On the Performance of Centralized DS-SS Packet Radio Networks with Random Spreading Code Assignment

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***Abstract*— This paper presents a random spreading code assignment scheme for enhancing channel efficiency in centralized DS-SS packet radio networks which employ a multiple-capture receiver for each code channel. Compared to the common code case, this approach requires modest increase in receiver complexity, but the number of distinct spreading codes being used is considerably less than the number of radios in the network. A general theoretical framework for evaluation of collision-free packet performance in each code channel is described, in which the possibility of collision-free transmission is conservatively estimated using a combinatorial method, and the effects of asynchronous multiple-access interference are characterized in terms of the primary and secondary user interferences. At the link level, the capture and throughput performances are evaluated for a proper set of codes, and compared with the results from the common code scheme. It is shown that the use of a random assignment scheme with more than one code results in a higher performance gain, and most of this gain can be achieved with just two distinct spreading codes.**

I. Introduction

FOR MANY YEARS, various forms of networking studies have been done on proposing channel access schemes and evaluating network performance such as throughput and delay under over-simplified radio link models, i.e., ALOHA assumption or single capture model. At the link level, Davis and Gronemeyer [1] analyzed a single capture model for a slotted random access spread-spectrum network with star topology. It achieves excellent performance characteristics in some respects, but neglects the possibility of more than one packet being captured at the same time. Polydoros and Silvester [2] developed a general model for performance studies in slotted spread-spectrum multiple-access networks. With the exception of retention models, they also analyzed the single capture models. At the network level, Sousa and Silvester [3] proposed novel spreading code assignment strategies for distributed spread-spectrum packet radio networks. For evaluation of network performance, they made the ALOHA assumption that ignores the possibility of multiple capture.

This paper is concerned with a multiple-capture model which allows two or more packets to succeed on a single spreading code if there are sufficient time offsets between them. We concentrate on centralized direct-sequence spread-spectrum (DS-SS) packet radio networks in which all radios can utilize power control to have equal received signal strength at the central node and perform range measurements to remove the effect of propagation times. With this somewhat limited topology, we are able to accurately, though not exactly, develop the multiple-capture model [4] in which the multiple capture is assumed to occur whenever some number of packets are collision-free, concurrently received interfering signals are treated like noise, and their headers are correctly identified.

There are two basic approaches, namely, the common code and transmitter-oriented code schemes [3] to achieve multiple simultaneous successful transmissions in the centralized networks. The former is easy to implement, but increases the packet loss because of the possibility of collision under heavy traffic conditions. On the other hand, the latter assures perfectly collision-free transmission, but increases the receiver complexity because of a large set of distinct spreading codes. In order to overcome the problems and take advantage of them, we propose a random spreading code assignment scheme which increases the possibility of collision-free transmission with modest receiver complexity [5].

In the next section, the random code assignment scheme is described, and Section III gives the system model with the necessary assumptions. The performance analysis is done in three different parts. In Section IV, we derive the general expression for the probability distribution of the number of collision-free transmissions. Section V outlines the radio channel model and characterizes the asynchronous multiple-access interference. In Section VI, expressions for the average number of packet captures and system throughput are derived, and results are presented in section VII. Finally, Section VIII provides concluding remarks.

II. Random Spreading Code Assignment

We propose a random spreading code assignment scheme in which the number of distinct spreading codes being used by all radio terminals is considerably less than the number of radios in the network. In the random assignment scheme,

Paper approved by B. Aazhang, the Editor for Spread Spectrum of the IEEE Communications Society. Manuscript received August 3, 1993; revised March 22, 1994. This work was supported in part by the Army Research Office under Contract DAAL-03-88-K-0059. This paper was presented in part at the IEEE MILCOM'91, Washington, DC, November 1991.

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IEEE Log Number 9413813.

every radio terminal randomly chooses a spreading code from a set of prespecified spreading codes for the transmission of a particular packet. This scheme allows multiple successful transmissions for those packets which are initiated by the terminals employing different transmitting codes in a given time interval. We refer to it as the *multiple-access capability*. When two or more packets using the same transmitting code arrive at the receiver with sufficient time offsets, there will be collision-free packets in the sense that concurrently received signals are treated like wideband noise, some of which will be received successfully with high probability. We refer to it as the *multiple-capture capability*. But if there is insufficient time offset between such packets to permit the receiver to distinguish between them, collision will occur and the collided packets will be destroyed. This is because there exists a strong correlation between overlapped signals within a capture time, normally a chip time T_c, and hence an uncorrectable number of errors will be introduced in the collided packets.

Compared to the common code case, this scheme is suitable for heavy traffic conditions, since the possibility of capture increases in proportion to the number of distinct spreading codes available. For a low level of traffic, however, the common code scheme also has a high probability of capture and is probably more desirable because of its simplicity. Generally, there exists a tradeoff between the order of receiver complexity, namely, the number of codes to be used, and the performance gain that results. We note that the effect of interfering packets on the capture of a collision-free packet is almost the same for all spreading code schemes. Hence we can enhance the channel efficiency only by increasing the possibility of collision-free transmission.

III. System Model

The network consists of K packet radios which communicate with a single central node in a slotted random-access mode, so every radio can initiate his transmission in the beginning of a time slot. But we introduce some amount of random delay in the transmission to randomize the time of arrival at the central node, which leads to an asynchronous communication at the bit-time level. This is because the possibility of capture is increased by differentiating the packet arrival times through this randomization technique.

There are V distinct spreading codes available for encoding a packet in a given slot at each radio terminal where V is much less than K. According to the random selection policy, any one code of period N is selected with equal probability $1/V$ and multiplied to the packet signal successively every bit time T_b in order to generate a bit-length encoded waveform. We consider spreading codes which are chosen from the pseudonoise or maximal-length binary sequences (m-sequences) with good autocorrelation and cross-correlation properties. As the modulation format, we adopt a hybrid system which employes DS/binary phase-shift keying for the common header and DS/differential phase-shift keying for the data packet, since this system does not require knowledge of the phase at the receiver and also enables us to derive theoretical results.

The channel introduces two sources of interference, one of which is thermal noise and the other is multiple-access interference. For the latter interference, we have two kinds of user interference, one of which is the primary user interference caused by the interfering packets existing in the same code channel as a desired packet, the other is the secondary user interference resulting from the concurrent transmissions on different code channels. As we know, the primary user interference acts as a packet collider if this hits the desired signal within a capture time, or otherwise as a wideband noise. The secondary user interference is simply treated as another wideband noise. The effect of thermal noise can be minimized by increasing signal strength, but multiple-access interference does not depend on signal strength. Hence we investigate network performance under the assumption that packet errors result only from multiple-access interference.

