Random Assignment/Transmitter-Oriented Code Scheme for Centralized DS/SSMA Packet Radio Networks

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***Abstract*— We address an issue of channel sharing among users by using a random assignment/transmitter-oriented (RA/T) code scheme which permits the contention mode only in the transmission of a header while avoiding collision during the data packet transmission. Once the header is successfully received, the data packet is ready for reception by switching to one of programmable matched-filters. But the reception may be blocked due to a limited number of matched-filters so that this effect is taken into account in our analysis. We also consider an acknowledgment scheme to notify whether the header is correctly detected and the data packet can be processed continuously, which aims at reducing the interference caused by unwanted data transmission. For realistic analysis, we integrate detection performance at the physical level with channel activity at the link level through a Markov chain model. It is shown that compared to classical code-division multiple-access (CDMA) systems, a reduction in receiver complexity of half is allowed by choosing a proper number of RA/T codes without losing performance quality in view of the normalized throughput.**

I. Introduction

MOST direct-sequence/spread-spectrum multiple-access (DS/SSMA) packet radio networks are usually allowed to change their spreading code sequences while transmitting. If so, system throughput and complexity are largely affected by selection of spreading code sequences. In spread-spectrum networks, transmission protocol as a rule of determining such selection is specified by a number of factors, such as transmitter, intended receiver, transmission time, and priority of message, etc. Up to now, there have been several studies which address this issue [1], [2], but a few results have been reported to provide detailed analysis of performance [3], [4].

This paper proposes a random assignment/transmitter-oriented code scheme for centralized DS/SSMA packet radio networks. Here RA/T refers to selection of two spreading codes to be used for transmission of the header and data portion of a packet. When a terminal is ready to send a packet to a central node, it chooses randomly one out of L spreading codes [4] for transmission of the header in which L is considerably smaller than the number K of users. The data packet is then transmitted using a distinct spreading code to avoid collision among contending packets. In the RA/T code scheme, correct detection of the header mainly determines system throughput, which enables us to continuously process the data packet by switching to one of G programmable matched-filters. But if we consider G much less than K to reduce system complexity, some of data packets may be blocked even though their preceding headers are correctly detected. System throughput is then affected by a complicated function of detection performance at the physical level and channel activity at the link level.

In the RA/T code scheme, a number of terminals may transmit their headers using the same spreading code which will cause collision leading to failure reception of these headers. Here the multiuser interference among them is referred to as the primary user interference. If the time delay between the first two arrivals is sufficient to distinguish between them, the first one may be captured [5], [6] using a high time resolution property of direct-sequence spread-spectrum signals after taking correlation. On the other hand, the multiuser interference caused by simultaneous transmissions using distinct spreading codes is referred to as the secondary user interference, which does not involve collision because of the multiple-access capability of our system and increases only the bit error rate.

For evaluation of system throughput, we obtain an approximation to the probability of minipacket success by taking into account the bit-to-bit error dependence within a minipacket [7], [8], and model the network state as a Markov chain. At the link level, we account for a collision event caused by the primary user interference and a blocking event due to the limited number of matched-filters in deriving the state transition probability of the Markov chain. At the physical level, the effects of the secondary user interference and error-correction coding are considered when we evaluate the probability of minipacket success. Thus, system throughput normalized by a code rate and its bandwidth shows the performance of the RA/T code scheme which reflects the characteristics of physical level and also the complexity of central node.

For the RA/T code scheme, a central node cannot continuously receive the data packet unless its preceding header is correctly detected, since the central node identifies a source address used for data demodulation by decoding the header. Also, in the case of $G \ll K$, the number of successful transmissions in a slot can not exceed G so that the data packet may not be received whenever all G matched-filters

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are preoccupied for data reception. If a terminal is transmitting such unsuccessful data packet, then it will increase only the secondary user interference. In order to remove unwanted interference, we may adopt an acknowledgment scheme to inform a receiving status immediately after processing the header. In this case, the terminal waits an acknowledgment after sending the header of a packet and then send the following data packet if the acknowledgment is received. Otherwise, it will stop sending and attempt retransmission later, which allows to reduce the interference and increase the multiple-access capacity of the proposed RA/T code scheme.

The organization of the paper is as follows. In Section II, we provide an overview of channel sharing using the RA/T code scheme along with system model. An accurate, simple Gaussian method is applied to obtain the approximation for the probability of minipacket success in Section III. In Section IV, an overall throughput is analyzed by accounting for detection performance and channel activity, and the analysis is extended to the case of using an acknowledgment in Section V. Numerical results are presented in Section VI with concluding remarks.

II. System Model and RA/T Code Scheme

There are K potential user terminals communicating with a single central node in a centralized DS/SSMA packet radio network. A packet is divided into M minipackets, each of which contains l coded bits, and the first one serves as the header including a source address. We call it the header minipacket (HMP), and if l is large compared to the header length, then it may contain the data portion of a packet. The remaining $M-1$ minipackets convey a real data, which is called the data minipacket (DMP). The packet transmission is allowed every slot whose interval is equal to one minipacket length, so the network is operated in a slotted manner.

In a given slot, the central node searches for HMP's by monitoring the L code channels allocated to the headers, while there are at most G code channels being used for reception of DMP's. We note that the L code channels are fixed, but the G code channels are time varying according to the receiving status of DMP's every slot. Here the G code channels are specified by properly decoding the source addresses of HMP's that are correctly detected in the previous $M-1$ slots. This operation can be realized by using a programmable matched-filter whose impulse response may be changed by each transmitting code of DMP's, that is obtained from decoded source addresses. Hence, if the number of code channels needed for reception of HMP's and DMP's is less than the number of users, i.e., $L+G<K$, then the RA/T code scheme is more efficient than the transmitter-oriented code scheme [3] in view of system complexity.

