Code Division Multiple Access

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***Abstract*—A novel modulation format is proposed for cellular direct-sequence CDMA systems where a user-specific spreading sequence is binary pulse position and biorthogonally modulated to form a set of biorthogonal spreading sequences. The modulation scheme trades the signal space used for spreading sequences with that for modulation while a global space is fixed. The interference is mainly determined by the cross correlation properties among sequences, but also affected by modulation. The effect is taken into account to evaluate the multi-user performance of the combined modulation. Compared to M-ary orthogonal modulation, the performance is shown to be almost the same while resulting in a simpler receiver structure.**

***Index Terms*—Binary PPM/biorthogonal modulation, biorthogonal receiver, multi-user performance, signal space.**

I. Introduction

We are concerned with enhanced multi-user performance of cellular direct-sequence code division multiple access (CDMA) systems [1] by employing M-ary signaling while receiver complexity is minimized. Unlike M-ary orthogonal signaling [2], we propose the adoption of binary pulse position modulation (PPM) that is embedded in the chip waveform, and also antipodal signaling for the Walsh/Hadamard orthogonal codes that results in biorthogonal modulation. We shall refer to this new signaling scheme as combined binary PPM/biorthogonal modulation which requires only $M/4$-ary orthogonal codes instead of M-ary codes. By exploiting this relationship, the receiver complexity may be greatly reduced when implementing the maximum-likelihood sequence detector.

A fully coherent receiver for the proposed PPM scheme is not easily obtained, while M-ary orthogonal signals can be detected using a noncoherent receiver if quadrature direct-sequence spreading is employed for the Walsh/Hadamard orthogonal codes [1]. In reality, a pilot-symbol assisted coherent demodulation is being realized for the mobile environment [3], and hence the technique may be used to detect binary PPM/biorthogonal signals. It should be pointed out that coherent demodulation[1] of M-ary orthogonal signals usually requires a pilot symbol (or tone) for the mobile environment. Therefore, the proposed binary PPM/biorthogonal modulation can still provide the advantage of simpler receiver structure.

For sufficiently large M, biorthogonal signaling yields almost the same multi-user performance compared to orthogonal signaling, and also binary PPM compensates for the loss in the processing gain by reducing the interference by half. Here, the signal space used for spreading sequences is reduced by a factor of two because of the binary PPM, given a global space is fixed. To investigate the performance, we characterize statistics on the interference caused by other users in view of the sequence properties and modulation.

With no bandwidth expansion and a constraint on the coding complexity, we may apply trellis coding techniques [4] to a set of biorthogonal spreading sequences generated by the binary PPM/biorthogonal modulation. Such techniques will produce additional coding gains, but it is beyond the scope of the current work to extend the analysis to the case with error correction.

II. Signal Characteristics

A combined binary PPM/biorthogonal modulation can be generated as follows. First, the binary PPM is embedded in the chip waveform, that is

$$v_k(t) = \phi(t - \lambda_k T_p), \quad t \in [0, T_c) \tag{1}$$

where $\phi(t)$ is any pulse of duration T_p, occupied in $[0, T_p)$, for a chip time $T_c = 2T_p$, and the chip rate T_c^{-1} is reduced by half with fixed bandwidth. Here λ_k represents the pulse position, taking the value of zero or one that is capable of conveying one-bit data of the kth user.

The following biorthogonal modulation is performed on the Walsh/Hadamard orthogonal codes by antipodal signaling that results in biorthogonal codes. It can be expressed in the form

$$w_k(t) = \sum_{m=0}^{M/4-1} a_k h_m^{(k)} \sum_{l=0}^{L-1} v_k(t - (l + mL)T_c), \quad t \in [0, T) \tag{2}$$

where a_k denotes the kth user's binary data ± 1, and the $M/4$-ary orthogonal codes $\mathbf{h}^{(k)} = (h_0^{(k)}, h_1^{(k)}, \cdots, h_{M/4-1}^{(k)})$, $h_m^{(k)} \in \{1, -1\}$, are generated by the Hadamard matrix. Note that $\{a_k \mathbf{h}^{(k)}\}$ become biorthogonal data sequences associated with the kth user's $M/2$-ary data.