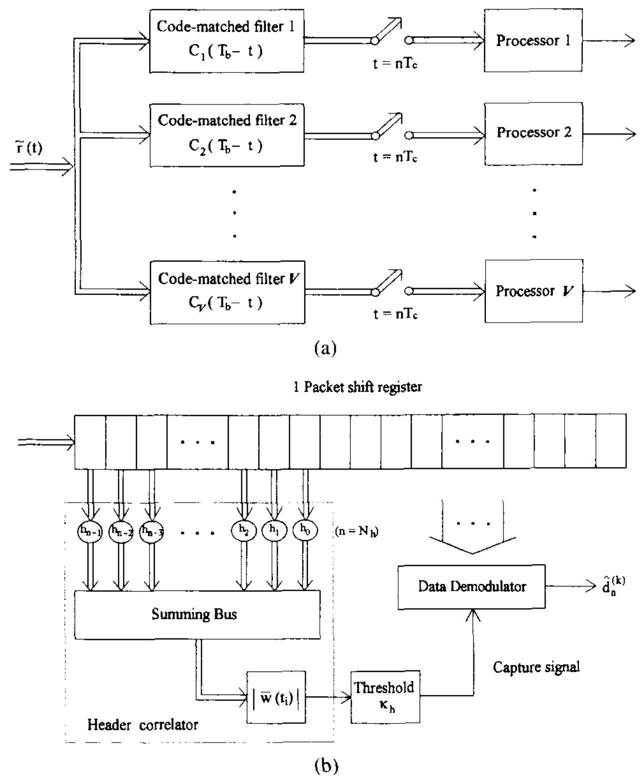

Fig. 1. Central receiver employing V different spreading codes. (a) Central receiver block diagram. (b) Realization of the processor.

At the central node, the receiver consists of a bank of V different code-matched filters and their following processors for envelope header detection and differential data demodulation. The vth code-matched filter has impulse response $c_v(T_b - t)$ where $c_v(t)$ denotes the vth spreading code waveform for $1 \leq v \leq V$. Each processor includes the header correlator that is a kind of digital filter matched to a common header sequence with good correlation properties. Here the central receiver can be viewed as the multi-receiver having both the multiple-capture capability that allows multiple successful transmissions using the same spreading code and the multiple-access capability of receiving the collision-free packets of each spreading code, the number of these quite possibly being larger than V and conceivably being larger than N. Fig. 1 shows the central receiver being equivalently modeled as a complex baseband.

In case of multipath, multiple copies of a packet may arrive at the receiver with relative delays of the order of a few chip times. It is possible for some of them to incur collision or capture the receiver, but this event is a complicated function of traffic, relative signal strength, and specific receiver design, etc. Thus, we here do not account for the effect of multipath and the analysis in the sequel is applicable to the cases without multipath.

IV. Collision-Free Transmission

At the central node, some number g of total m transmissions attempted in a given slot will be on a particular ith spreading code channel. In this case, collision will probably occur among these g transmissions as the primary user group in the ith code channel, and we need to evaluate the effect of the primary user interference by properly estimating the possibility of collision-free transmission. In order to derive theoretical results, we make the following assumptions:

1) The packet arrival time τ_k modulo T_b for the kth radio using the ith spreading code is uniformly distributed among a set of L discrete times with equal spacing in $[0, T_b)$.
2) The packet arrival times associated with different radios are statistically independent.
3) The received signal strength is the same for all radios.

The packet collision is mainly caused by the strong correlation between overlapped signals in the same code channel within a capture time Δ of the order of T_c. Thus, it is assumed that collision at the ith code channel occurs when any two of the g transmissions hit within Δ, i.e., $\min_{j:j\neq k}\min_n |\tau_k - \tau_j + nT_b| < \Delta, 1 \leq j \leq g$. Based on this assumption, we derive the probability distribution of the number of collision-free transmissions among the g transmissions. The following *Principle of Inclusion and Exclusion* [6] and Claim 1 will be useful in deriving this probability distribution.

Given $\mathcal{U}$ is a set and $\mathcal{A}$ is an index set, we define that for each $\alpha \in \mathcal{A}, \mathcal{P}_\alpha = \{x \in \mathcal{U} | x \text{ has property } \alpha\}$. Then the number of elements satisfying neither property is

$$N_o = \left|\mathcal{U} - \bigcup_{\alpha\in\mathcal{A}} \mathcal{P}_\alpha\right| = |\mathcal{U}| + \sum_{k\geq 1}(-1)^k \sum_{\substack{\mathcal{J}\subset\mathcal{A}\\ |\mathcal{J}|=k}} \left|\bigcap_{\alpha\in\mathcal{J}} \mathcal{P}_\alpha\right|. \quad (1)$$

Let $f(x)$ denote the number of properties that an element x of $\mathcal{U}$ satisfies. Then the number of elements satisfying exactly f properties is

$$\begin{aligned} N(f) &= |\{x \in \mathcal{U}\,|f(x) = f\}| \\ &= \sum_{k\geq 0}(-1)^k \binom{f+k}{f} \sum_{\substack{\mathcal{J}\subset\mathcal{A}\\ |\mathcal{J}|=f+k}} \left|\bigcap_{\alpha\in\mathcal{J}} \mathcal{P}_\alpha\right| \end{aligned} \quad (2)$$

for $f = 1, 2, \cdots, |\mathcal{A}|$.

Claim 1: Let $S(n, w, v)$ denote the number of ways of placing v identical balls into n labeled boxes in which empty is allowed, but no more than w balls in the same box. Then we find that

$$S(n, w, v) = \sum_{j\geq 0}(-1)^j \binom{n}{j} \left\langle {n \atop v - j(\omega+1)} \right\rangle \quad (3)$$

where $\binom{n}{r} \triangleq n!/(n-r)!r!$ and $\left\langle {n \atop r} \right\rangle \triangleq \binom{n+r-1}{r}$.

Proof of Claim 1: See Appendix A.

Define $\mathcal{U} = \{x | \mathcal{A} \mapsto \mathcal{L}$ where order counts, but repetition is allowed$\}$ where an index set $\mathcal{A}$ denotes the g labeled arrivals at the ith code channel in the slot and an index set $\mathcal{L}$ denotes the L discrete times with equal spacing $\Delta/w+1$ (some integer $w \geq 1$) in $[0, T_b)$, and $|\mathcal{U}| = L^g$ with $L = \lfloor (w+1)T_b/\Delta \rfloor$. (Here $\lfloor x \rfloor$ denotes the largest integer not exceeding x). The set $\mathcal{U}$ is equivalent to the collection of all possible ways in which the g labeled arrivals may be arranged among the L discrete times with repetition. For each $\alpha \in \mathcal{A}$, we define $\mathcal{P}_\alpha = \{x \in \mathcal{U} | x$ has the property that the αth packet is collision-free$\}$. The event $\min_{\beta:\beta\neq\alpha}\min_n |\tau_\alpha - \tau_\beta + nT_b| \geq \Delta, 1 \leq \beta \leq g$ is then equivalent to $\mathcal{P}_\alpha$, i.e., the subset of $\mathcal{U}$ such that if we arrange the L discrete times in a circle, none of adjacent arrivals lie within w discrete times from the αth arrival. Hence the event of exactly f packets being collision-free corresponds to the subset $\{x \in \mathcal{U} | f(x) = f\}$ in (2).

