Counting Collision-Free Transmissions in Common-Code SSMA Communications

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Abstract— **This paper presents a technique for computing the probability that f of m transmissions will be collision-free at a given receiver in a spread-spectrum multiple-access (SSMA) radio network in which all transmitters employ identical wideband symbol waveforms for signalling. Separation of signals (collision-freedom) is based on the fine time-resolution characteristics of pulse compression receivers operating on wideband waveforms. A taxonomy of collision-free reception events is presented for a low-complexity sampling receiver operating in the absence of significant resolvable multipath, and combinatorial techniques are used to count collision-free reception events. A method for embedding this calculation in receiver performance analyses is given.**

Keywords— **spread spectrum, multiple access, collisions, spread ALOHA.**

I. Introduction

A receiver in a spread spectrum multiple-access radio network [1], [2], [3] generally has the capability to receive more than one simultaneous transmission at a time. If each transmission is assigned a separate pseudorandom code to control spread-spectrum modulation, then to accomplish multiple signal reception, a conventional receiver must construct separate signal generators, correlators, and acquisition and tracking circuits for each desired signal, and the system must maintain a pseudorandom code assignment scheme, etc. The advantage of such a scheme is that interfering signals look like noise (called *multiple-access noise*) in the process of reception of a desired signal, and the processing gain properties of spread-spectrum techniques can make this noise level tolerable [4]. Even more complex receivers are required to fully exploit all the knowledge available about the structure of the multiple-access noise [5].

An alternative to the scheme described above is one referred to as common-code spread-spectrum multiple access [6], [7] or spread-Aloha [8], and in the time-synchronous case as code time-division multiple access [9]. In these systems, every transmitter uses exactly the same modulation format, including the same pseudorandom code generator, and signals are distinguished only by differences in the time-of-arrival of their common wideband symbol waveform. The advantages of this scheme are relatively simple multiple-access receiver design and significant reduction of multiple-access noise by inverse filtering of the common-code waveform [9], [10] at the cost of occasional occurrences of catastrophic reception events called *collisions.*

Two identical wideband waveforms can be resolved, i.e., separated by receiver signal processing [11], [12], if their times of arrival at the receiver differ by more than an appropriately defined reciprocal bandwidth. A collision event between two or more signals occurs when the signals cannot be resolved because their symbol waveforms arrive in near synchronism. As the bandwidth of the symbol waveform is increased, the maximum arrival-time mismatch at which two symbol waveforms collide decreases and, in asynchronous operation, collisions become less likely.

A collision event affects the demodulation of the colliding signals in a common code system during the time interval over which approximate symbol synchronism is maintained between the colliding signals' symbol clocks at the receiver. This can be affected by clock stability, relative motions between transmitters and receiver, clock randomization protocols, etc. One system, in which the duration of a collision event is well-defined, is a slotted packet communication system in which each transmitter chooses a new random initial transmission time from within an interval of approximately one symbol time duration, in every transmission slot. In this case, the effect of such a collision on the involved signals persists for the duration of the time slot.

The purpose of this paper is to develop a method for describing and analyzing the effects of collisions on common-code spread-spectrum system performance. The tacit assumption is that the number of active transmitters typically does not fluctuate to a significant extent over the duration of a collision, and the calculations presented here are applicable over this time interval. The analysis of slotted packet radio communication systems using a common-code format benefits directly from the techniques described in this paper, but the results contained here may also be applicable to other common-code communication systems.

The analysis to follow is developed for wideband channels without multipath. It is generally applicable for the purpose of counting collisions in communication environments in which significant resolvable multipath is not present.

II. The Multiple-Access Signal Model

The signal transmitted by the k-th radio in this multiple-access communication system can be modelled as

$$s_k(t) = \mathrm{Re}\left\{x_k(t)\exp(j\omega_c t)\right\}, \tag{1}$$

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where Re{·} denotes the real-part operator, and the baseband modulation $x_k(t)$ on the k-th transmitted signal is given by

$$x_k(t) = \sum_n d_n^{(k)} c(t - nT_s)\,. \tag{2}$$

In this signal model, $\{d_n^{(k)}\}$ is the data sequence transmitted by the k-th radio, and $c(t)$ is a fixed waveform of duration T_s. The variable t and the parameter ω_c are the time and frequency, as determined from the k-th transmitter's clocks and oscillators.

When m such radios are transmitting, the signal observed at a given receiver is of the form

$$\begin{aligned} r(t) = & \sum_{k=1}^{m} \mathrm{Re}\{ A_k(t) \sum_n d_n^{(k)} c(t-\tau_k - nT_s) \\ & \exp(j\omega_c t) \} + n(t) \\ = & \mathrm{Re}\{ \tilde{r}(t) \exp(j\omega_c t) \}\,, \end{aligned} \tag{3}$$

where

$$\tilde{r}(t) = \sum_{k=1}^{m} A_k(t) \sum_n d_n^{(k)} c(t - \tau_k - nT_s) + \tilde{n}(t) \tag{4}$$

and the time t in this equation is determined by the receiver's clocks and oscillators. The amplitude and phase-shift effects of the channel from the k-th transmitter to the receiver on the k-th signal's carrier are indicated by the complex function factor $A_k(t)$, and the transmitter-receiver clock differences and propagation delay effects on the k-th transmitted modulation are lumped into the delay parameter τ_k. Extraneous signals and receiver noise are represented by $n(t)$, or by the equivalent baseband signal $\tilde{n}(t)$.

We assume that data transmissions to any given receiver from participating radios are asynchronous at the symbol-time level. Hence the random variables $\tau_k \bmod T_s$, $k = 1, \ldots, m$, are assumed independent, and each is uniformly distributed over the interval $[0, T_s]$.