To permit signal separation among users, the user-specific spreading sequences $\{c_n^{(k)}\}$ $(k = 1, 2, \cdots, K)$ are employed, in which the kth user's baseband spread-spectrum signal is shown in Fig. 1 as $(n = l + mL)$

$$\tilde{s}_k(t) = \sum_{m=0}^{M/4-1} \sqrt{2}\, a_k h_m^{(k)} \sum_{l=0}^{L-1} c_{l+mL}^{(k)} \phi(t - (l + mL)T_c - \lambda_k T_p) \tag{3}$$

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[1] It offers about 30% increase in user capacity at the uncoded bit-error rate (BER) of interest (e.g., 10^{-1}–10^{-2}) compared to noncoherent demodulation [1].

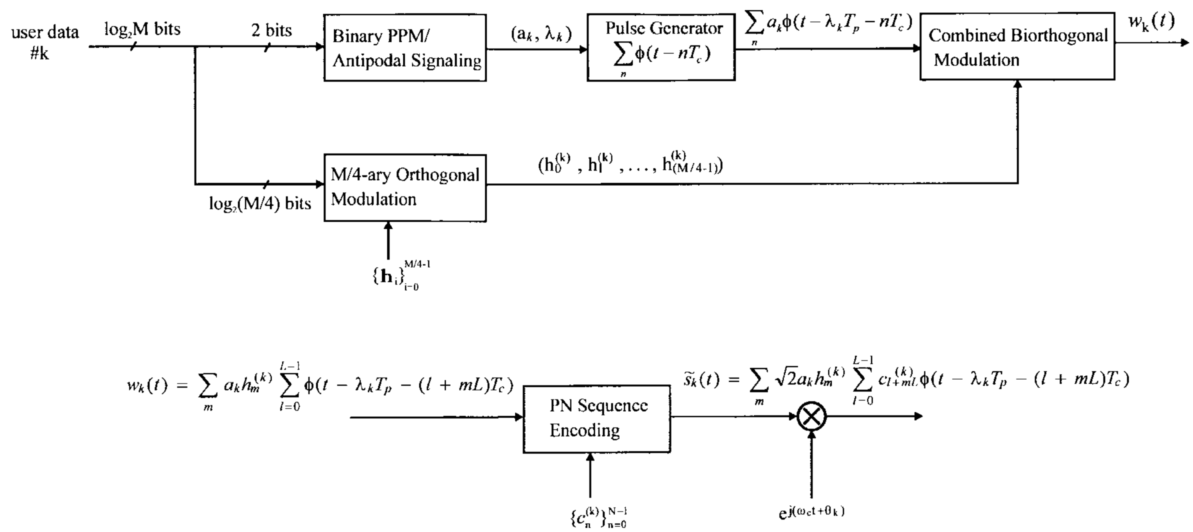

Fig. 1. Combined binary PPM/biorthogonal modulation for the kth user.