For the RA/T code scheme, every user terminal shares L spreading codes $\{c_j^{ra}\}_{j=1}^{L}$ for transmission of HMP's, which are referred to as random assignment (RA) codes. However, a near-orthogonal transmitting code c_k^t is assigned to the kth terminal only for transmission of its DMP's, which allows to avoid collision with other HMP's and DMP's using different spreading codes. Hence we require K distinct transmitting codes for transmission of DMP's in which at most G of them can be dynamically selected depending on correct detection and decoding of their preceding HMP's. Such selected (maximum) G transmitting codes form a time-varying subset $\{c_{k_g}^t\}_{g=1}^{G}$ for some $k_g \in \{1,2,\ldots,K\}$, namely

$$c_{k_g}^t \in \{c_1^t, c_2^t, \ldots, c_K^t\}, \quad c_{k_g}^t \neq c_{k_{g'}}^t \quad \text{if } g \neq g'.$$

In this situation, we need some type of coordination between surrounding K terminals and the central node to properly determine the subset $\{c_{k_g}^t\}_{g=1}^{G}$ every slot. For this, the central node should have a list of the spreading codes $\{c_j^{ra}\}_{j=1}^{L}$ and $\{c_k^t\}_{k=1}^{K}$ for reception of HMP's and DMP's, respectively. The following procedure of transmitting a particular kth terminal's packet illustrates such coordination, i.e., the operations of transmission protocol and central node.

1) The central node monitors L RA code channels at the same time for reception of HMP's.
2) The kth terminal randomly chooses one from the code set $\{c_j^{ra}\}_{j=1}^{L}$ with equal probability $1/L$ for transmission of its HMP, and then sends its DMP's using the transmitting code c_k^t.
3) The central node detects the kth user's HMP, sets to c_k^t one of programmable matched-filters available by decoding the source address in HMP, and then demodulate the subsequent $M-1$ DMP's.

At the central node, the receiving scheme envisioned above can be implemented in Fig. 1, in which the received signal $r(t)$ passes through a bank of L RA code matched-filters (MF), and then try to lock onto those HMP's existing in solely occupied RA code channels. If two or more HMP's are placed into one RA code channel c_j^{ra} $(1 \leq j \leq L)$, it is unlikely to acquire sync information and properly decode their source addresses. By referring to a look-up table (ROM), each transmitting code $c_{k_g}^t$ $(1 \leq k_g \leq K)$ associated with correctly decoded source addresses determines the impulse response of unoccupied programmable matched-filters and then initiates demodulation of their subsequent DMP's. On the other hand, those previous DMP's under demodulation continue to occupy some of G programmable matched-filters until complete reception of them.

III. Probability of Minipacket Success

In general, some of the K potential users in a DS/SSMA network are likely to access the channel in a given time, in which the number J of active users remains constant throughout the slot time, since every user in a slotted network starts and ends his transmission at the boundary of a slot. Here all terminals initiate their packet transmissions synchronously at the minipacket level, but these packets are received asynchronously at the bit time level due to timing drifts and propagation delays. Hence the relative phases $\{\phi_j\}$ and delays $\{\tau_j\}$ of interfering users $(j=2,3,\cdots,J)$ from the first user (desired) can be modeled as identically and independently distributed over $[0,2\pi)$ and $[0,T_b)$ for a bit time T_b, respectively. Since the random phases $\{\phi_j\}$ and delays $\{\tau_j\}$ remain almost constant over the packet duration, the multiple-access interference (MAI) is correlated from bit to

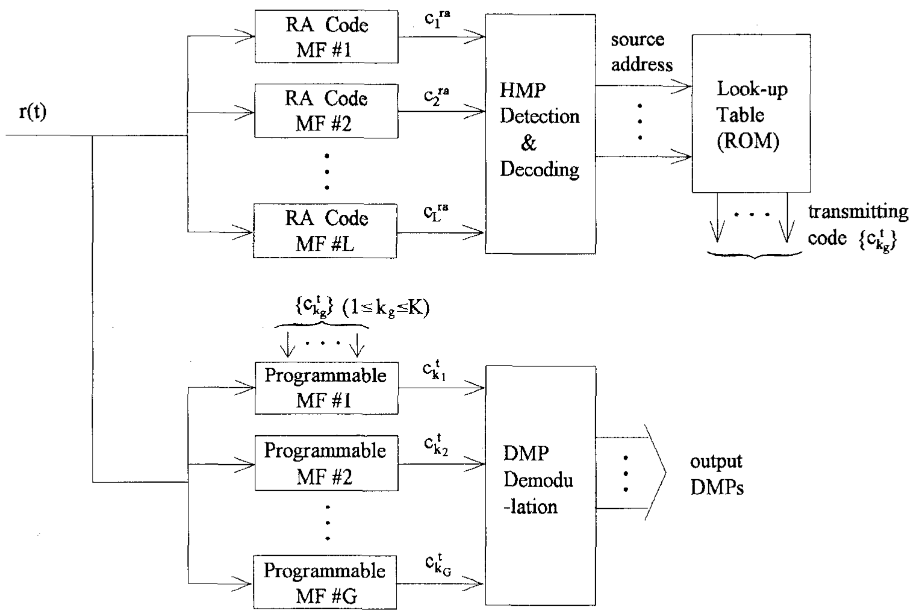

Fig. 1. Central receiver block diagram employing RA/T codes.

bit, which results in mutual dependence of data bit errors of a desired packet.

In this case, an exact evaluation of the probability of packet success involves computational difficulties, so there has been more emphasis on bounding or approximation techniques. Morrow and Lehnert derived an improved approximation with good accuracy for the probability of packet success by accounting for the bit-to-bit error dependence present in the MAI statistic [7]. The approach provides fairly accurate results compared to prior techniques but still requires computational complexity. Based on [7], Holtzman developed an accurate Gaussian approximation to the probability of data bit error which needs only calculations of the first two moments and so greatly reduces computational complexity [8]. Here we derive an approximation to the probability of minipacket success using the simple Gaussian method of [8].