We prepare Claim 2 to evaluate $N(f)$ for $f = 1, 2, \cdots, g$.

Claim 2: For a particular subset $\mathcal{J} = \{1, 2, \cdots, q\}$ with size $|\mathcal{J}| = q$, the number of occurrences of the event that at least this q are collision-free is given by

$$\left|\bigcap_{\alpha=1}^{q} \mathcal{P}_\alpha\right| = \begin{cases} \displaystyle\sum_{r=1}^{r_{\max}}\sum_{v=0}^{v_{\max}} (q-1)!\, L\binom{q}{r} S(q-r, w, v) \\ \left\langle {r \atop L-(w+1)(q+r)-v} \right\rangle \\ [L-(w+1)(q+r)+r-v]^{g-q} \\ \qquad \text{if } 0 < q < L/(w+1) \quad \text{and} \quad q < g, \\ L^g \qquad \text{if} \quad q = 0, \\ L(g-1)!\left\langle {g \atop L-(w+1)g} \right\rangle \\ \qquad \text{if} \quad q = g \leq L/(w+1), \\ 0 \qquad \text{otherwise}, \end{cases} \quad (4)$$

where $r_{\max} = \min\{q, \lfloor L/w+1 \rfloor - q\}$, $v_{\max} = \min\{w(q-r), L-(w+1)(q+r)\}$, and the formula (4) is equivalent to the corresponding one in [7] when $g = m$ and $w+1$ is replaced by $2l$.

Proof of Claim 2: Refer to [7].

Claim 2 is valid for any subset $\mathcal{J}(|\mathcal{J}| = q)$ of the index set $\mathcal{A}$ so that the number of occurrences of the event that exactly f of g packets are collision-free is expressed as $(q = f + k)$

$$N(f) = \sum_{k\geq 0}(-1)^k \binom{f+k}{f}\binom{g}{f+k}\left|\bigcap_{\alpha=1}^{f+k} \mathcal{P}_\alpha\right| \quad (5)$$

for $f = 1, 2, \cdots, g$.

Define

$$\begin{aligned} P_{F|G}(f|g) \triangleq\ & \Pr\{F = f \text{ collision-free packets}|G \\ & = g \text{ transmissions}\}. \end{aligned} \quad (6)$$

In reality, it is necessary that the time of arrival τ_k (modulo T_b) at the central node is assumed to have a continuously uniform distribution over $[0, T_b)$, so the probability distribution $P_{F|G}(f|g)$ must be derived based on this assumption. However, theoretical evaluation of this $P_{F|G}(f|g)$ appears intractable, and instead the combinatorial method is applied to derive a discrete-time approximation to $P_{F|G}(f|g)$

$$P_{F|G}^{(w)}(f|g) = \frac{N(f)}{L^g} \tag{7}$$

where w indicates the order of approximation. In the limit, it follows that $\lim_{w\to\infty} P_{F|G}^{(w)}(f|g) = P_{F|G}(f|g)$. Here we denote $P_{F|G}^{(w)}(f|g)$ as the collision-free probability of order w, which can be derived by combining (4), (5), and (7).

V. Asynchronous Multiple-Access Interference

We consider a time-slotted system in combination with asynchronous communication at the bit-time level, and hence the number of interfering transmissions is not always constant throughout the entire packet. Capture was assumed to occur if the header of a packet is correctly detected in the presence of other interfering transmissions. For the evaluation of capture performance, we should be able to characterize the effect of asynchronous multiple-access interference as wideband noise on the header detector.

At the central node, if some number g of total m transmissions attempted in a given slot are on the ith spreading code channel, then the complex envelope of the received signal can be expressed as

$$\tilde{r}(t) = \sum_{n=0}^{N_p-1} \sqrt{2P}\Bigg[\sum_{k=1}^{g} d_n^{(k)} c_i(t-\tau_k-nT_b)\exp[j\theta_k(t)] + \sum_{k=g+1}^{m} d_n^{(k)} c_{v_k}(t-\tau_k-nT_b)\exp[j\theta_k(t)]\Bigg] \tag{8}$$

where $\{1, 2, \cdots, g\}$ indicates the primary user group and $\{g+1, g+2, \cdots, m\}$ the secondary user group both in the ith code channel with distinct spreading codes $c_i(t)$ and $c_{v_k}(t)$ for $v_k \neq i, 1 \leq v_k \leq V$. In this signal model, N_p is the packet length, P is the received signal power, $d_n^{(k)}$ is the kth radio's differentially encoded binary data sequence whose first N_h elements $(d_0^{(k)}, d_1^{(k)}, \cdots, d_{N_h-1}^{(k)})$ is the common header sequence $(h_0, h_1, \cdots, h_{N_h-1}), \tau_k$ is the time of arrival of the kth radio's at the central node, and $\theta_k(t)$ is the kth radio's unknown signal phase.

Because of the randomization technique to enhance the capture effects, the time of arrival τ_k may be ranged over several number of bits exceeding the beginning of time slot, but for slotted operation at the packet level, τ_k is required to be small compared to the packet duration. Thus, we simply assume that the packet arrival times $\{\tau_k\}$ are uniformly distributed over a randomization time interval $[0, T_r]$ where $T_r = IT_b$ for some integer $I, I \ll N_p$. In this case, we look into the normalized signal output at a sampling time t_i of the ith code-matched filter/header correlator that takes the form

$$\overline{w}(t_i) = \sum_{k=1}^{g} PU_k(t_i, \tau_k, \theta_k) + \sum_{k=g+1}^{m} SU_k(t_i, \tau_k, \theta_k). \tag{9}$$

Here the signal output PU_k due to the kth primary user's packet can be written as

$$PU_k(t_i, \tau_k, \theta_k) = \frac{1}{T_b}\sum_{j=0}^{N_h-1} h_j[d_{n_k+j}^{(k)} f_i(\overline{\tau}_k) + d_{n_k+1+j}^{(k)} f_i(T_b-\overline{\tau}_k)]\exp(j\theta_k) \tag{10}$$

where $\overline{\tau}_k = \tau_k - t_i$ modulo $T_b, n_k = \lfloor t_i - \tau_k - N_h T_b/T_b \rfloor$ for which $N_h T_b$ is the decoding delay caused by the code-matched filter/header correlator, and the partial autocorrelation function $f_i(\tau)$ is defined by $f_i(\tau) = \int_0^\tau c_i(t)c_i(t-\tau+T_b)\,dt$. Similarly, the signal output SU_k due to the kth secondary user's packet is given by (10) with $f_i(\overline{\tau}_k)$ and $f_i(T_b-\overline{\tau}_k)$ replaced by $f_{iv_k}(\overline{\tau}_k)$ and $\hat{f}_{iv_k}(\overline{\tau}_k)$, respectively, in which the partial cross-correlation functions $f_{iv_k}(\tau)$ and $\hat{f}_{iv_k}(\tau)$ are defined by $f_{iv_k}(\tau) = \int_0^\tau c_i(t-\tau+T_b)c_{v_k}(t)\,dt$ and $\hat{f}_{iv_k}(\tau) = \int_\tau^{T_b} c_i(t-\tau)c_{v_k}(t)\,dt$.