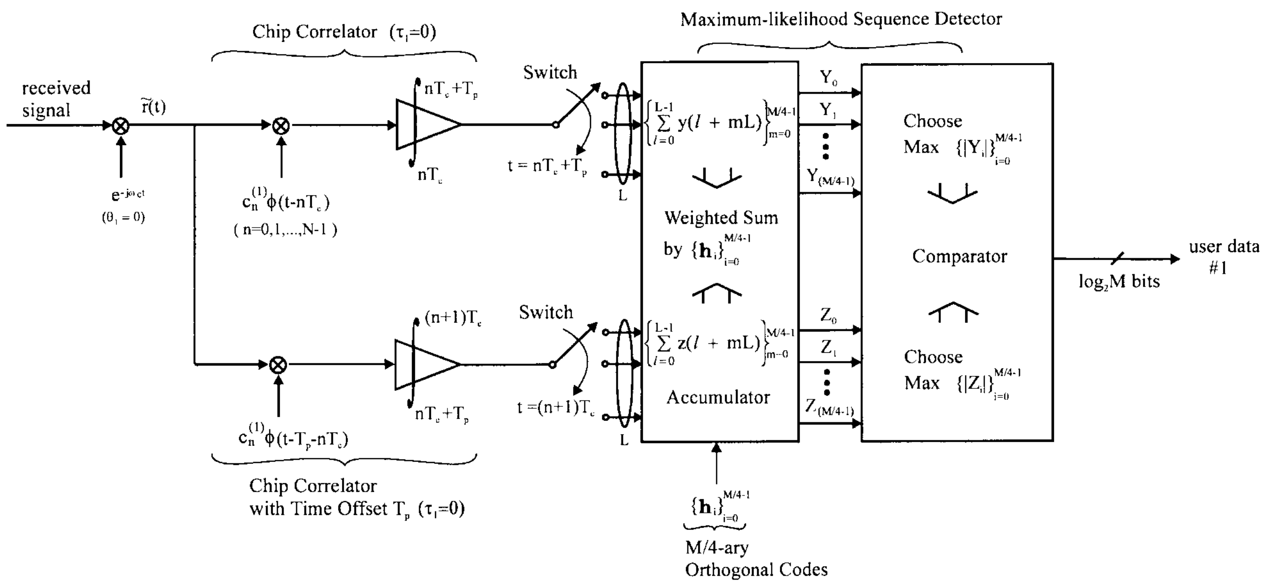

Fig. 2. Biorthogonal receiver structure implemented for the first user.

for $t \in [0, T)$. A symbol time T is equal to $T = (M/4)LT_c$ for which L indicates the amount of partial spreading by the spreading sequences.

III. Multi-User Performance

The received signal which includes noise and other-user interference can be expressed by

$$\tilde{r}(t) = \tilde{s}_1(t) + \sum_{k=2}^{K} \tilde{s}_k(t-\tau_k)\exp(j\theta_k) + \tilde{n}(t) \qquad (4)$$

where the first user is desired ($\tau_1 = \theta_1 = 0$); K denotes the number of users. The channel delay τ_k of the kth user is uniformly distributed over $[0, T)$, the unknown phase θ_k is uniformly distributed over $[0, 2\pi)$, and $\tilde{n}(t)$ is a complex-valued white Gaussian noise with two-sided power spectral density N_o.

A biorthogonal receiver in Fig. 2 consists of two chip correlators, operating every T_c with time offset T_p, and the maximum-likelihood sequence detector to find the pulse position λ_1 and the $M/4$-ary orthogonal code $\mathbf{h}^{(1)}$ where the comparator decides the binary data a_1.

Assuming $\lambda_1 = 0$, the output of the on-time chip correlator is

$$\begin{aligned} y(n) &= \frac{1}{\sqrt{2}} \int_{nT_c}^{nT_c+T_p} \mathrm{Re}\{\tilde{r}(t)\} c_n^{(1)} \phi(t-nT_c)\, dt \\ &= a_1 h_m^{(1)} E_p + \sum_{k=2}^{K} I_k(n) + \eta(n) \end{aligned} \qquad (5)$$

with $n = l + mL$. Here, the chip energy E_p is defined by $\int_0^{T_p} \phi^2(t)\, dt = E_p$; the noise term $\eta(n)$ is a zero-mean Gaussian random variable with variance $N_o E_p/2$; and the interference term due to the kth user becomes

$$I_k(n) = \begin{cases} [a_{k(n-l_k-1)} h^{(k)}_{[(n-l_k-1)/L]} c^{(k)}_{(n-l_k-1)} c^{(1)}_n \\ \quad \cdot R_\phi(\langle \bar\tau_k \rangle)\, \delta(\lambda_{k(n-l_k-1)} - 1) + a_{k(n-l_k)} \\ \quad \cdot h^{(k)}_{[(n-l_k)/L]} c^{(k)}_{(n-l_k)} c^{(1)}_n \hat R_\phi(\langle \bar\tau_k \rangle) \\ \quad \cdot \delta(\lambda_{k(n-l_k)})] \cos\theta_k, \quad \text{for } 0 \le \bar\tau_k < T_p \\ a_{k(n-l_k-1)} h^{(k)}_{[(n-l_k-1)/L]} c^{(k)}_{(n-l_k-1)} c^{(1)}_n \\ \quad \cdot [R_\phi(\langle \bar\tau_k \rangle)\, \delta(\lambda_{k(n-l_k-1)}) + \hat R_\phi(\langle \bar\tau_k \rangle) \\ \quad \cdot \delta(\lambda_{k(n-l_k-1)} - 1)] \cos\theta_k, \quad \text{for } T_p \le \bar\tau_k < T_c \end{cases} \tag{6}$$