For coherent binary phase shift keying (BPSK), decision statistic of the first user at the correlation receiver can be written as [8]

$$Z_1 = N + \sum_{j=2}^{J} \mathrm{MAI}_j \tag{1}$$

where Z_1 is normalized with respect to the chip time T_c for $N = T_b/T_c$ and all signal's received power $P = 2$. If the MAI is approximately Gaussian when conditioned on $\{\phi_j\}$ and $\{\tau_j\}$ $(j = 2, 3, \cdots, J)$, then the conditional probability of data bit error is approximated to

$$p_e = Q\left[\frac{N}{\sqrt{\Psi}}\right] \tag{2}$$

with Q given by $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \mathrm{e}^{-u^2/2} du$. In the above, the variance Ψ of the MAI is modeled as a function of random variables $\{\phi_j\}$ and $\{\tau_j\}$ $(j = 2, 3, \cdots, J)$ to account for the bit-to-bit error dependence.

For a DS/SSMA system employing t error-correction coding, the probability of minipacket success at the central node can be accurately approximated to [7]

$$P_s(J) = \mathbf{E}\left[g\left(Q\left[\frac{N}{\sqrt{\Psi}}\right]; l, t\right)\right] \tag{3}$$

$$= \int_0^\infty g\left(Q\left[\frac{N}{\sqrt{\psi}}\right]; l, t\right) f_\Psi(\psi)\, d\psi \tag{4}$$

where $f_\Psi(\psi)$ denotes the probability density function of Ψ and

$$g(p_e; l, t) = \sum_{i=0}^{t} \binom{l}{i} p_e^i (1 - p_e)^{l-i}. \tag{5}$$

Applying the simple, accurate approximation of [8] to (4) yields

$$\hat{P}_s(J) = \frac{2}{3} g\left(Q\left[\frac{N}{\sqrt{\mu}}\right]; l, t\right) + \frac{1}{6} g\left(Q\left[\frac{N}{\sqrt{\mu + \sqrt{3}\sigma}}\right]; l, t\right) + \frac{1}{6} g\left(Q\left[\frac{N}{\sqrt{\mu - \sqrt{3}\sigma}}\right]; l, t\right). \tag{6}$$

In the above, we have used the mean μ and variance σ^2 of Ψ which are given by [8]

$$\mu = \frac{(J-1)N}{3} \tag{7}$$

$$\sigma^2 = (J-1)\left[N^2 \frac{23}{360} + (N-1)\left(\frac{1}{20} + \frac{J-2}{36}\right)\right]. \tag{8}$$

IV. Throughput Analysis

A Markov chain model is introduced in evaluating throughput of a DS/SSMA packet radio network using the proposed RA/T code scheme. In case of a fixed packet length, the number of states is quite large and hence the analysis is intractable. For analysis, we simply assume a variable length of packet with geometric distribution, which leads to a Markov chain model with fewer states because of the memoryless property. Then the number M of minipackets within a packet is distributed as

$$\Pr[M = m] = q(1-q)^{m-1}, \quad m = 1, 2, \cdots \tag{9}$$

where the mean number of minipackets is given by $\bar{M} = \frac{1}{q}$. If a terminal is in idle state or transmitting the last DMP at the $(t-1)$th slot, the terminal is assumed to generate a new packet with probability p and then send at the beginning of the tth slot. First, an acknowledgment scheme is not considered in the analysis, but will be employed to further reduce the interference.

As central receiver operation is envisioned here, the path to successful reception of HMP's in a slot involves two events:

1) For a given RA code channel, only one terminal should be sending its HMP in the slot, otherwise the primary MAI will cause collision.
2) The HMP must contain t or less data bit errors, otherwise proper decoding will not be allowed because of the secondary MAI.

In the above, we have assumed zero capture model which yields conservative results, since it is possible for one to be captured even though two or more terminals are sending HMP's in the same RA code channel.

The operation of terminals in the system can be represented as the Markov chain with four states, namely, the state (H) of sending HMP, the state (D^s) of sending DMP's received, the state (D^f) of sending DMP's not received, the idle state (I). If the HMP sent by a terminal is not correctly detected, or there is none of the matched-filters available to receive the DMP's after proper decoding of the HMP, the terminal is in the state D^f by sending the DMP's not received.

Each of K terminals in the network is considered to be identical and independent, which enable us to model the network state as the Markov chain with three state variables $z_t = (h_t, d_t^s, d_t^f)$ during the tth slot. Here h_t, d_t^s, and d_t^f indicate the number of terminals belonging to the states H, D^s, and D^f, respectively. The state space of z_t is

$$\mathbf{Z} = \{(n_1, n_2, n_3): n_1 \geq 0,\ 0 \leq n_2 \leq G,\ n_3 \geq 0, \text{ and } n_1 + n_2 + n_3 \leq K\} \tag{10}$$

where the number of states is given by $|\mathbf{Z}| = \eta(K) - \eta(K - G - 1)$ for a function $\eta(x) = 1/6x^3 + x^2 + 1/6x + 1$.