III. SS Multiple-Capture Receiver

The combination of linear data modulation and use of a common symbol waveform $c(t)$ makes the receiver's front-end processing simple in principle (see Figure 1). A receiver can recover the modulation function $\tilde{r}(t)$ corresponding to $r(t)$ and filter it to create a complex-valued (quadrature) baseband signal of the form

$$z(t) = \sum_{k=1}^{m} A_k(t) \sum_n d_n^{(k)} c_{\mathrm{rec}}(t - \tau_k - nT_s) + \tilde{n}_{\mathrm{rec}}(t)\,, \tag{5}$$

where

$$c_{\mathrm{rec}}(t) = \int_{-\infty}^{\infty} c(\alpha) h(t-\alpha)\, d\alpha \tag{6}$$

and

$$\tilde{n}_{\mathrm{rec}}(t) = \int_{-\infty}^{\infty} \tilde{n}(\alpha) h(t-\alpha)\, d\alpha\,. \tag{7}$$

We assume that $A_k(t)$ is slowly varying enough that the approximation (5) is valid.

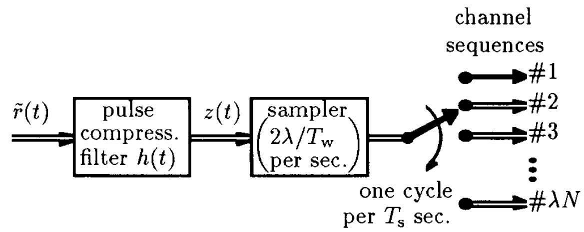


Fig. 1. Block diagram of receiver processing, illustrating the formation of channel sequences.

The filter with impulse response $h(t)$ is a *pulse compression* filter for the waveform $c(t)$. For example, if the receiver is thermal-noise limited, then $h(t)$ optimally may represent a matched filter, and $c_{\mathrm{rec}}(t)$ then is the autocorrelation function of the $c(t)$ waveform (which must be wideband to achieve the desired pulse compression effect [11]). On the other hand, if the receiver is multiple-access noise limited, then the filter $h(t)$ may represent an approximation to an inverse filter whose output $c_{\mathrm{rec}}(t)$ is nearly impulse-like, with very small residual signal outside the peak region [9], [10], [14]. In any case, we assume that the receiving filter $h(t)$ compresses an input symbol waveform $c(t)$ to an output $c_{\mathrm{rec}}(t)$ which consists of a narrow peak (*the mainlobe*) of duration approximately T_w seconds ($T_w \ll T_s$), and a relatively low residual signal (*the sidelobes*) outside the peak interval. We will refer to $c_{\mathrm{rec}}(t)$ as the *waveform response function.*

Let's define T_{peak} to be the time location of the center of the mainlobe of the waveform response function $c_{\mathrm{rec}}(t)$, and hence the mainlobe is the interval $(T_{\mathrm{peak}} - \frac{1}{2}T_w\,,\ T_{\mathrm{peak}} + \frac{1}{2}T_w)$. The k-th transmitted signal will cause peaks of the waveform response function, multiplied by data modulation, to appear at times $\tau_k + T_{\mathrm{peak}} + nT_s$ for integer n, in the filter output $z(t)$. Hence, t is a time for which $z(t)$ is in a mainlobe of the k-th filtered signal if and only if $t \bmod T_s \in \mathcal{M}_k$, where

$$\mathcal{M}_k = \{t : t \in [0, T_s] \text{ and } |t - \tau_k - T_{\mathrm{peak}} - nT_s| \tag{8}$$

$$< \frac{1}{2} T_w \text{ for some integer } n\}\,. \tag{9}$$

We will refer to $\mathcal{M}_k$ as the *peak support set for the k-th signal* (see Figure 2). A fundamental assumption for this common-code multiple-access system is that, if there exists a time t^* with the property that t^* is near the peak time in $\mathcal{M}_k$ and $t^* \notin \mathcal{M}_j$ for all $j \neq k$, then under typical operating conditions the data $\{d_n^{(k)}\}$ modulating the k-th transmitted signal can be recovered from samples of the filter output signal $z(t)$ taken at the times $t^* + nT_s$, n integer. On the other hand, if $t^* \notin \mathcal{M}_k$, then these samples are of little utility in determining the data modulating the k-th transmitted signal.

The remaining signal processing, which must be done individually for each transmitted signal, can be done digitally. To digitize the filter output $z(t)$ in a reasonably information-lossless manner, $z(t)$ is sampled at a rate of λ samples per $T_w/2$ seconds, λ integer, thereby guaranteeing

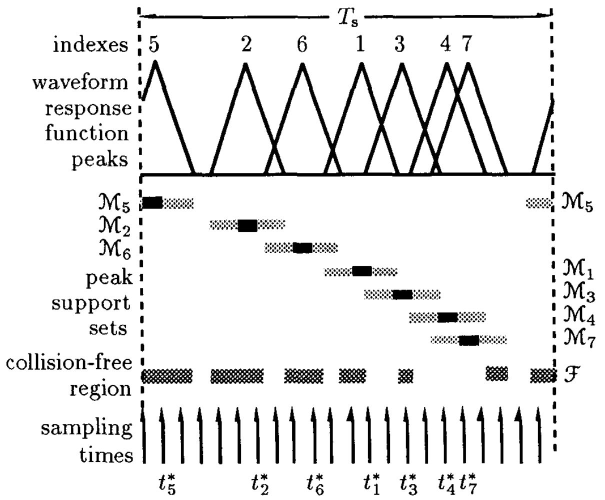


Fig. 2. Time diagram illustrating idealized waveform response function peaks and associated collision-free events. Classification of arrivals, as developed in Section 4B and Table 1, are as follows: The dark centers of the peak support sets indicate the regions that must be totally within the collision-free region $\mathcal{F}$ for the given transmission to be GECF. Transmission 5 is fully collision-free; transmissions 2, 5, and 6 are GECF; transmissions 2, 3, 5, and 6 are essentially collision-free. All transmissions except 4 are partially collision-free; transmission 4 is fully collided.

at least 2λ sample times (modulo T_s) in the peak support set of each of the m transmitted signals. Furthermore, with probability one each peak support set will have one sample time (modulo T_s) within $T_w/4\lambda$ of the peak value of its corresponding waveform response function.

As illustrated in Figure 1, successive samples are cyclically switched into λN distinct registers (or *channels*) whose contents will be referred to as *channel sequences*. The quantity N indicates the number of half-widths of the waveform response function's peak that occur in one symbol time, and so

$$N = \frac{2T_s}{T_w}. \tag{10}$$

In a direct sequence spread-spectrum modulation design with matched filter detection, $T_w/2$ would correspond to the chip time of the system, and N would be the number of chips per symbol.