where $l_k T_c \le \tau_k < (l_k+1) T_c$ for an integer l_k. The delay term $\bar\tau_k = \tau_k - l_k T_c$ is uniformly distributed over $[0, T_c)$ and $\langle \bar\tau_k \rangle = \bar\tau_k$ modulo T_p. The following functions are defined: $[x]$ is an integer part of x; the delta function $\delta(u)$ is defined by $\delta(u) = 0$ if $u \ne 0$ and $\delta(0) = 1$; and the partial autocorrelation functions for $\phi(t)$ are defined by $R_\phi(\tau) = \int_0^\tau \phi(t)\phi(t + T_p - \tau)\, dt$ and $\hat R_\phi(\tau) = \int_\tau^{T_p} \phi(t)\phi(t - \tau)\, dt$ for $0 \le \tau \le T_p$. Note that $a_{k(m)}$ and $\lambda_{k(m)}$ represent the previous data symbols if $m < 0$, and otherwise current symbols.

Similarly, the output of the T_p-offset chip correlator is

$$\begin{aligned} z(n) &= \frac{1}{\sqrt{2}} \int_{nT_c+T_p}^{(n+1)T_c} \mathrm{Re}\{\tilde r(t)\} c^{(1)}_n \phi(t - nT_c - T_p)\, dt \\ &= \sum_{k=2}^{K} I^*_k(n) + \eta^*(n) \end{aligned} \tag{7}$$

where the noise term $\eta^*(n)$ is identically distributed as $\eta(n)$, mutually independent, and the interference term $I^*_k(n)$ is shown to be

$$I^*_k(n) = \begin{cases} a_{k(n-l_k)} h^{(k)}_{[(n-l_k)/L]} c^{(k)}_{(n-l_k)} c^{(1)}_n \\ \quad \cdot [R_\phi(\langle \bar\tau_k \rangle)\, \delta(\lambda_{k(n-l_k)}) + \hat R_\phi(\langle \bar\tau_k \rangle) \\ \quad \cdot \delta(\lambda_{k(n-l_k)} - 1)] \cos\theta_k, \quad \text{for } 0 \le \bar\tau_k < T_p \\ [a_{k(n-l_k-1)} h^{(k)}_{[(n-l_k-1)/L]} c^{(k)}_{(n-l_k-1)} c^{(1)}_n \\ \quad \cdot R_\phi(\langle \bar\tau_k \rangle)\, \delta(\lambda_{k(n-l_k-1)} - 1) + a_{k(n-l_k)} \\ \quad \cdot h^{(k)}_{[(n-l_k)/L]} c^{(k)}_{(n-l_k)} c^{(1)}_n \hat R_\phi(\langle \bar\tau_k \rangle) \\ \quad \cdot \delta(\lambda_{k(n-l_k)})] \cos\theta_k, \quad \text{for } T_p \le \bar\tau_k < T_c. \end{cases} \tag{8}$$

The maximum-likelihood sequence detector produces decision variables which are the sums of correlated outputs weighted by the $M/4$-ary orthogonal codes $\{\mathbf{h}_i\}$, $\mathbf{h}_i = (h_{i,0}, h_{i,1}, \cdots, h_{i,M/4-1})$ $(i = 0, 1, \cdots, M/4 - 1)$. The decision variables are defined by

$$Y_i = \sum_{m=0}^{M/4-1} h_{i,m} \sum_{l=0}^{L-1} y(l + mL) \tag{9}$$

$$Z_i = \sum_{m=0}^{M/4-1} h_{i,m} \sum_{l=0}^{L-1} z(l + mL) \tag{10}$$

for $i = 0, 1, \cdots, M/4 - 1$. If we model $\{Y_i\}$ and $\{Z_i\}$ as Gaussian random variables, the following approximations for their statistics hold:

$$\mathrm{var}(Y_i) \cong \mathrm{var}(Z_i) \cong \frac{1}{2}(K-1) N \mathbf{E}\{R^2_\phi(\langle \bar\tau_k \rangle)\} + \frac{1}{2} E_s N_o \tag{11}$$