Let us define $P(z_t \mid z_{t-1})$ by the probability of transition from state z_{t-1} to state z_t, that is,

$$P(z_t \mid z_{t-1}) = \Pr\{z_t = (h_t, d_t^s, d_t^f) \mid z_{t-1} = (h_{t-1}, d_{t-1}^s, d_{t-1}^f)\}. \tag{11}$$

First, as the conditional for derivation of $P(z_t \mid z_{t-1})$, consider the event that i of h_{t-1} terminals in the state H, j of d_{t-1}^s in the state D^s, and k of d_{t-1}^f in the state D^f enter into the state I at the beginning of the tth slot by ending their transmissions. For such event denoted by $C_{i,j,k}$, we find that

$$\Pr[C_{i,j,k}] = B(i; h_{t-1}, q)\, B(j; d_{t-1}^s, q)\, B(k; d_{t-1}^f, q) \tag{12}$$

where $B(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$ and we have used the memoryless property of geometric distribution. Conditioned on the event $C_{i,j,k}$, the transition probability $P(z_t \mid z_{t-1})$ can be related to

$$P(z_t \mid z_{t-1}) = \sum_{i=0}^{h_{t-1}} \sum_{j=0}^{d_{t-1}^s} \sum_{k=0}^{d_{t-1}^f} \Pr[C_{i,j,k}] \Pr[\Gamma \mid C_{i,j,k}] \tag{13}$$

where Γ denotes the event of making transition from z_{t-1} to z_t.

When we look into the network at the beginning of the tth slot, there are $K_H = h_{t-1} - i$ terminals in the state H and $K_I = K - (h_{t-1} + d_{t-1}^s + d_{t-1}^f) + (i + j + k)$ terminals in the state I. In order to make the event Γ occur, we require two conditions, i.e., i) h_t of K_I terminals have to send their HMP's, ii) given K_H terminals, $s = d_t^s - (d_{t-1}^s - j)$ of them should enter into the state D^s while $f = d_t^f - (d_{t-1}^f - k)$ enter into the state D^f. Since the K_H terminals must enter into either D^s or D^f, we have the relation $K_H = s + f$. Because of the independence of i) and ii), the probability of both events' occurrence at the same time is given by the product of each probabilities where the first one is simply evaluated as $B(h_t; K_I, p)$. To evaluate the probability of occurrence of ii), we define $P_1(s \mid h_{t-1}, K_H, E_t)$ by the probability that s of K_H terminals enter into the state D^s, given h_{t-1}, K_H, and E_t matched-filters available in the tth slot. Here E_t is equal to $G - (d_{t-1}^s - j)$ under the above conditions.

The probability $P_1(s \mid h_{t-1}, K_H, E_t)$ is derived in Appendix A

$$P_1(s \mid h_{t-1}, K_H, E_t) = \begin{cases} P_1(s \mid h_{t-1}, K_H) & \text{if } s < \min(K_H, E_t), \\ \sum_{n=\min(K_H, E_t)}^{K_H} P_1(n \mid h_{t-1}, K_H) & \text{if } s = \min(K_H, E_t) \end{cases} \tag{14}$$

where for $K_H \leq E_t$, we have

$$P_1(s \mid h_{t-1}, K_H) = \left(\frac{1}{L}\right)^{h_{t-1}} \sum_{w=s}^{K_H} \binom{K_H}{w} \binom{L}{w} w! \cdot B(s; w, P_s^H) \sum_{r=0}^{K_H - w} (-1)^r \binom{K_H - w}{r} \cdot \binom{L - w}{r} r! (L - w - r)^{h_{t-1} - w - r}. \tag{15}$$

In the above, P_s^H stands for the probability that the HMP is correctly detected in the presence of the secondary MAI caused by the $(h_{t-1}+d_{t-1}^{s}+d_{t-1}^{f}-1)$ interfering packets. Under the random sequence model, we can apply (6) to obtain the approximation

$$P_s^H \approx \hat{P}_s(h_{t-1}+d_{t-1}^{s}+d_{t-1}^{f}). \tag{16}$$

Therefore, applying the above results to (13) yields

$$P(z_t \mid z_{t-1}) = \sum_{(i,j,k)\in\Omega(z_{t-1},z_t)} \Pr[C_{i,j,k}]\, B(h_t; K_I, p) \cdot P_1(s \mid h_{t-1}, K_H, E_t) \tag{17}$$

where $\Omega(z_{t-1},z_t) = \{(i,j,k): 0 \le i \le h_{t-1},\ 0 \le j \le d_{t-1}^{s},\ 0 \le k \le d_{t-1}^{f},\ \text{and } K_H = s+f\}$.

Let us denote the transition matrix of the Markov chain by $\mathbf{P} = [P(z_t \mid z_{t-1}): z_{t-1}, z_t \in \mathbf{Z}]$, then the steady-state probability distribution $\{\pi(z): z \in \mathbf{Z}\}$ can be derived by solving the formula

$$\pi = \pi\mathbf{P}, \quad \sum_{z\in\mathbf{Z}} \pi(z) = 1 \tag{18}$$

with the row vector $\pi = [\pi(z): z \in \mathbf{Z}]$.

Now, by using the probability distribution $\{\pi(z)\}$, we can evaluate throughput for the DS/SSMA packet radio network that employs the RA/T code scheme. Here throughput is defined as the mean number of successful minipackets per slot. When the network is in the steady-state $z = (h, d^s, d^f)$, there is a contribution to throughput by some of the h HMP's whose entire packets are correctly decoded and some of the d^s DMP's which give rise to t or less data bit errors.