If a desired packet, say the first one, is collision-free in the ith code channel, then other primary user transmissions can be treated as wideband noise so that the relative time delays $\{\overline{\tau}_k\}_{k=2}^{g}$ are required to be uniformly distributed over $[T_c, T_b - T_c]$. But the secondary user transmissions can be always considered to be wideband noise, so the relative time delays $\{\overline{\tau}_k\}_{k=g+1}^{m}$ are uniformly distributed over $[0, T_b]$. The two types of multiple-access interference are characterized by evaluating their mean-square values taken with respect to all parameters involved. In Appendix B, the total mean-square value of asynchronous multiple-access interference accumulated over the header duration is derived and given by

$$\sigma_{h_i}^2(t_i) = \sigma_{I_P}^2(t_i) + \sigma_{I_S}^2(t_i) \tag{11}$$

$$= N_h[(g-1)\overline{\sigma}_{I_P}^2(t_i) + (m-g)\overline{\sigma}_{I_S}^2(t_i)] \tag{12}$$

where $\sigma_{I_P}^2(t_i)$ and $\sigma_{I_S}^2(t_i)$ are the mean-square values of the primary and secondary user interferences with the normalized value $\overline{\sigma}_{I_P}^2(t_i)$and $\overline{\sigma}_{I_S}^2(t_i)$ per bit per user, respectively.

We proceed to evaluate the probability of the header of the desired collision-free packet being detected at the correct sampling time $t_i = \tilde{\tau}_1 + N_h T_b$ in which $\tilde{\tau}_1$ is the nearest sampling time to τ_1. With the chip-rate sampling considered here, the capture time Δ can be chosen to be $3T_c/2$ that meets the condition of $T_c \leq \overline{\tau}_k \leq T_b - T_c$ whenever $\min_n |\tau_1 - \tau_k + nT_b| > \Delta$ for $2 \leq k \leq g$. In this case, we can show that the normalized sync time offset $\epsilon_i \triangleq |\tau_1 - \tilde{\tau}_1|/T_c$ is uniformly distributed over $[0, \frac{1}{2}]$. Now, for possible theoretical results, we invoke the Gaussian assumption on the multiple-access interference accumulated over the header duration in which the quadrature components of $\overline{w}(t_i)$ become independent Gaussian random variables when conditioned on (τ_1, θ_1). The probability of correct header detection at t_i is defined by

$$P_{h_i}(t_i) = \Pr\{|\overline{w}(t_i)| \geq \kappa_h\} \tag{13}$$

where κ_h denotes the threshold level for header detection. Conditioned on ϵ_i, this can be approximated to [4]

$$P_{h_i}(\epsilon_i|\tilde{\tau}_1) \cong Q\left(\frac{N_h(1-\epsilon_i)}{\sigma_{h_i}(\tilde{\tau}_1 + N_h T_b)}, \frac{\kappa_h}{\sigma_{h_i}(\tilde{\tau}_1 + N_h T_b)}\right) \tag{14}$$

where $Q(\cdot,\cdot)$ is the Marcum Q-function and the envelope of the desired signal PU_1 is given by $|PU_1(\tilde{\tau}_1+N_hT_b,\tau_1,\theta_1)| \cong N_h(1-\epsilon_i)$. By taking the average with respect to $(\epsilon_i,\tilde{\tau}_1)$, we derive that

$$\overline{P}_{h_i}(g,m)=\frac{1}{I}\sum_{\alpha_i=0}^{I-1}\boldsymbol{E}_{\epsilon_i}[P_{h_i}(\epsilon_i|\alpha_iT_b\le\tilde{\tau}_1\le(\alpha_i+1)T_b)] \tag{15}$$

where $\boldsymbol{E}_{\epsilon_i}$ denotes the expectation with respect to ϵ_i and (g,m) explicitly implies the dependency of $\overline{P}_{h_i}$ on the number of primary and secondary users in the ith code channel through the formula (12).

VI. Capture and Throughput Performance

We analyze the performance of a centralized DS-SS network which adopts the random assignment policy at the radio site and employs the multiple-capture receiver for each code channel at the central node. Given the m simultaneous transmissions attempted in a given slot, there will be some number g of transmissions on the ith code channel and $m-g$ transmissions on different code channels. Let a random variable G denote the number of transmissions on the ith code channel in a slot. We then obtain

$$\Pr\{G=g|M=m\}=\binom{m}{g}\left(\frac{1}{V}\right)^g\left(1-\frac{1}{V}\right)^{m-g}\quad \text{for}\quad 0\le g\le m. \tag{16}$$

We define $P_{C_i|G,M}(c|g,m)$ as the probability that some number c of packets using the ith spreading code are captured at the ith filter/correlator, conditioned on the g transmissions on the ith code channel and the m transmissions in the slot. Combining $P_{C_i|G,M}(c|g,m)$ and (16), we have the multiple-capture probability at the ith filter/correlator

$$\begin{aligned}P_{C_i|M}(c|m)&=\Pr\{C_i=c|M=m\}\\&=\sum_{g=c}^{m}P_{C_i|G,M}(c|g,m)\\&\quad\cdot\binom{m}{g}\left(\frac{1}{V}\right)^g\left(1-\frac{1}{V}\right)^{m-g}\\&\quad\text{for}\quad 0\le c\le m.\end{aligned} \tag{17}$$

We proceed to the evaluation of $P_{C_i|G,M}(c|g,m)$ which accounts for the collision-free transmissions among the g transmissions on the ith code channel and the multiple-access interference caused by the $m-1$ interfering packets, given a collision-free packet at the ith filter/correlator. We obtain that for $0\le c\le g$

$$\begin{aligned}P_{C_i|G,M}(c|g,m)=&\sum_{f=c}^{g}\binom{f}{c}[\overline{P}_{h_i}(g,m)]^c\\&\cdot[1-\overline{P}_{h_i}(g,m)]^{f-c}P_{F|G}(f|g).\end{aligned} \tag{18}$$

This is because the multiple-access interferences for each collision-free packet are assumed independent due to the randomization of arrival times.