Samples taken one symbol-time apart can be viewed as being placed in the same channel, and hence we assume that ideally N and λ are integer. Equivalently, samples taken at a fixed time (modulo T_s) in the peak support set of one of the transmissions will appear as successive entries in one of the channels (which will be said to be *occupied by the transmission*). On the other hand, samples of the waveform response sidelobes usually will fall in other channels and act as *multiple-access noise* in those channels occupied by other transmissions, but may also fall in the same channel and act as *intersymbol interference.*

In the receiver envisioned here, each channel is separately searched to detect and demodulate signals. The price paid for single filter processing of all received transmissions (prior to digitizing) is that the individual received signals are not processed in perfectly synchronous fashion, either in terms of carrier phase tracking or in symbol-time sampling the output of the filter processor. Hence compensation for frequency and phase mismatches will have to be accomplished digitally in each channel, and the number of channels must be dense enough to minimize symbol-time sampling errors. Drifts between a transmitter's and the receiver's symbol clocks will cause the peak samples to migrate from channel to channel, and good clock stability will be required to make this a relatively slow process compared to processing times of interest, e.g., code-word durations.

IV. Multiple Capture Performance

There are two primary mechanisms that may cause a communication failure in this scheme:

- *A collision event.* When more than one signal occupies a given channel, then each provides significant interference to the other(s), and it is unlikely that any signal can be demodulated correctly from the data in the given channel. The signals occupying the given channel are said to have *collided* in that channel.
- *Multiple-access noise.* The accumulation of sidelobe correlation samples from other transmissions, along with receiver noise, may be large enough to cause a failure in the reception of a signal, even in the absence of collisions. In a packet communication system, this failure may be either a header detection failure (the packet goes undetected), or too many data symbol errors in the packet (i.e., a decoder failure), or a header false alarm (saying that a header sequence is present in a sequence position where it actually is not). A header false alarm may mask a later packet arrival in the same file (causing frame mis-synchronization of the real packet) or may simply contribute to the overloading of any commonly shared equipment, e.g., a packet decoder.

We will see how multiple-access noise and collisions affect various performance measures.

A. Reception of a Given Transmission in a Given Channel

Let's examine the contents of one of the channels in more detail. Consider a channel whose samples are taken at times $t^* + iT_s$, with i ranging over the integers and t^* a fixed time in the interval $[0, T_s)$. Using (5), the samples are of the form

$$z(t^* + iT_s) = \sum_{k=1}^{m} A_k(t^* + iT_s) \sum_n d_n^{(k)} c_{\text{rec}}(t^* + (i-n)T_s - \tau_k) + \tilde{n}_{\text{rec}}(t^* + iT_s). \tag{11}$$

We will refer to the above samples as *the t^* channel sequence.*

Now suppose that t^* is in the peak support set $\mathcal{M}_j$ of the j-th transmitter's signal. Then from (9), there exists an integer n_j and a δ_j such that t^* can be represented in the form

$$t^* = T_{\text{peak}} + \tau_j + n_j T_s + \delta_j \quad \text{with} \quad |\delta_j| < \frac{1}{2} T_w . \qquad (12)$$

For the purposes of modelling the detection of the j-th transmitter's signal, let the *set* $\mathcal{C}_j(t^*)$ denote the set of indices k, $k \neq j$, for which $t^* \in \mathcal{M}_k$. Then the terms in the sample representation (11) for the t^* channel can be grouped as

$$\begin{aligned} z(t^* + iT_s) = \; & \underbrace{A_j(t^* + iT_s) d^{(j)}_{n_j+i} c_{\text{rec}}(T_{\text{peak}} + \delta_j)}_{\text{desired signal}} \\ & \underbrace{+ A_j(t^* + iT_s) \cdot \sum_{n \neq n_j + i} d^{(j)}_n c_{\text{rec}}(T_{\text{peak}} + \delta_j + [n_j + i - n]T_s)}_{\text{intersymbol interference}} \\ & \underbrace{+ \sum_{k \in \mathcal{C}_j(t^*)} A_k(t^* + iT_s) \cdot \sum_n d^{(k)}_n c_{\text{rec}}(T_{\text{peak}} + \delta_k + [n_k + i - n]T_s)}_{\text{collision signals}} \\ & \underbrace{+ \sum_{k \notin \mathcal{C}_j(t^*) \cup \{j\}} A_k(t^* + iT_s) \cdot \sum_n d^{(k)}_n c_{\text{rec}}(t^* + [i - n]T_s - \tau_k)}_{\text{multiple-access noise}} \\ & + \underbrace{\tilde{n}_{\text{rec}}(t^* + iT_s)}_{\text{other noise}} . \end{aligned} \qquad (13)$$

As the multiple-access system operation is envisioned here, successful reception of the j-th signal in the t^* channel is likely only if the set $\mathcal{C}_j(t^*)$ is empty. When $\mathcal{C}_j(t^*)$ is empty, we say that the j-th *signal is collision-free in channel* t^*.

The probability $P_{\text{cor}}(j, t^*)$ of correct reception of the j-th signal in the t^* channel can be bounded very simply by

$$\begin{aligned} P_{\text{cor}}(j, t^*) & \triangleq \Pr\{\mathcal{R}_j(t^*)\} \\ & \geq \Pr\{\mathcal{R}_j(t^*),\, \mathcal{C}_j(t^*) = \varphi,\, t^* \in \mathcal{M}_j\} \\ & = \Pr\{\mathcal{R}_j(t^*) \,|\, \mathcal{C}_j(t^*) = \varphi,\, t^* \in \mathcal{M}_j\} \\ & \quad \cdot \Pr\{\mathcal{C}_j(t^*) = \varphi\} \Pr\{t^* \in \mathcal{M}_j\} , \end{aligned} \qquad (14)$$

where $\mathcal{R}_j(t^*)$ is the desired correct reception event and φ is the empty set. Using the fact that the delay variables τ_k mod T_s, $k = 1, \ldots, m$, are independent and uniformly distributed on $[0, T_s)$, and that the peak support sets all have width T_w, it follows immediately that

$$\begin{aligned} P_{\text{cor}}(j, t^*) \geq \; & \Pr\{\mathcal{R}_j(t^*) \,|\, \mathcal{C}_j(t^*) = \varphi,\, t^* \in \mathcal{M}_j\} \\ & \cdot \left(1 - \frac{T_w}{T_s}\right)^{m-1} \frac{T_w}{T_s} . \end{aligned} \qquad (15)$$

The first factor on the right side of (15), namely, the probability of correct reception of the j-th transmitter's signal in a specified channel t^*, given that the j-th transmitter's signal is the sole occupant of that channel, often can be evaluated by modelling the multiple-access noise as a Gaussian random process (e.g., see [4]). Even in relatively ideal circumstances, the details of this calculation generally depend on the signal design $c(t)$, the filter design $h(t)$, the information structure and coding, etc., and for accuracy, this conditional probability must be averaged under the assumption that δ_j is a uniformly distributed variable over $(-T_w/2, T_w/2)$.