First, we define $\beta_h(z)$ by the mean number of such HMP's, that is, $\beta_h(z) = \mathbf{E}\{V\}$, where $\mathbf{E}$ denotes an expectation and V is the corresponding random variable. Then the random variables I and J are introduced to represent the numbers of idle terminals from the previous states H and D^s, respectively. Conditioned on the event $\{I = i, J = j\}$, $\beta_h(z)$ can be rewritten by

$$\begin{aligned}\beta_h(z) &= \mathbf{E}\{\mathbf{E}\{V = v \mid I = i, J = j\}\} \\ &= \sum_{i=0}^{h}\sum_{j=0}^{d^s} p_I(i)\, p_J(j) \sum_{v=0}^{h} v p_V(v \mid I = i, J = j).\end{aligned} \tag{19}$$

In the above, the probability density functions $p_I(i)$ and $p_J(j)$ are given by $B(i; h, q)$ and $B(j; d^s, q)$ because the packet length has geometric distribution. The conditional probability density function $p_V(v \mid I = i, J = j)$ is shown in Appendix B to be

$$p_V(v \mid I = i, J = j) = \sum_{(s,c)\in\Omega(v)} P_2(s, c \mid h, K_H, E) \tag{20}$$

where $K_H = h - i$, $E = G - (d^s - j)$, $\Omega(v) = \{(s,c): 0 \le s \le K_H,\ 0 \le c \le i, \text{ and } v = s + c\}$, and

$$P_2(s, c \mid h, K_H, E) = \begin{cases} P_2(s, c \mid h, K_H) & \text{if } s < \min(K_H, E), \\ \sum_{n=\min(K_H,E)}^{K_H} P_2(n, c \mid h, K_H) & \text{if } s = \min(K_H, E). \end{cases} \tag{21}$$

With $K_H \le E$, we obtain that

$$\begin{aligned}P_2(s, c \mid h, K_H) &= \left(\frac{1}{L}\right)^h \sum_{w=s}^{K_H}\sum_{g=c}^{i} \binom{K_H}{w}\binom{L}{w} w! \binom{i}{g}\binom{L-w}{g} g! \\ &\cdot B(s; w, P_s^H)\, B(c; g, P_s^H) \sum_{r=0}^{h-w-g} (-1)^r \binom{h-w-g}{r} \\ &\cdot \binom{L-w-g}{r} r!\,(L-w-g-r)^{h-w-g-r}\end{aligned} \tag{22}$$

where $P_s^H \approx \hat{P}_s(h+d^s+d^f)$. Substituting them into (19) gives

$$\beta_h(z) = \sum_{i=0}^{h}\sum_{j=0}^{d^s} B(i; h, q)\, B(j; d^s, q) \cdot \sum_{s=0}^{K_H}\sum_{c=0}^{i} (s+c) P_2(s, c \mid h, K_H, E). \tag{23}$$

Next, if we define $\beta_d(z)$ by the mean number of successful DMP's, then we have

$$\beta_d(z) = d^s P_s^D. \tag{24}$$

Here P_s^D, the probability of DMP success, is well approximated to $\hat{P}_s(h + d^s + d^f)$.

Conditioned on the steady-state z, the normalized throughput $\bar{\beta}(z)$ with respect to the code rate and bandwidth expansion has the expression

$$\bar{\beta}(z) = \frac{1}{N}[r_h\beta_h(z) + r_d\beta_d(z)]. \tag{25}$$

The r_h and r_d indicate the code rates associated with the HMP and DMP, respectively. In case of $r_h \ne r_d$, if we are using block codes for forward error correction, t in (6) needs to be replaced by the corresponding values from the Varsharmov-Gilbert lower bound [9]

$$r \ge 1 - H\left(\frac{d_{\min} - 2}{l}\right) \tag{26}$$

where $H(x) = -x\log_2 x - (1-x)\log_2(1-x)$ and $d_{\min} \ge 2t+1$. Finally, taking the average with respect to z yields

$$\bar{\beta} = \sum_{z\in\mathbf{Z}} \bar{\beta}(z)\,\pi(z). \tag{27}$$

V. RA/T WITH HMP ACKNOWLEDGMENT

We adopt an acknowledgment (ACK) scheme to reduce the secondary MAI caused by unwanted data transmission. To simplify the analysis, a terminal is assumed to attempt retransmission with the same probability p as for new transmission in idle state so that retransmission state is merged into idle state. So we take the composite traffic model with transmission probability p in which an ideal feedback channel is assumed for the ACK to ignore its effect on delay and throughput.

In the RA/T code scheme employing HMP acknowledgment, there is no need for the state D^f which indicates the terminals sending those DMP's not received. So the operation of terminals can be modeled as the Markov chain with three states such as H, $D = D^s$, and I. Similarly as in Section IV, the network state can be represented by $z_t = (h_t, d_t)$ at the beginning of the t-th slot. Here h_t and d_t denote the number of terminals in the states H and D, respectively. The state space of z_t becomes

$$\mathbf{Z} = \{(n_1, n_2): n_1 \geq 0,\ 0 \leq n_2 \leq G, \text{ and } n_1 + n_2 \leq K\} \tag{28}$$

with $|\mathbf{Z}| = 1/2(G+1)(2K+2-G)$.

The transition probability is defined by

$$P(z_t \mid z_{t-1}) = \Pr\{z_t = (h_t, d_t) \mid z_{t-1} = (h_{t-1}, d_{t-1})\}. \tag{29}$$

Then it follows that

$$P(z_t \mid z_{t-1}) = \sum_{i=0}^{h_{t-1}} \sum_{j=0}^{d_{t-1}} \Pr[C_{i,j}]\, B(h_t; K_I, p) \cdot P_1(s \mid h_{t-1}, K_H, E_t) \tag{30}$$

where $\Pr[C_{i,j}] = B(i; h_{t-1}, q)\, B(j; d_{t-1}, q)$, $K_H = h_{t-1} - i$, $K_I = K - (h_{t-1} + d_{t-1}) + (i + j)$, and $E_t = G - (d_{t-1} - j)$. To evaluate $P_1(s \mid h_{t-1}, K_H, E_t)$, (16) is replaced by $P_s^H \approx \hat{P}_s(h_{t-1} + d_{t-1})$.