TABLE I
The Proper Spreading Code Sets for $V\le 3$ and $N=63,127$: $\mathcal{G}$ Denotes the Octal Representation of a Primitive Polynomial $g(x)$ That Generates the Spreading Code with Initial Sequence $\vec{\alpha}_0$

proper	$N=63$		$N=127$	
codes	$\mathcal{G}$	$\vec{\alpha}_0$	$\mathcal{G}$	$\vec{\alpha}_0$
$\mathcal{C}_1$	147	(1,0,0,0,1,1)	203	(1,1,0,1,1,0,1)
$\mathcal{C}_2$	103	(0,0,0,0,1,0)	247	(0,0,1,0,1,1,1)
$\mathcal{C}_3$	133	(1,1,0,0,0,1)	211	(0,0,1,0,0,0,0)

Since channel traffic can be modeled as a binomial random variable with parameters K and δ (transmission probability), the average number of packet captures $\overline{C}_i$ in the ith code channel is given by the expression

$$\overline{C}_i=\sum_{m=1}^{K}\sum_{c=1}^{m}cP_{C_i|M}(c|m)f_M(m)\quad\text{packets/slot} \tag{19}$$

where $f_M(m)=\binom{K}{m}\delta^m(1-\delta)^{K-m}$ for $m\le K$. Applying (17)–(19), we derive

$$\begin{aligned}\overline{C}_i=&\sum_{m=1}^{K}\sum_{c=1}^{m}\sum_{g=c}^{m}\binom{m}{g}\left(\frac{1}{V}\right)^g\left(1-\frac{1}{V}\right)^{m-g}\\&\cdot cP_{C_i|G,M}(c|g,m)f_M(m)\quad\text{packets/slot.}\end{aligned} \tag{20}$$

If the packet captures are assumed to occur independently in different code channels, the average number of packet captures $\overline{C}$ at the central node then becomes

$$\overline{C}=\sum_{i=1}^{V}\overline{C}_i\qquad\text{packets/slot.} \tag{21}$$

Next, we evaluate the expected number of packets successfully received on the ith spreading code, i.e., the system throughput of the ith spreading code channel based on a threshold model for the channel, in which a data packet after being captured is assumed to be successfully received if the probability of data bit error is maintained below a specified bit-error rate P_b^*, and destroyed otherwise.

For differential detection [8], the probability of data bit error for a collision-free packet in the ith code channel has the approximation

$$\begin{aligned}&\overline{P}_{b_i}(g,m)\\&\quad\cong\boldsymbol{E}_{\epsilon_i}\left[\frac{1}{2}\exp\left[\frac{-(1-\epsilon_i)^2}{2[(g-1)\overline{\sigma}^2_{I_P}(t_i)+(m-g)\overline{\sigma}^2_{I_S}(t_i)]}\right]\right]\end{aligned} \tag{22}$$

where the normalized mean-square values $[\overline{\sigma}^2_{I_P}(t_i),\overline{\sigma}^2_{I_S}(t_i)]$ at a sampling time t_i can be directly calculated by combining (27), (33), and (34), (36) with $N_h=1,(d^{(k)}_{n_k})^2=(d^{(k)}_{n_k+1})^2=1$, respectively.

The probability distribution of the number of packet successes S_i at the ith filter/processor can be determined subject to

TABLE II
THE SECOND-ORDER MOMENTS PER DATA BIT $\overline{\sigma}^2_{I_P}, \overline{\sigma}^2_{I_{S_i}}$ $(10^{-3}, i = 1, 2)$ OF THE PRIMARY AND SECONDARY USER INTERFERENCES FOR $V \leq 3$ AND $N = 63, 127$

no codes	$N = 63$			$N = 127$		
V	$\bar{\sigma}^2_{I_P}$	$\bar{\sigma}^2_{I_{S_1}}$	$\bar{\sigma}^2_{I_{S_2}}$	$\bar{\sigma}^2_{I_P}$	$\bar{\sigma}^2_{I_{S_1}}$	$\bar{\sigma}^2_{I_{S_2}}$
1	1.183	–	–	0.632	–	–
2	1.293	4.638	–	0.708	2.480	–
3	1.422	5.424	5.302	0.740	2.562	2.605

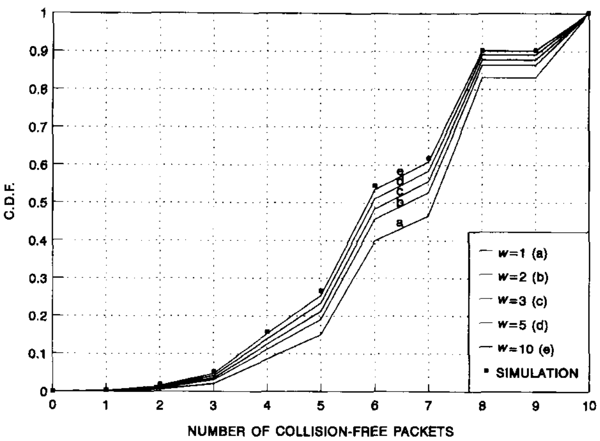

Fig. 2. Cumulative distribution function $\Pr[F \leq f | G = g]$ when $g = 10$, $N = 63$.

a constraint on the bit-error probability $\overline{P}_{b_i}(g, m)$ as follows: If $\overline{P}_{b_i}(g, m) \leq P_b^*$, then we have

$$P_{S_i|G,M}(s|g,m) = P_{C_i|G,M}(s|g,m) \qquad \text{for} \quad 0 \leq s \leq g, \tag{23}$$

and otherwise, 1 for $s = 0$ and 0 for $1 \leq s \leq g$. Thus, by replacing $P_{C_i|G,M}(c|g,m)$ in (20) with $P_{S_i|G,M}(s|g,m)$, the average number of packet successes $\overline{S}_i$ in the ith code channel can be expressed as

$$\overline{S}_i = \sum_{m=1}^{K} \sum_{s=1}^{m} \sum_{g=s}^{m} \binom{m}{g} \left(\frac{1}{V}\right)^g \left(1 - \frac{1}{V}\right)^{m-g} \cdot s P_{S_i|G,M}(s|g,m) f_M(m) \quad \text{packets/slot.} \tag{24}$$

Finally, the overall system throughput $\overline{S}$ becomes

$$\overline{S} = \sum_{i=1}^{V} \overline{S}_i \quad \text{packets/slot.} \tag{25}$$

VII. RESULTS

As for the distinct spreading codes, we choose a proper code set as follows. Given the code period N and number V of codes being used, we first compute the second-order moments per data bit $\overline{\sigma}^2_{I_P}$ and $\overline{\sigma}^2_{I_S}$ for a set of m-sequences (AO/LSE) listed in [9], and then choose V of the m-sequences which give the smallest $(\overline{\sigma}^2_{I_P}, \overline{\sigma}^2_{I_S})$. In Table I we list the proper spreading code sets for $V \leq 3$ and $N = 63, 127$, and their values of the second-order moments per data bit $\overline{\sigma}^2_{I_P}, \overline{\sigma}^2_{I_{S_i}}$ $(i = 1, 2)$ are presented in Table II, in which the index i denotes the ith secondary user with respect to a given Vth primary user.