B. Events Related to Collisions

The analysis of Section 4A is not sufficient to estimate the behavior of a network using this multiple-access radio scheme. It usually is necessary to have knowledge of the probability distribution of the number of collision-free signals among a given number of transmissions in a slot to adequately assess system performance.

Considering the continuum of times in an interval of duration T_s, let the *collision-free region* $\mathcal{F}$ be the set of those times (modulo T_s) at which the filter output $z(t)$ can be sampled, such that exactly one of the m transmitted signals is sampled in the mainlobe of the waveform response functions that make up its contribution to the filter output $z(t)$. Equivalently, in terms of the peak support sets (9),

$$\mathcal{F} = \bigcup_{k=1}^{m} \left[\mathcal{M}_k - \bigcup_{j: j \neq k} (\mathcal{M}_j \cap \mathcal{M}_k) \right] . \qquad (16)$$

Using (16) and (9), this collision-free region is determined by the set $\{\tau_k\}$ of random delay variables. See Figure 2 for an example of a collision-free region. Several collision-free events are defined mathematically in Table I, from the point of view of receiving the i-th transmitter's signal. The abbreviation CF stands for collision-free.

TABLE I. EVENTS ASSOCIATED WITH DEGREES OF COLLISION FREEDOM.

Quality of i-th signal	Event	
Fully CF	$\mathcal{M}_i \subset \mathcal{F}$	(17)
$\Downarrow$	$\Downarrow$	
Guaranteed Essentially CF	$\Delta \leq \min_{j: j \neq i} \min_k |\tau_i - \tau_j + kT_s|$	(18)
$\Downarrow$	$\Downarrow$	
Essentially CF	$t_i^* \in \mathcal{F}$	(19)
$\Downarrow$	$\Downarrow$	
Partially CF	$\mathcal{M}_i \cap \mathcal{F} \neq \varphi$	(20)

The best possible circumstance is that the i-th peak support set $\mathcal{M}_i$ is *fully* within the collision-free region $\mathcal{F}$, thereby guaranteeing that the i-th signal will be collision-free in every t^* channel with $t^* \in \mathcal{M}_i$. On the other hand, if only a portion of $\mathcal{M}_i$ is in the collision-free region, then we call the i-th signal *partially collision-free* (PCF), and there may or may not exist a sample time t^* in $\mathcal{M}_i \cap \mathcal{F}$. If $\mathcal{M}_i \cap \mathcal{F}$ is empty, then we say that the i-th transmission is fully collided, and its reliable reception is very unlikely.

However, all that really is necessary to receive the i-th transmission reliably is that it be collision-free in one channel, preferably one in which the samples of the peak of its waveform response functions are largest. Toward this objective, we define t_i^* to be the sample time that is closest to the center of signal i's peak support set $\mathcal{M}_i$, where we expect large sample values of the i-th signal. If the i-th signal is collision-free in the t_i^* channel, then we say that the i-th signal is *essentially collision-free.*

The events that transmission i is fully collision-free (17), partially collision-free (20), or fully collided, do not depend on knowledge of the sample times used to fill the channel sequences. Determination of the event that the i-th transmission is essentially collision-free (19), which is a more natural measure of collision freedom in a sampled receiver, requires this additional knowledge of the relationship of sample times to the signal arrival-time delay parameters $\{\tau_k\}$. Since sampling is done at a rate of one every $T_w/2\lambda$ seconds, it is possible to guarantee that the i-th transmission is essentially collision-free by insuring that an interval of duration $T_w/2\lambda$ seconds, centered at $T_{\text{peak}} + \tau_i \bmod T_s$ is within the collision-free region $\mathcal{F}$ (see Figure 2). This occurs when

$$\begin{aligned} \min_{j:j\neq i} \min_k |\tau_i - \tau_j + kT_s| &\geq \frac{T_w}{4\lambda} + \frac{T_w}{2} \\ &= \frac{T_w}{2}\left(1 + \frac{1}{2\lambda}\right) \triangleq \Delta . \end{aligned} \tag{21}$$

When the event (21) occurs, we say that the i-th transmission is a *guaranteed essentially collision-free* (GECF) event and the critical value Δ given in (21) as the *minimum time offset*. The key feature of a guaranteed essentially collision-free event is that no knowledge of the sampling time t_i^* is required in its determination.

C. Counting GECF Transmissions

The number F of GECF transmissions from m active transmitters is a random variable that is a deterministic function of the set of random delay parameters τ_k, $k = 1, \dots, m$, the minimum time offset Δ, and the symbol time T_s. The probabilistic characterization of F is difficult to evaluate directly when the delay parameters are continuous variables, so we will discretize the symbol-time interval $[0, T_s]$ to a set $\mathcal{T} = \{t_0, t_1, \dots, t_{L-1}\}$ of L uniformly-spaced discrete times,

$$t_n = n\left(\frac{\Delta}{2l}\right), \tag{22}$$

with l a fixed integer. We then assume that $\tau_k \bmod T_s$, $k = 1, \dots, m$, are independent and each uniformly distributed on $\mathcal{T}$. Counting techniques can now be employed to develop the probabilistic model for F. Using (10) and (21), the number L of elements in the discrete time set $\mathcal{T}$ can be rewritten as

$$L = T_s\left(\frac{2l}{\Delta}\right) = \frac{4l\lambda N}{2\lambda + 1}, \tag{23}$$

and the parameter l can be chosen to make L integer. In this discrete model, as l tends to infinity, the discrete time model approaches the continuous arrival-time model in $[0, T_s)$.