Based on the above results, the probability distribution $\{\pi(z): z \in \mathbf{Z}\}$ can be derived through (18). Looking at the steady-state $z = (h, d)$, we find that $\beta_h(z)$ and $\beta_d(z)$ are given by (23) and (24) with $d^s = d$ and $d^f = 0$, respectively. Then the normalized throughput $\bar{\beta}$ can be evaluated by using (25) and (27).

VI. Results and Conclusion

Based on the RA/T code scheme, we find a proper number L of RA codes to be used for transmission of HMP's and at the same time, determine an optimum number G of matched-filters required for reception of DMP's through numerical analysis. Because of computational complexity, we consider relatively small values of K and N in which the normalized throughput $\bar{\beta}$ is evaluated for two cases, namely without or with HMP acknowledgment. Note that for numerical computation, the number of coded bits is chosen to be $l = 100$ b/minipacket and the mean number of minipacket/packet is given by $\bar{M} = 10$, which implies 1000 b/packet on the average.

Fig. 2 shows the probability $\hat{P}_s(J)$ of minipacket success as a function of the number J of simultaneous users for varying t when the number of chips/bit is $N = 11$. We see that with a single-error correction capability, i.e., $t = 1$, $\hat{P}_s(J)$ is very close to one in the range of $J = 3-5$. Thus, we choose $t = 1$ to evaluate $\bar{\beta}$ for small K and N considered here.

In Figs. 3 and 4, we plot $\bar{\beta}$ versus the transmission probability p for various L and G when $K = 12$, N =7, assuming the case without HMP acknowledgment. First, if we increase L up to 6 for fixed $G = 4$, then $\bar{\beta}$ becomes saturated in which most of the performance gain is achieved with only two RA codes, i.e., $L = 2$. So we may select $L = 2$ to reduce system complexity without causing any performance loss while for fixed $L = 2$, the number of matched-filters is changed from $G = 3$ to $G = 5$. Then we also observe that $\bar{\beta}$ becomes saturated near at $G = 4$, which leads to an optimum choice of

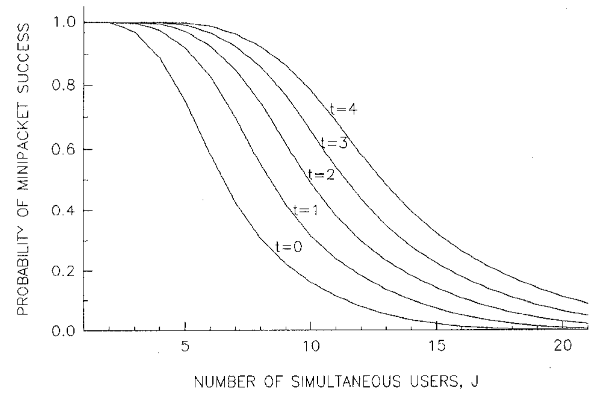

Fig. 2. Probability $\hat{P}_s(J)$ of minipacket success versus J for various t when $N = 11$.

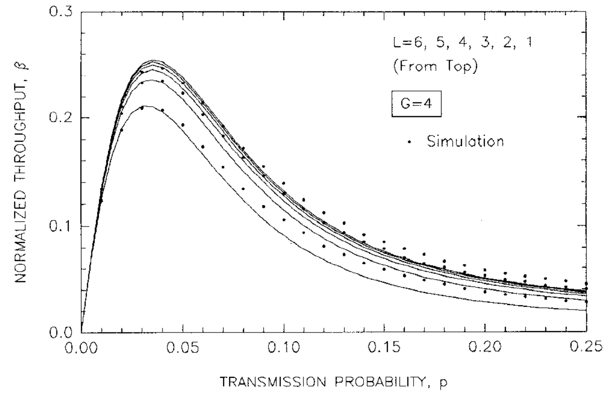

Fig. 3. Normalized throughput $\bar{\beta}$ versus p for various L when $K = 12, N = 7$, (without HMP acknowledgment).

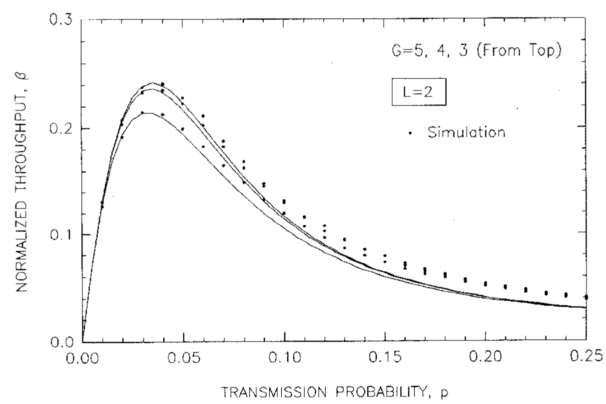

Fig. 4. Normalized throughput $\bar{\beta}$ versus p for various G when $K = 12, N = 7$, (without HMP acknowledgment).

$L = 2$ and $G = 4$. This implies that about half reduction in receiver complexity is allowed by employing the RA/T code scheme requiring $L + G = 6$ correlators when compared to the classical CDMA with $K = 12$ ones.

Simulation results were also provided in Figs. 3 and 4 to validate theoretical results obtained from the Markov chain model on the collision and blocking events. Note that the two results are well in accord near at those p yielding the peak

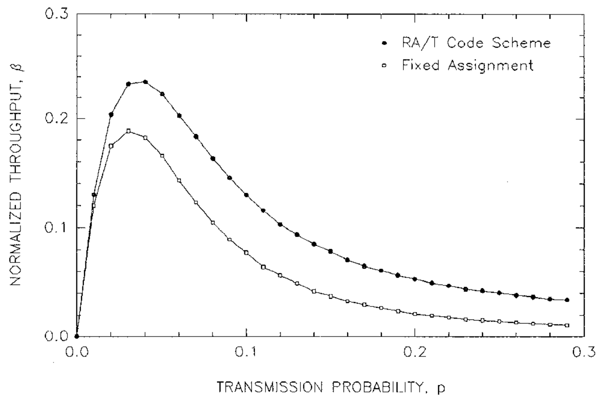

Fig. 5. Normalized throughput $\bar{\beta}$ versus p for $L = 2, G = 4$ when $K = 12, N = 7$.