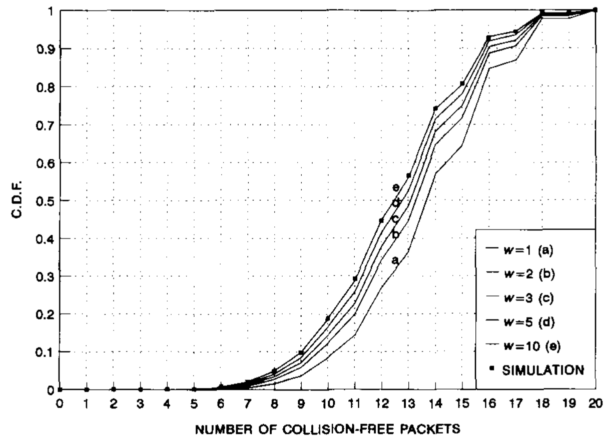

Fig. 3. Cumulative distribution function $\Pr[F \leq f | G = g]$ when $g = 20$, $N = 127$.

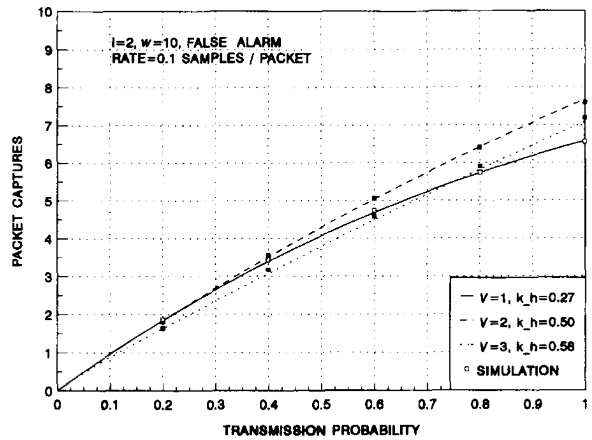

Fig. 4. Average number of packet captures when $K = 10$, $N = 63$, $N_h = 13$.

In order to determine the order w of discrete-time approximation $P^{(w)}_{F|G}(f|g)$ well in accordance with the collision-free probability $P_{F|G}(f|g)$, we plot the cumulative distribution function $\Pr[F \leq f | G = g]$ as a function of w in Figs. 2 and 3 when $G = 10, N = 63$ and $G = 20, N = 127$, respectively. We see that the discrete-time approximations of order $w = 10$ closely approach the simulation results on $P_{F|G}(f|g)$. Thus, instead of $P_{F|G}(f|g)$ whose theoretical evaluation appears to be intractable, $P^{(w)}_{F|G}(f|g)|_{w=10}$ can be used with good accuracy in evaluating the link performances.

Figs. 4–7 show the average number of packet captures for both the random assignment and common code schemes when $K = 10, 20, N = 63, 127$, and $N_h = 13, 42$. When the long frame sync word of length $N_h = 42$ [10] is employed as the header sequence along with the randomization time $I = 6$, it is quite obvious that the random assignment scheme with $V = 2, 3$ performs better than the common code scheme. We see that if two distinct codes ($V = 2$) are used for the packet transmissions at the transmitting radios, the performance gain

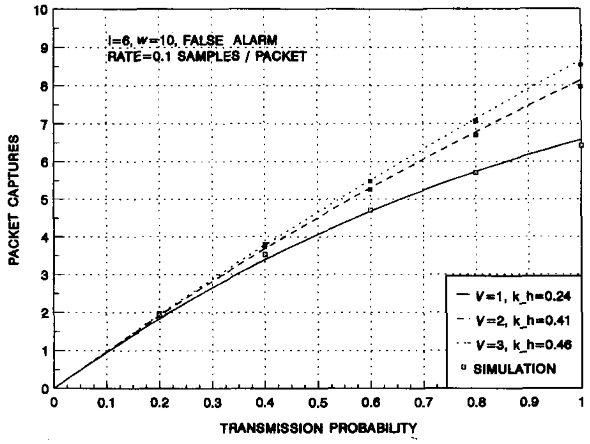

Fig. 5. Average number of packet captures when $K = 10$, $N = 63$, $N_h = 42$.

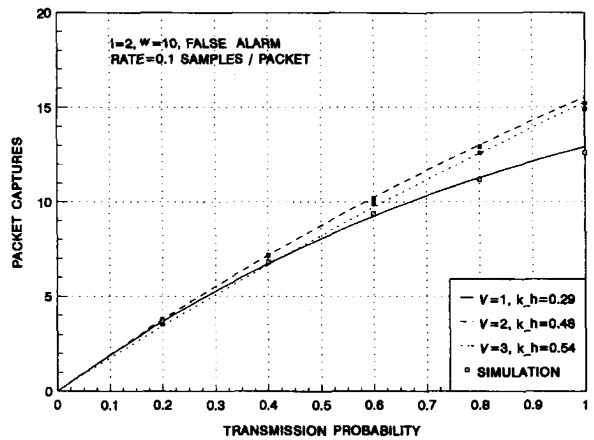

Fig. 6. Average number of packet captures when $K = 20$, $N = 127$, $N_h = 13$.

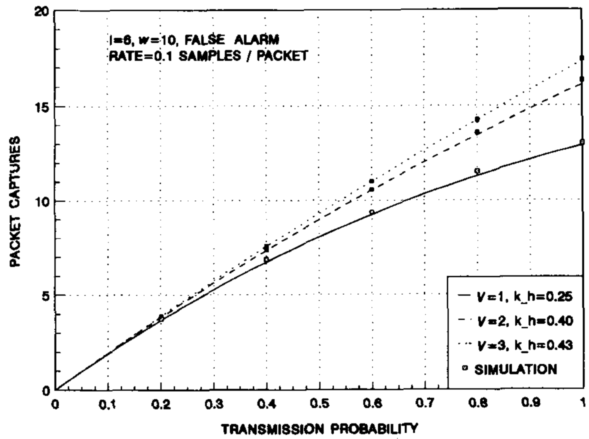

Fig. 7. Average number of packet captures when $K = 20$, $N = 127$, $N_h = 42$.