The i-th signal is guaranteed essentially collision-free if and only if all other arrival delays are at least $2l$ discrete time units apart (mod L). That is, if the discretized delays are given (modulo T_s) by elements of the form (22),

$$\tau_k \bmod T_s = t_{a_k} = a_k\left(\frac{\Delta}{2l}\right), \qquad k = 1, \dots, m, \tag{24}$$

then, using (18) and (23), it is possible to simplify the test for the i-th transmission being GECF to

$$\begin{aligned} &\text{the } i\text{-th transmission is GECF} \\ \iff\ & \Delta \leq \min_{j:j\neq i} \min_k \left| a_i\left(\frac{\Delta}{2l}\right) - a_j\left(\frac{\Delta}{2l}\right) + kL\left(\frac{\Delta}{2l}\right)\right| \\ \iff\ & 2l \leq \min_{j:j\neq i} \min_k |\, a_i - a_j + kL| . \end{aligned} \tag{25}$$

We will refer to $a_1, \dots, a_m$, as discrete arrival times. Any two signals whose discrete arrival times are less than $2l$ discrete time units apart, can be identified as partially collision-free, fully collided, or even essentially collision-free signals.

Let $\mathcal{A} = \{1, \dots, m\}$ denote the set of m transmitter indexes whose discrete arrival-time variables $a_1, \dots, a_m$, can take on values in the set $\mathcal{L} = \{0, 1, \dots, L-1\}$, and let x denote a mapping of $\mathcal{A}$ into $\mathcal{L}$. Then we define the set of all such possible mappings as

$$\mathcal{U} = \{\, x \mid \mathcal{A} \to \mathcal{L}\} . \tag{26}$$

Certainly the number of such mappings in $\mathcal{U}$ is L^m. The set $\mathcal{U}$ is equivalent to the collection of all possible ways in which the m labeled arrivals may be arranged among the L discrete times with repetition.

For each i, $1 \leq i \leq m$, let

$$\mathcal{P}_i = \{x \in \mathcal{U} \mid x \text{ makes the } i\text{-th transmission GECF}\} . \tag{27}$$

Hence, $\cup_i \mathcal{P}_i$ contains all mappings that produce at least one GECF transmission, and the size of this set could be evaluated by the *Principle of Inclusion and Exclusion.* Here we wish to precisely evaluate the number $N(f)$ of mappings $x \in \mathcal{U}$ such that exactly f of sets $\mathcal{P}_i$ contain x. That is, $N(f)$ is the number of mappings that produce exactly f GECF transmissions. A generalization (e.g., see Theorem 62 of [13]) of the Principle of Inclusion and Exclusion can be applied directly to this counting problem to give

$$N(f) = \sum_{k\geq 0}(-1)^k \binom{f+k}{f} W(f+k), \tag{28}$$

where $W(q)$ is the sum of the number of mappings in each possible intersection of exactly q of the sets $\mathcal{P}_i$, $i = 1, \ldots, m$. More precisely,

$$W(f) \triangleq \sum_{\substack{\mathcal{J} \subset \mathcal{A} \\ |\mathcal{J}| = q}} \left| \bigcap_{i \in \mathcal{J}} \mathcal{P}_i \right| = \binom{m}{q} \cdot \left| \bigcap_{i=1}^{q} \mathcal{P}_i \right| . \tag{29}$$

The last equality above is a result of symmetry, i.e., the asynchronous properties of each received signal are identical in this system model.

In the evaluation of $|\bigcap_{i=1}^{q} \mathcal{P}_i|$ to follow, we will need the following result.

Lemma: Let $S(q', l', v)$ denote the number of ways inserting v balls into q' labeled boxes with no more than l' balls in each box. Then

$$S(q', l', v) = \begin{cases} \sum_{i \geq 0} (-1)^i \binom{q'}{i} \left\langle \begin{matrix} q' \\ v - i(l'+1) \end{matrix} \right\rangle & \text{if } 1 \leq v \leq q'l' \\ 1 & \text{if } v = 0. \end{cases} \tag{30}$$

where $\binom{n}{r} \triangleq \frac{n!}{(n-r)!r!}$ and $\left\langle \begin{matrix} n \\ r \end{matrix} \right\rangle \triangleq \binom{n+r-1}{r}$.

This lemma can also be proven using Theorem 62 of [13]. In this case the i-th subset of the collection of all possible ways to load the q' boxes is the set of loadings which place more than l' balls in the i-th box. The objective is to evaluate the number of loadings which are not in any of the subsets. Details are left to the reader.

We are now in a position to prove the following proposition.

Proposition: Let $\mathcal{A} = \{1, \ldots, m\}$ denote the set of m transmitter indexes whose discrete arrival-time variables $a_1, \ldots, a_m$, can take on values in the set $\mathcal{L} = \{0, 1, \ldots, L-1\}$ (with L given by (23)) , and let $2l$ be the minimum discrete-time offset required to make a transmission GECF (as stated in (25)). Then the number of mappings of $\mathcal{A}$ to $\mathcal{L}$, i.e., choices of the set $\{a_1, \ldots, a_m\}$ of discrete arrival times, such that at least transmissions 1 through q of the m transmissions are GECF, is given by

$$\left| \bigcap_{i=1}^{q} \mathcal{P}_i \right| = \begin{cases} L^m & \text{if } q = 0\,, \\ \sum_{r=1}^{r_{\max}} \sum_{v=0}^{v_{\max}} f_{r,v}(L,q,l)[L - 2l(q+r) + r - v]^{m-q} & \text{if } 0 < q < L/2l \text{ and } q < m\,, \\ L(m-1)! \left\langle \begin{matrix} m \\ L - 2lm \end{matrix} \right\rangle & \text{if } q = m \leq L/2l\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{31}$$