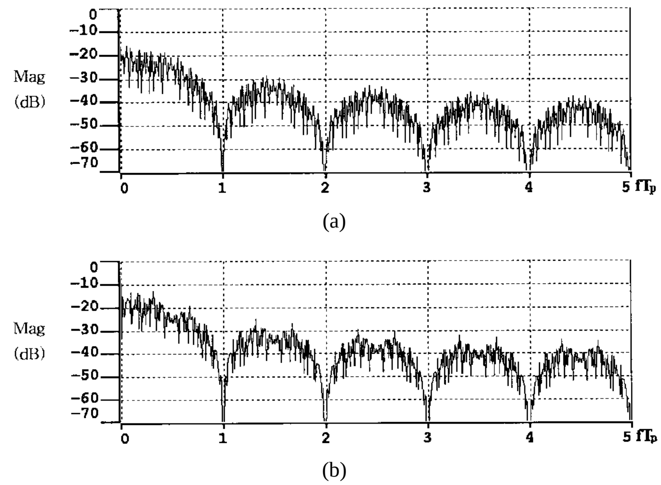

Fig. 3. Spectral characteristics for the combined and M-ary orthogonal modulations when a rectangular chip pulse of duration T_p is used.

$$\mathrm{cov}(Y_i, Z_i) \cong \frac{1}{4}(K-1) N \mathbf{E}\Big\{ R_\phi(\langle \bar\tau_k \rangle) \hat R_\phi(\langle \bar\tau_k \rangle) \Big\}. \tag{12}$$

The cross terms have been ignored because of long PN spreading sequences that are modeled as random binary sequences, and the large number $K \ge 30$ of users considered.

Now the average probability of symbol error can be expressed by

$$\begin{aligned} P(\epsilon) &= \mathbf{E}\{P(\epsilon \,|\, Y_0 = \gamma > 0)\} \\ &= 1 - \mathbf{E}\Bigg\{ \Pr\Bigg[\bigcap_{i=1}^{M/4-1} \{|Y_i| < \gamma\} \\ &\qquad \cdot \bigcap_{j=0}^{M/4-1} \{|Z_j| < \gamma\} \;\Big|\; Y_0 = \gamma > 0 \Bigg] \Bigg\}. \end{aligned} \tag{13}$$

We have assumed that the symbol $\{a_1 = 1, \lambda_1 = 0, \mathbf{h}^{(1)} = \mathbf{h}_0\}$ was sent for the first user, and the expectation $\mathbf{E}$ is taken with respect to $\gamma > 0$. If $\{|Y_i| < \gamma, |Z_i| < \gamma\}$ can be approximately modeled as a set of independent events given Gaussian assumptions, then $P(\epsilon)$ is approximated by

$$\begin{aligned} P(\epsilon) \cong 1 - \int_0^\infty & \prod_{i=1}^{M/4-1} \Pr[|Y_i| < \gamma, |Z_i| < \gamma] \\ & \cdot \Pr[|Z_0| < \gamma, Y_0 = \gamma]\, d\gamma. \end{aligned} \tag{14}$$

The Gaussian approximation, along with the joint statistics in (11) and (12), yields

$$\begin{aligned} \Pr[|Y_i| < \gamma, |Z_i| < \gamma] \cong & \int_{-\gamma}^{\gamma} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{z_i^2}{2\sigma^2}\right) \\ & \cdot \left[1 - 2Q\left(\frac{\gamma + \rho z_i}{\sigma\sqrt{1-\rho^2}} \right) \right] dz_i \end{aligned} \tag{15}$$

$$\begin{aligned} &\Pr[|Z_0| < \gamma, Y_0 = \gamma] \\ &\quad \cong \begin{cases} \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp[-(\gamma - E_s)^2/2\sigma^2] \\ \quad \cdot [Q(\alpha) - Q(\beta)] \quad \text{if } \gamma < \rho E_s/(1+\rho), \\ \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp[-(\gamma - E_s)^2/2\sigma^2] \\ \quad \cdot [1 - Q(-\alpha) - Q(\beta)] \quad \text{if } \gamma \ge \rho E_s/(1+\rho) \end{cases} \end{aligned} \tag{16}$$