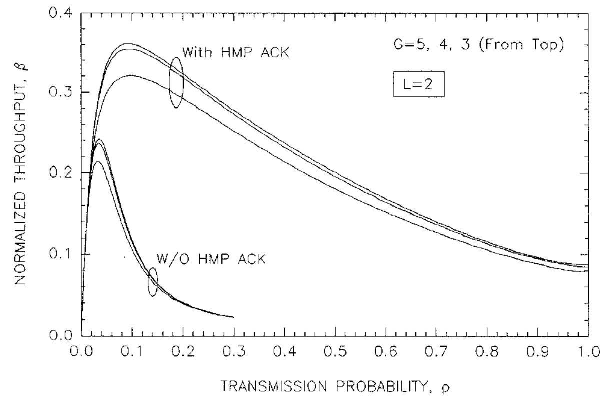

Fig. 7. Normalized throughput $\bar{\beta}$ versus p for various G when $K = 12, N = 7$.

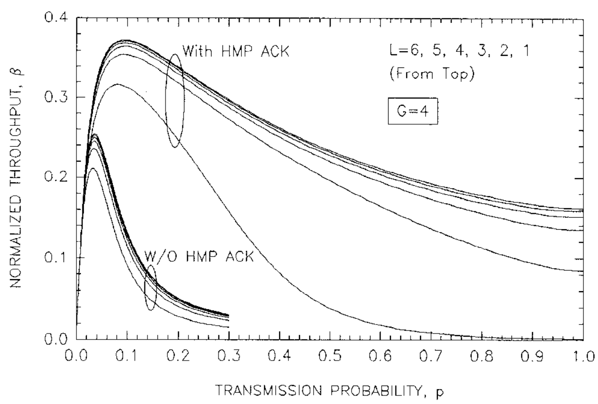

Fig. 6. Normalized throughput $\bar{\beta}$ versus p for various L when $K = 12, N = 7$.

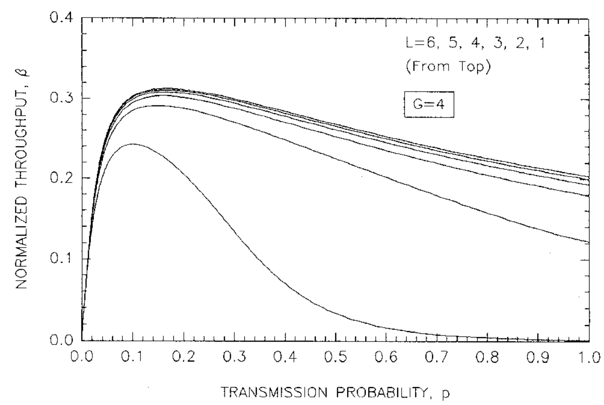

Fig. 8. Normalized throughput $\bar{\beta}$ versus p for various L when $K = 12, N = 11$, (with HMP acknowledgment).

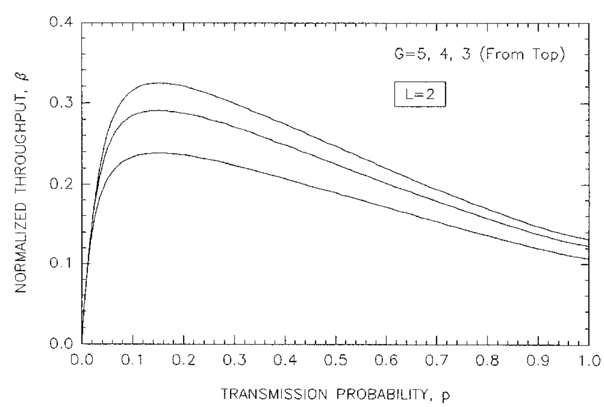

Fig. 9. Normalized throughput $\bar{\beta}$ versus p for various G when $= 12, N = 11$, (with HMP acknowledgment).

value of $\bar{\beta}$, and the saturation behavior is also preserved for both L and G.

To compare the RA/T code scheme with possible ones based on classical CDMA in terms of throughput performance, we may consider a fixed assignment of the subset of $L + G$ distinct codes to K terminals [11] without RA code and HMP acknowledgment. Here $\bar{\beta}$ is evaluated through simulation for the fixed assignment, assuming all subsequent DMP's not received when the HMP is involved in collision [12]. With the optimum choice of $L = 2, G = 4$ for $K = 12, N = 7$, we notice that the RA/T code scheme outperforms the fixed assignment employing $L + G = 6$ codes in Fig. 5.

Figs. 6 and 7 show $\bar{\beta}$ versus p for the same parameters when the HMP acknowledgment is adopted. It is obvious that throughput can be greatly enhanced by using the HMP acknowledgment which allows to reduce the unwanted secondary MAI. In addition, a similar behavior is observed in view of $\bar{\beta}$ when we change the parameters L and G. So the above optimum choice is valid regardless of the HMP acknowledgment. Figs. 8 and 9 also show $\bar{\beta}$ for the same parameters only except $N = 11$ with HMP acknowledgment. It is shown that the effect of secondary MAI is not so severe, and hence the normalized throughput is somehow reduced compared to the case of $N = 7$. This is true when the number of active users is small near at $p = 0.1$ where $\bar{\beta}$ reaches its maximum value, and the unwanted secondary MAI is greatly reduced through the HMP acknowledgment. In this situation, as we increase G for fixed $L = 2$, the above saturation behavior is less prominent because of relatively small secondary MAI.