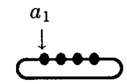
where

$$f_{r,v}(L,q,l) = (q-1)!\, L \binom{q}{r} S(q-r, 2l-1, v)$$

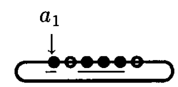
$$\cdot \left\langle \begin{matrix} r \\ L - 2l(q+r) - v \end{matrix} \right\rangle , \tag{32}$$

$$r_{\max} = \min\left\{ q, \left\lfloor \frac{L}{2l} \right\rfloor - q \right\} , \tag{33}$$

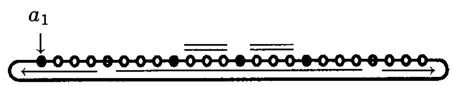
$$v_{\max} = \min\{(2l-1)(q-r),\ L - 2l(q+r)\} . \tag{34}$$



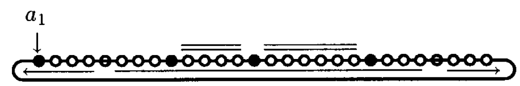
(a) Display of q (= 4) labelled GECF arrival indices (elements of $\mathcal{J} = \{a_1, \ldots, a_q\}$) on a circle with index 1 in a marked position.



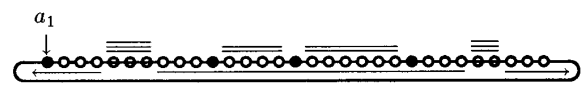
(b) Partition of order r (=2) (indicated by underlining) with r partition points (lined circles) inserted to separate sets of the partition. The partition points indicate where the remaining $m - q$ transmissions can be inserted, relative to the q labelled transmissions.



(c) Expanded partition with minimum number $2l - 1$ (=3) of additional time points inserted to make the q marked arrivals GECF. Note the end-around partition containing a_1. Interior arrival-free (IAF) subsets of the partition sets are double overlined.



(d) Further expanded $q - r$ (=2) interior arrival-free (IAF) subsets of the partition, by addition of t (=4) additional discrete times, not to exceed $2l - 1$ (=3) additional points in any given IAF subset.



(e) Insertion of the remaining $L - 2l(q+r) - t$ (=3 for L=31) possible arrival times by expansion of the r partition points. The r expanded partition point subsets are triple overlined.

Fig. 3. Step-by-step illustration of the construction of an arrival index set for which a prescribed set of q arrivals are GECF, regardless of the collision-free properties of the remaining $m - q$ transmissions.

Proof of Proposition: We will first prove the result for the case in which $0 < q < \lfloor L/2l \rfloor$. (Here $\lfloor x \rfloor$ denotes the largest integer not exceeding x). In this proof, we will count the number of ways that it is possible to construct a discrete arrival-time set $\{a_1, \ldots, a_m\}$ such that the first q transmissions are GECF. An example of the construction process is given in Figure 3. Notice that if we view the elements of $\mathcal{L}$ as being L points spaced one unit apart around a circle, then the quantity $\min_k |a_i - a_j + kL|$ of (25) measures the shortest distance (a Lee metric) between a_i and a_j in units around the circle.

The construction consists of the following steps.

Step (a): Place discrete arrival time a_1 on the circle (L possible ways) and to choose the order in which $a_2, \ldots, a_q$

will follow a_1, reading clockwise around the circle ($(q-1)!$ possible ways).

Step (b): Partition the q ordered discrete arrival times into r ($1 \le r \le q$) subsets which we call *partition sets*, and place single partition points between partition sets to indicate the ordered location of possible additional discrete arrival times a_j, $j > q$. For a given r and q, there are $\binom{q}{r}$ ways to construct such an r-th order partition. In the remaining steps, it is assumed that it is impossible to place an arrival time a_j, $j > q$, between elements of a partition set without destroying the desired GECF character of some transmissions with discrete arrival times in the partition set.

Step (c): Add the minimal number $(2l-1)$ of unused discrete time points between each adjacent $q+r$ pairs of points already on the circle. There is only one way to do this, and at this point in the construction, there are exactly $2l(q+r)$ points that have been placed on the circle. We define *interior arrival-free (IAF) subsets* to be the sets of unused discrete time points that are located between discrete arrival times in the same partition set. Notice that there are $q-r$ IAF subsets.

Step (d): Apportion v of the remaining $L-2l(q+r)$ discrete times among the IAF subsets such that each IAF subset contains at most $2(2l-1)$ points. This insures that no additional arrival inserted between them could be GECF. Since each of the $q-r$ IAF subsets constructed in (c) contains $2l-1$ unused discrete time points, at most an additional $2l-1$ discrete time points can be added to each IAF subset in this construction step. By the lemma (30), there are $S(q-r, 2l-1, v)$ ways of doing this for each value of v in the range $0 \le v \le \min\{L-2l(q+r), (2l-1)(q-r)\}$.

Step (e): Use the remaining $L-2l(q+r)-v$ discrete times to expand the r original partition point locations (from Step (b)) where the remaining $m-q$ arrivals are allowed. There are $\left\langle {r \atop L-2l(q+r)-v} \right\rangle$ ways of using these final discrete-time points to expand the partition point locations. Then the remaining $m-q$ discrete arrival times $a_{q+1}, \ldots, a_m$, can be placed in the total collection of $r+L-2l(q+r)-v$ partition points in $[r+L-2l(q+r)-v]^{m-q}$ ways.

To summarize, for a given number of partition sets r and a given number v of extra points appropriately inserted in the IAF sets, let $f_{r,v}(L,q,l)$ denote the number of ways of arranging $a_1, \ldots, a_q$ on the circle, such that the corresponding q signals are GECF, and providing an expanded set of $r+L-2l(q+r)-v$ partition points at which the remaining $m-q$ discrete arrival times can be placed. Annotating the contribution of each step in the construction, we have by the product rule,

$$f_{r,v}(L,q,l) = \underbrace{(q-1)!\,L}_{(a)}\underbrace{\binom{q}{r}}_{(b)}\underbrace{S(q-r,2l-1,v)}_{(d)} \cdot \underbrace{\left\langle {r \atop L-2l(q+r)-v} \right\rangle}_{(e)}, \quad (35)$$

which is (32). Multiplying (32) by $[r+L-2l(q+r)-v]^{m-q}$ to account for the number of ways that the remaining $m-q$ arrivals can be assigned to points in the expanded set of partition points, and summing over the allowed values of r and v, gives the appropriate result in (31).