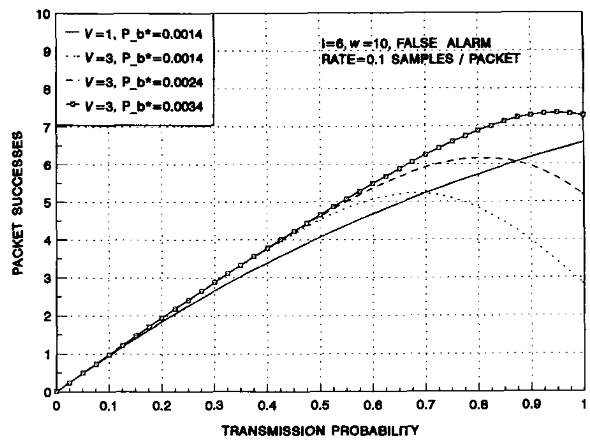

Fig. 8. Average number of packet successes when $K = 10$, $N = 63$, $N_h = 42$.

is almost achieved compared to the case of $V = 3$ in the random assignment scheme. On the other hand, when we employ the shorter Barker sequence of $N_h = 13$ along with $I = 2$, it is observed that the performance gain using $V = 3$ is smaller rather than that of $V = 2$, since the threshold level k_h here increases in proportion to the number V of distinct codes and is relatively higher compared to the case of $N_h = 42$.

In Figs. 8 and 9 we plot the average number of packet successes as a function of the transmission probability δ for both schemes when $K = 10, 20, N = 63, 127$, and $N_h = 42$. In this case, we adopted the threshold approximation that the data packet is successfully received if $P_b(k,m) \leq 1.4 \times 10^{-3}(1.3 \times 10^{-3})$, and discarded otherwise. As the number V of distinct codes increases, the effect of the multiple-access interference usually builds up when we choose the proper code set as mentioned above. Specially for the random assignment scheme with $V = 3$, we find that the average number of packet successes severely degrades in the region of heavy traffic because of the multiple-access interference. We note that if we allow the data bit-error rate up to $2.4 \times 10^{-3}(2.2 \times 10^{-3})$ or $3.4 \times 10^{-3}(3.2 \times 10^{-3})$ for $V = 3$, the average number of packet successes further increases, and this will become exactly equal to the average number of packet captures in Fig. 5 (Fig. 7) when $P_b(k,m) = 6.5 \times 10^{-3}(6.8 \times 10^{-3})$ is allowed, because the data bit-error rate $P_b(k,m)$ is always less than $6.5 \times 10^{-3}(6.8 \times 10^{-3})$ for all k, m.

VIII. Conclusions

In this paper, we have shown that the random assignment scheme using two or three distinct spreading codes provides greater performance gain with respect to the common code scheme. This was accomplished through the increased possibility of collision-free transmission in each code channel and construction of the proper spreading code set with good correlation properties. We also found that the additional performance gain resulting from employing more than two distinct codes was not as significant, and we may prefer to the random assignment scheme with just two distinct codes which causes modest increase in receiver complexity. In addition, simulation results were provided to validate the utility of the combinatorial analysis introduced for the discrete-time approximation to the collision-free probability, and the

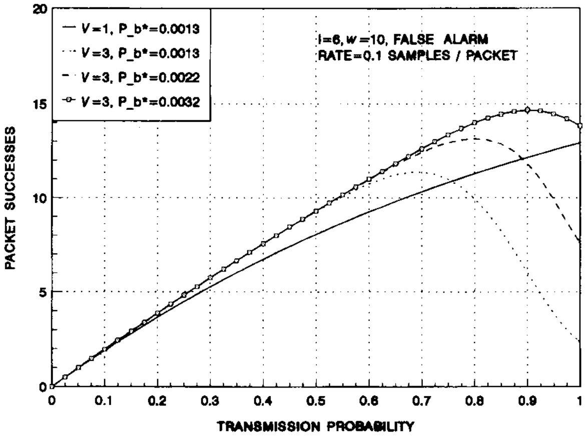

Fig. 9. Average number of packet successes when $K = 20$, $N = 127$, $N_h = 42$.

Gaussian approximation invoked in evaluating the header detection probability. Finally, we observed that the effect of the multiple-access interference was pronounced in the region of heavy traffic and resulted in severe degradation of system throughput. For this, we suggested that an efficient combining of a coding scheme with the random assignment scheme allows higher data bit-error rate in the packet decoder and hence assures the full performance gain achieved in the header detection process.

APPENDIX A
PROOF OF CLAIM 1

Let $\mathcal{U}$ denote the collection of all possible ways of placing v balls into n boxes in which some box may be empty and an index set $\mathcal{A}$ denote the n labeled boxes. $\mathcal{P}_\alpha$ is defined as the subset of $\mathcal{U}$ such that the αth box has more than w balls. Then $S(n, w, v)$ is given by N_o in (1), and using the redundant combination completes the proof.

APPENDIX B
MEAN-SQUARE VALUES $\sigma^2_{I_P}(t_i)$ AND $\sigma^2_{I_S}(t_i)$

Consider the mean-square value of the kth primary user's signal output

$$\boldsymbol{E}[\mathrm{Re}^2\{PU_k(t_i,\tau_k,\theta_k)\}] = \frac{1}{T_b^2}\boldsymbol{E}\left[\left(\sum_{j=0}^{N_h-1} h_j[d^{(k)}_{n_k+j} f_i(\overline{\tau}_k) + d^{(k)}_{n_k+1+j} f_i(T_b-\overline{\tau}_k)]\cos(\theta_k)\right)^2\right] \tag{26}$$

where θ_k is assumed a uniform random phase over $[0, 2\pi)$. In the above, the cross terms can be ignored because of the following reasons: a) good aperiodic autocorrelation property of header sequence, b) randomness of real data due to the random arrival time over $[0, IT_b]$, c) $\boldsymbol{E}[f_i(\overline{\tau}_k)f_i(T_b-\overline{\tau}_k)] \to 0$ for larger N. So the mean-square value can be approximated by

$$\boldsymbol{E}[\mathrm{Re}^2\{PU_k(t_i,\tau_k,\theta_k)\}] \cong \frac{1}{2T_b^2}\sum_{j=0}^{N_h-1}\boldsymbol{E}[(d^{(k)}_{n_k+j})^2 f_i^2(\overline{\tau}_k) + (d^{(k)}_{n_k+1+j})^2 f_i^2(T_b-\overline{\tau}_k)] \tag{27}$$