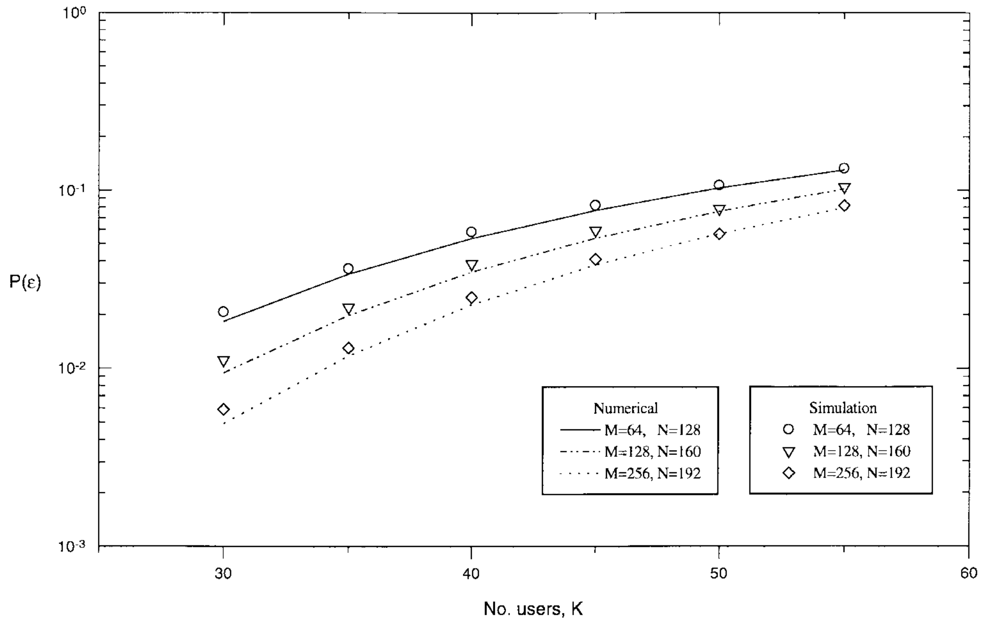

Fig. 4. Average symbol error probability $P(\epsilon)$ as a function of K for the combined modulation when a rectangular chip pulse is used and $E_b/N_o = 10$ dB.