When $q = m \le L/2l$, then the partitioning process (Step (b)) in the above construction is not appropriate because no partition points are required for the insertion of additional discrete arrival-times. In this case, after completing Steps (a) and (c) in the construction, the remaining $L-2lm$ discrete-time points may be inserted arbitrarily between the m discrete arrival times in $\left\langle {m \atop L-2lm} \right\rangle$ ways. A count of this construction yields the remaining result of (31). ∎

Equations (29) and (31) can be substituted into (28) to evaluate the number $N(f)$ of ways of arranging the m labeled arrivals among the L discrete times so that exactly f transmissions are GECF. Let the random variable M denote the total number of simultaneous transmissions in a given interval, and recall that the random variable F can be defined as the number of GECF transmissions in the same interval. Assuming that the L^m possible choices of sets of m discrete arrival times are all equally likely, it follows that the probability $P_{F|M}(f|m)$ which some number f of m simultaneously transmitted signals are GECF is given by

$$P_{F|M}(f|m) = \Pr\{F = f | M = m\} \quad (36)$$

$$= \frac{N(f)}{L^m} \quad \text{for } 0 \le f \le m. \quad (37)$$

D. *Multiple-Capture Probability*

The counting techniques of Section 4.3 provide an estimate of the probability that f transmissions are GECF, given that m transmitters of the multiple-access system are active. However, this does not mean that f transmissions will be successfully received. A *capture event*, or *successful reception event*, defined by an appropriate receiver action must be used as a measure of successful system performance. For example, a capture event for a given transmission may be defined in a slotted spread-Aloha packet communication system as the detection of the transmission's common packet header in the proper time position in a channel which the packet occupies. In this case, a false-detection probability constraint must be placed on the system, or else the capture event will become a certainty as the detection threshold is lowered, and unwanted erroneous packet detections increase.

With no processing constraints, a receiver in the above slotted packet communication system may assume that a packet is present in every channel, starting in every sample position. Hence, the burden of determining the presence or absence of the packet is left up to the packet decoder which is designed with the primary failure mode being an announced failure to decode. The secondary failure mode, namely erroneously decoding a packet, or announcing a packet decoding when none is present, must be very much less likely than the primary failure mode, to make the event extremely unlikely that incorrect information exits the receiver. In this situation, one might define a successful reception event for a given transmission in the above slotted packet communication system as the event that the packet decoder recovers the transmitted information without error in any one of the channels in any time position in which the packet signal is present. This broad notion of successful reception even encompasses the effects of transmission over multipath and fading channels.

The results of the previous section can be used to develop an estimate of the number of transmissions in a given time interval that are both GECF and captured. Let $\mathcal{G}$, $\mathcal{G} \in \mathcal{A}$, denote the set of transmitter indices of GECF transmissions, and let $\mathcal{K}(\mathcal{G})$ denote the set of transmission indices in $\mathcal{G}$ of captured (or successfully received) signals. Both $\mathcal{G}$ and $\mathcal{K}(\mathcal{G})$ represent random events that depend on arrival-time variables, etc. Let's define C to be the number of transmissions that are both GECF and captured, i.e.,

$$C = |\mathcal{K}(\mathcal{G})|. \quad (38)$$

Then the probabilistic description of the random variable C, given that the random number of M simultaneous transmissions is m, is given by

$$\begin{aligned} P_{C|M}(c|m) &= \sum_{\mathcal{G}\subset\mathcal{A}} \sum_{\mathcal{K}(\mathcal{G}):|\mathcal{K}(\mathcal{G})|=c} \Pr\{\mathcal{K}(\mathcal{G})\} \\ &= \sum_{f=c}^{m} \sum_{\mathcal{G}:\mathcal{G}\subset\mathcal{A},|\mathcal{G}|=f} \Pr\{\mathcal{G}\} \\ &\quad \cdot \sum_{\mathcal{K}(\mathcal{G}):|\mathcal{K}(\mathcal{G})|=c} \Pr\{\mathcal{K}(\mathcal{G})\,|\,\mathcal{G}\}. \quad (39) \end{aligned}$$

At this point, an assumption generally is made that whether or not a particular GECF transmission is captured, is independent of the capture properties of all other transmissions, and only depends on the number m of active transmitters (which determines the multiple-access noise level incurred in the reception of the desired signal). Furthermore, each GECF signal is assumed equally likely to be captured. These assumptions are justifiable if the multiple-access interference and noise random variables appearing in the processing of each channel file are independent and identically distributed from one channel file to the next. Let's define $P_{\text{cap}}(m)$ be the probability that any given GECF transmission is captured when m transmitters are active. Then the last sum in (39) is the probability of a binomial random variable $|\mathcal{K}(\mathcal{G})|$, given that $\mathcal{G}$ and hence its size is known, and (39) reduces to

$$\begin{aligned} P_{C|M}(c|m) &= \sum_{f=c}^{m} \sum_{\substack{\mathcal{G}\,:\,\mathcal{G}\subset\mathcal{A},\\ |\mathcal{G}|=f}} \Pr\{\mathcal{G}\}\Pr\Big\{|\mathcal{K}(\mathcal{G})| = c\,\Big|\,\mathcal{G}\Big\} \\ &= \sum_{f=c}^{m} \sum_{\substack{\mathcal{G}\,:\,\mathcal{G}\subset\mathcal{A},\\ |\mathcal{G}|=f}} \Pr\{\mathcal{G}\}\binom{f}{c} P_{\text{cap}}(m)^c(1-P_{\text{cap}}(m))^{f-c} \\ &= \sum_{f=c}^{m} P_{F|M}(f|m)\binom{f}{c} P_{\text{cap}}(m)^c[1-P_{\text{cap}}(m)]^{f-c} \quad (40) \end{aligned}$$

where $P_{F|M}(f|m)$ was derived in the previous section.