where n_k and $\overline{\tau}_k$ are random variables depending on τ_k given t_i. For $(\alpha_i + N_h)T_b \le t_i \le (\alpha_i + 1 + N_h)T_b$, some integer α_i, the mean-square value can be bounded by

$$\begin{aligned}\boldsymbol{E}[\mathrm{Re}^2\{PU_k(t_i,\tau_k,\theta_k)\}] &\ge \frac{1}{2IT_b^2}\sum_{r=1}^{\alpha_i} N_h[\boldsymbol{E}[f_i^2(\overline{\tau}_k)] + \boldsymbol{E}[f_i^2(T_b-\overline{\tau}_k)]] \\ &+ \frac{1}{2IT_b^2}\sum_{r=\alpha_i+1}^{I}[(N_h+\alpha_i-r)\boldsymbol{E}[f_i^2(\overline{\tau}_k)] \\ &+ (N_h+\alpha_i+1-r)\boldsymbol{E}[f_i^2(T_b-\overline{\tau}_k)]]\end{aligned} \tag{28}$$

and

$$\begin{aligned}\boldsymbol{E}[\mathrm{Re}^2\{PU_k(t_i,\tau_k,\theta_k)\}] &\le \frac{1}{2IT_b^2}\sum_{r=1}^{\alpha_i+1} N_h[\boldsymbol{E}[f_i^2(\overline{\tau}_k)] + \boldsymbol{E}[f_i^2(T_b-\overline{\tau}_k)]] \\ &+ \frac{1}{2IT_b^2}\sum_{r=\alpha_i+2}^{I}[(N_h+\alpha_i+1-r)\boldsymbol{E}[f_i^2(\overline{\tau}_k)] \\ &+ (N_h+\alpha_i+2-r)\boldsymbol{E}[f_i^2(T_b-\overline{\tau}_k)]].\end{aligned} \tag{29}$$

In the above, we have used that $(d^{(k)}_{n_k+j})^2 = (d^{(k)}_{n_k+1+j})^2 = 1$ $(0 \le j \le N_h - 1)$ for $\tau_k \le t_i - N_hT_b$ and otherwise, these values depend on the relative delay of t_i and $\tau_k \in [0, IT_b]$. We take the average of two extremes to obtain

$$\boldsymbol{E}[\mathrm{Re}^2\{PU_k(t_i,\tau_k,\theta_k)\}] \cong \frac{1}{T_b^2}\left[N_h - \frac{I}{4}\left[\left(1-\frac{\alpha_i}{I}\right)^2 + \left(1-\frac{\alpha_i+1}{I}\right)^2\right]\right]\boldsymbol{E}[f_i^2(\overline{\tau}_k)]. \tag{30}$$

Therefore, we derive that

$$\sigma^2_{I_P}(t_i) = \sum_{k=2}^{g}\boldsymbol{E}[\mathrm{Re}^2\{PU_k(t_i,\tau_k,\theta_k)\}] \tag{31}$$

$$= N_h \cdot (g-1) \cdot \overline{\sigma}^2_{I_P}(t_i) \tag{32}$$

where the normalized mean-square value per bit per user $\overline{\sigma}^2_{I_P}(t_i)$ is defined by

$$\overline{\sigma}^2_{I_P}(t_i) = \frac{1}{N_h}\boldsymbol{E}[\mathrm{Re}^2\{PU_k(t_i,\tau_k,\theta_k)\}]. \tag{33}$$

Similarly, the mean-square value of the kth secondary user's signal output becomes

$$\begin{aligned}\boldsymbol{E}\,[\mathrm{Re}^2\{SU_k(t_i,\tau_k,\theta_k)\}] &\cong \frac{1}{2T_b^2}\sum_{j=0}^{N_h-1}\boldsymbol{E}[(d^{(k)}_{n_k+j})^2 f^2_{iv_k}(\overline{\tau}_k)\\ &\quad + (d^{(k)}_{n_k+1+j})^2 \hat{f}^2_{iv_k}(\overline{\tau}_k)]. \qquad (34)\end{aligned}$$

Using the average of the two extremes in (28) and (29), we find that

$$\begin{aligned}\boldsymbol{E}[\mathrm{Re}^2\{SU_k(t_i,\tau_k,\theta_k)\}] &\cong \frac{1}{2T_b^2}\left[\left[N_h-\frac{I}{2}\left(1-\frac{\alpha_i}{I}\right)^2\right]\boldsymbol{E}[f^2_{iv_k}(\overline{\tau}_k)]\right.\\ &\quad\left.+\left[N_h-\frac{I}{2}\left(1-\frac{\alpha_i+1}{I}\right)^2\right]\boldsymbol{E}[\hat{f}^2_{iv_k}(\overline{\tau}_k)]\right]. \qquad (35)\end{aligned}$$

If we define the normalized mean-square value per bit per user $\overline{\sigma}^2_{I_S}(t_i)$ by

$$\overline{\sigma}^2_{I_S}(t_i) = \frac{1}{N_h}\boldsymbol{E}[\mathrm{Re}^2\{SU_k(t_i,\tau_k,\theta_k)\}] \qquad (36)$$

and assume this value approximately equal for all secondary users in the ith code channel, it follows that

$$\begin{aligned}\sigma^2_{I_S}(t_i) &= \sum_{k=g+1}^{m}\boldsymbol{E}[\mathrm{Re}^2\{SU_k(t_i,\tau_k,\theta_k)\}] \qquad (37)\\ &= N_h\cdot(m-g)\cdot\overline{\sigma}^2_{I_S}(t_i). \qquad (38)\end{aligned}$$

In both cases, for evaluation of the second-order moments such as $\boldsymbol{E}[f_i^2(\overline{\tau}_k)], \boldsymbol{E}[f^2_{iv_k}(\overline{\tau}_k)]$, and $\boldsymbol{E}[\hat{f}^2_{iv_k}(\overline{\tau}_k)]$, refer to [4].

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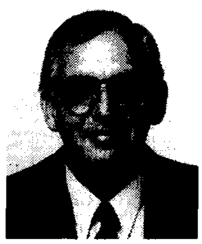

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Dr. Scholtz was elected Fellow of the IEEE "for contributions to the theory and design of synchronizable codes for communications and radar systems." In 1983, he received the Leonard G. Abraham Prize Paper Award for the historical article, "The Origins of Spread Spectrum Communications"; this same paper received the 1984 Donald G. Fink Prize Award given by the IEEE. His paper "Acquisition of Spread Spectrums Signals by an Adaptive Array" with D. M. Dlugos received the 1992 Senior Award of the IEEE Signal Processing Society. He has been an active member of the IEEE for many years, heading important organizational posts, including Finance Chairman for the 1977 National Telecommunications Conference, Program Chairman for the 1981 International Symposium on Information Theory, and Board of Governors positions for the Information Theory Group and Communications Society.