In reality, the performance of CDMA system is only interference limited, and operated under such optimum condition which gives a maximum normalized throughput for given system parameters. In this case, there exists a saturation behavior in view of $\bar{\beta}$ as seen in Figs. 3, 4 and 6, 7 when we increase the parameters L and G. Using the RA/T code scheme, it is possible to minimize system complexity by choosing a proper L and G without losing performance quality in view of $\bar{\beta}$. We note that a proper value of L depends on mainly the effect of the primary MAI on collision event, while an optimum value of G is usually determined by the effect of the secondary MAI on detection of DMP's.

APPENDIX A
DERIVATION OF $P_1(s \mid h_{t-1}, K_H, E_t)$ IN (14)

Suppose $K_H \le E_t$, then anyone of K_H terminals entering into the state D_f is only when its HMP being sent is not received. In this case, we define $P_1(s \mid h_{t-1}, K_H)$ by the probability that s of the K_H terminals successfully send their HMP's. It is assumed that $w \ge s$ of the K_H terminals send their HMP's using different RA code channels, i.e., free of the primary MAI, while the remaining $K_H - w$ terminals cause collisions among them by sending their HMP's in the presence of the primary MAI. Consider the event A_w that s of the w terminals send their HMP's within t data bit errors in the presence of the secondary MAI, then we have

$$P_1(s \mid h_{t-1}, K_H) = \Pr\left[\bigcup_{w=s}^{K_H} A_w\right] = \sum_{w=s}^{K_H} \Pr[A_w]. \quad (31)$$

Note that $\{A_w : s \le w \le K_H\}$ forms a set of mutually independent events.

To derive the probability of occurrence of A_w, first assign different RA code channels to the w terminals in $\binom{K_H}{w}\binom{L}{w}w!$ ways. Next, consider an assignment of the remaining $L - w$ channels to cause collisions among the $K_H - w$ terminals, in which the number N_o of such assignments can be computed by using the principle of inclusion and exclusion [10] as follows. Suppose any r of the $K_H - w$ terminals are assigned different RA code channels, while the $h_{t-1} - w - r$ terminals the remaining $L - w - r$ channels at random. Then the number of such assignments is given by $\binom{K_H - w}{r}\binom{L-w}{r}r!\,(L - w - r)^{h_{t-1}-w-r}$. Using the above principle, we find that

$$N_o = \sum_{r=0}^{K_H - w} (-1)^r \binom{K_H - w}{r}\binom{L - w}{r} \cdot r!\,(L - w - r)^{h_{t-1}-w-r}. \quad (32)$$

Since any s of the w terminals successfully send their DMP's with probability $B(s; w, P_s^H)$, and the number of total assignments is $L^{h_{t-1}}$, the probability $\Pr[A_w]$ becomes

$$\Pr[A_w] = \left(\frac{1}{L}\right)^{h_{t-1}} \binom{K_H}{w}\binom{L}{w} w!\,B(s; w, P_s^H)\,N_o. \quad (33)$$

Now, if we combine (32) and (33) with (31), it reduces to (15).

Next, suppose $K_H > E_t$, then we may have some change into the state D_f when the matched-filters are not available. Using the above results, the probability $P_1(s \mid h_{t-1}, K_H, E_t)$ is easily evaluated as (14).

APPENDIX B
DERIVATION OF $p_V(v \mid I = i, J = j)$ IN (20)

Suppose $K_H \le E$. Then we define $P_2(s, c \mid h, K_H)$ by the probability that s of the K_H terminals and c of the i terminals send their HMP's successfully. Consider the event $A_{w,g}$ that $w \ge s$ of the K_H terminals send their HMP's free of the primary MAI and s of them send their HMP's within t data bit errors, while $g \ge c$ of the i terminals send free of the primary MAI and c of them send within t errors. Since $\{A_{w,g} : s \le w \le K_H, c \le g \le i\}$ forms a set of mutually independent events, then

$$P_2(s, c \mid h, K_H) = \sum_{w=s}^{K_H} \sum_{g=c}^{i} \Pr[A_{w,g}]. \quad (34)$$

For evaluation of the number of occurrence of $A_{w,g}$, first assign different RA code channels to the w and g terminals in $\binom{K_H}{w}\binom{L}{w}w!\binom{i}{g}\binom{L-w}{g}g!$ ways. Next, the remaining $L - w - g$ channels are assigned to cause collisions among the $h - w - g$ terminals, then the number N_o of such assignments is given by

$$N_o = \sum_{r=0}^{h-w-g} (-1)^r \binom{h - w - g}{r}\binom{L - w - g}{r} \cdot r!\,(L - w - g - r)^{h-w-g-r}. \quad (35)$$

Note that any s of the w terminals successfully send their HMP's with probability $B(s; w, P_s^H)$, while any c of the g terminals with probability $B(c; g, P_s^H)$. Since they are mutually independent, and the number of total assignments is L^h, we obtain

$$\Pr[A_{w,g}] = \left(\frac{1}{L}\right)^h \binom{K_H}{w}\binom{L}{w} w! \binom{i}{g}\binom{L - w}{g} \cdot g!\,B(s; w, P_s^H)\,B(c; g, P_s^H)\,N_o. \quad (36)$$

Now substituting (36) into (34) with (35) gives (22).

Suppose $K_H > E$, then we should account for the event that the DMP's may be blocked even though the preceding HMP's are successfully received. In this case, the probability in (34) is replaced by $P_2(s, c \mid h, K_H, E)$ which can be evaluated as (21). Based on the above results, we can derive $p_V(v \mid I = i, J = j)$ in (20) by summing up all pairs (s, c) of $P_2(s, c \mid h, K_H, E)$ satisfying the relation $v = s + c$ for $0 \le s \le K_H$ and $0 \le c \le i$.

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