The quantity $P_{\text{cap}}(m)$ depends critically on the details of the signal structure, receiver filtering, receiver noise level, etc., in addition to the number m of active transmitters. In the notation of (15),

$$\begin{aligned} P_{\text{cap}}(m) &= \Pr\Big\{\mathcal{R}_j(t_j^*)\,|\,t_j^* \in \mathcal{F}\Big\} \\ &= \Pr\Big\{\mathcal{R}_j(t_j^*)\,|\,\mathcal{C}_j(t_j^*) = \varphi,\, t_j^* \in \mathcal{M}_j\Big\}, \quad (41) \end{aligned}$$

which is the same for all choices of j, where the conditioning is on the event that the desired j-th transmission is essentially collision-free. The conditioning event $t_j^* \in \mathcal{M}_j$ is used here to show the similarity to the quantity in (15), and is not necessary in the mathematical statement, because the sample time t_j^* closest to the center of signal j's peak support set $\mathcal{M}_j$ is guaranteed to be in the peak support set.

V. Sample Calculations

To develop the counting techniques of Section 4.3, we had to employ a discrete-time model for arrivals, in which $2l$ discrete times occurred per minimum time offset Δ. As l increases, the model will more accurately estimate the GECF signal count probabilities of the multiple-access system. Figures 4 and 5 show the effect of increasing l for two different choices of processing gain N and active transmitters m, for $\lambda = 1$ channel per chip time, i.e., a total of N channels. Notice that for smaller values of l, GECF count probabilities are more optimistic than reality (l large), and by the time that l reaches the value 5 in these examples, the distribution of the GECF signal count F is relatively close to the results of a more accurate computer simulation. The simulation also separately counted partially collision-free (PCF) signals, which gives a more optimistic assessment of the number of collision-free transmissions.

In the real system, as the parameter λ increases, then the minimum time offset Δ decreases toward $T_w/2$, making GECF signal transmissions more easily achievable. This effect is displayed in Figures 6 and 7 for two choices of processing gain N and active transmitters m, with the arrival-time model's sampling parameter l set at 5. When λ gets large, the GECF transmission probabilities should correspond to a receiver that has the capability to sample directly at the center of the peak support set of each arriving signal, perhaps with a signal capture scheme that is somewhat different from the λN channel processing scheme suggested in this paper.

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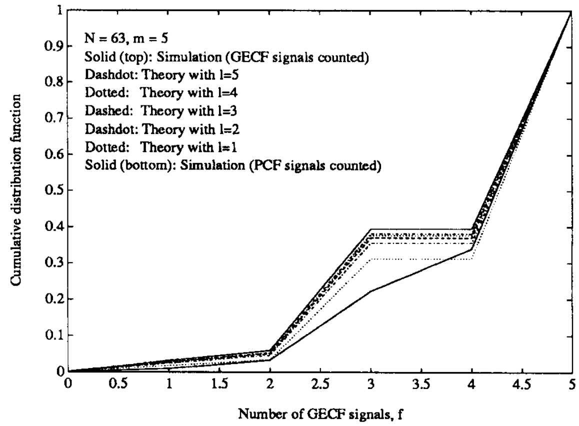

Fig. 4. Cumulative distribution function $P_{F|M}(f|m)$ for $m = 5$, $N = 63$ and $\lambda = 1$, for different model discretization parameter values l. Equivalent simulation results for GECF signals and PCF signals for comparison.

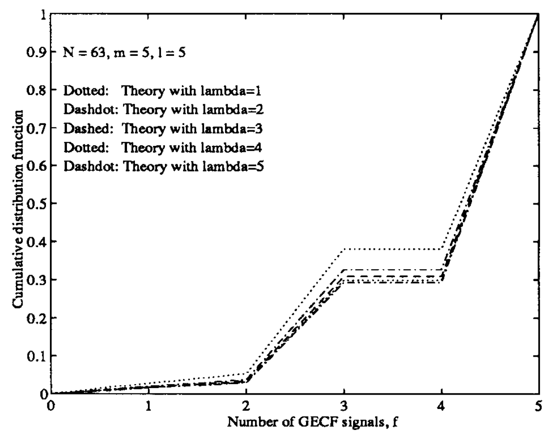

Fig. 6. Cumulative distribution function $P_{F|M}(f|m)$ for $m = 5$, $N = 63$ and $l = 5$, for sampling rate parameter values $\lambda = 1, 2, \cdots, 5$.

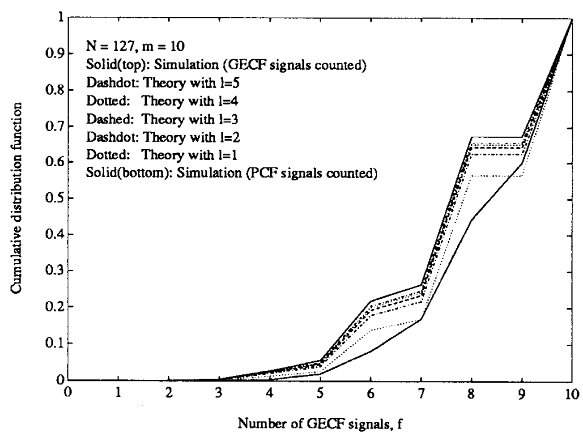

Fig. 5. Cumulative distribution function $P_{F|M}(f|m)$ for $m = 10$, $N = 127$ and $\lambda = 1$, for different model discretization parameter values l. Equivalent simulation results for GECF signals and PCF signals for comparison.

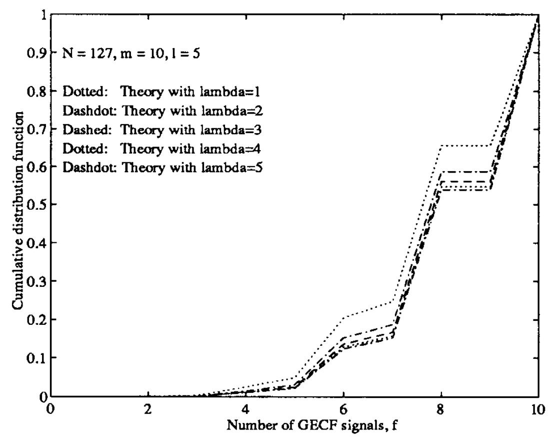

Fig. 7. Cumulative distribution function $P_{F|M}(f|m)$ for $m = 10$, $N = 127$ and $l = 5$, for sampling rate parameter values $\lambda = 1, 2, \cdots, 5$.

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