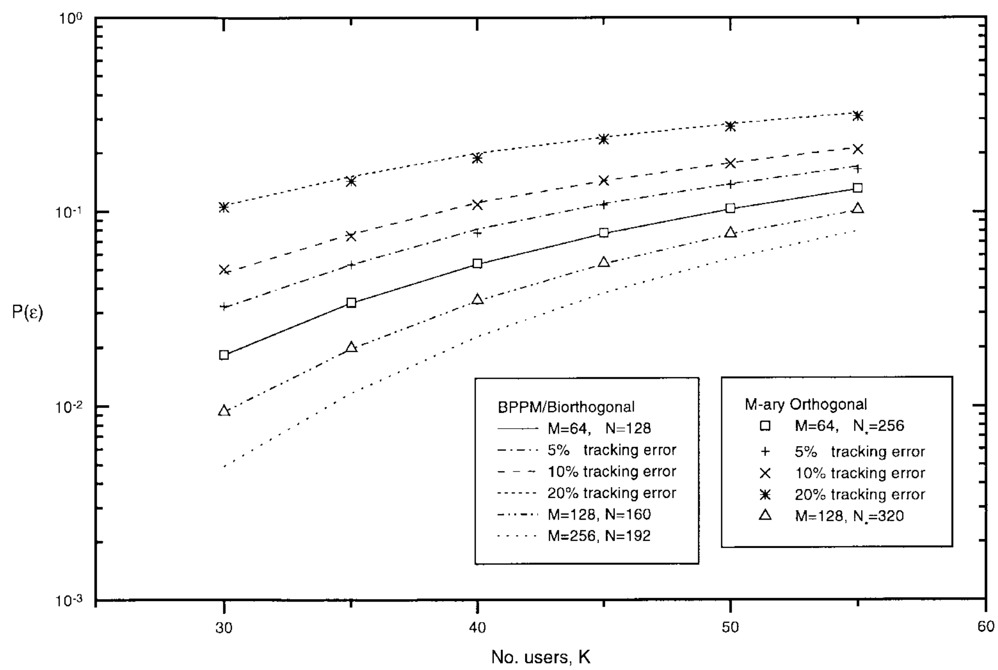

Fig. 5. Comparisons of $P(\epsilon)$ as a function of K for the combined and M-ary orthogonal modulations with and without time tracking errors when $E_b/N_o = 10$ dB.

where $\sigma^2 = \mathrm{var}(Y_i) = \mathrm{var}(Z_i)$ and $\rho\sigma^2 = \mathrm{cov}(Y_i, Z_i)$, $\alpha = \frac{\rho E_s - \gamma(1+\rho)}{\sigma\sqrt{1-\rho^2}}$ and $\beta = \frac{\rho E_s + \gamma(1-\rho)}{\sigma\sqrt{1-\rho^2}}$ for the symbol energy $E_s = NE_p$ $(N = LM/4)$, and $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$.

Finally, combining (14)–(16), with numerical integration, we can evaluate the multi-user performance for the binary PPM/biorthogonal modulation scheme.

IV. Results

To observe some of the spectral characteristics of the proposed binary PPM/biorthogonal modulation, the resulting waveform was simulated in the frequency domain by taking a 2048-point fast Fourier transform (16 samples/T_p), seen in Fig. 3. It is observed that the spectral characteristics remain almost the same as for M-ary orthogonal modulation for sufficiently large N in view of the null-to-null bandwidth and sidelobes.

For $M = 64, 128, 256$, $N = 128$, 160, 192 $(L = 8, 5, 3)$, and $K = 30 - 55$, the average symbol error probability $P(\epsilon)$ is plotted in Fig. 4 when $E_b/N_o = 10$ dB (E_b bit energy). The simulation results agree well with the analysis, in which the number of chips/symbol N is adjusted to have the same bandwidth, i.e., $N_* = 2N$ for the M-ary orthogonal modulation.

To investigate the effect of time tracking errors on $P(\epsilon)$, we performed simulations when the tracking errors were set to 5, 10, and 20% of the pulse duration T_p. Fig. 5 shows $P(\epsilon)$ versus K, number of users in which the original M-ary scheme performs slightly better in the presence of the corresponding tracking errors (separation more pronounced for a 20% mismatch). Synchronization for the proposed demodulator is an unsolved problem and it might in fact be more difficult to acquire the signal in the proposed system than it would be in a conventional system. Due to this fact, the amount of timing mismatch would be different for two schemes, even under the same operating conditions.

With no tracking error, the two modulation schemes were also compared in Fig. 5 where the receiver complexity is on the order of $(M/4)(2N)/\log_2 M$ for the combined modulation and $MN_*/\log_2 M$ for orthogonal modulation. It is observed that the combined modulation performs well for large $M = 256$ with $N = 192$ by employing $M/4 = 64$-ary orthogonal codes, which is comparable to $M = 64$-ary orthogonal modulation with $N_* = 256$ in view of complexity. Besides, the two schemes present almost the same performance even with fixed $M = 64$, 128 and $N_* = 2N$.

V. Conclusion

A biorthogonal modulation combined with binary PPM has been proposed for direct-sequence CDMA systems that need enhanced multi-user performance with applications to cellular systems. The scheme provides a simpler receiver structure by adopting reduced $M/4$-ary orthogonal codes while maintaining almost the same performance relative to M-ary orthogonal modulation. With some constraints on the complexity, we can achieve higher CDMA capacity by increasing M, N to a certain limit in the combined modulation. Tradeoff between complexity and capacity offered by the combined modulation provides flexibility when designing a cellular CDMA system.

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